Curſus Mathematicus:

OR, A

Compleat Courſe

OF THE

MATHEMATICKS.

Containing all the Uſeful PARTS thereof.

VOL. III.

Comprehends

GEOMETRY and FORTIFICATION.

Written in *French* by Monſieur *OZANAM*, Profeſſor of the MATHEMATICKS at *Paris*.

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PREFACE.

IN the *First* Volume of this *Cursus*, and in the beginning of the *Second*, I have treated of Quantity consider'd abstractly; and towards the end thereof, I have handled it, as it may be aplied to Matter, and as it discovers to us the Properties and Distances of the most remote Bodies. In this *Third* Volume, I consider Quantity as a Property inseparable from Matter, without which it could have neither Figure nor Situation. Trigonometry, which was the Subject of the last Part of the preceding Volume, is likewise in some Measure the Subject of Geometry, as being a Science treating of the Triangle, which is the first and simplest of all Rectilineal Figures. The Geometry which I am now going to handle, takes in all other Figures, examines their Properties, divides them, compares them, and observes their several Relations to each other.

Since many Gentlemen study Geometry, with no other intent than for the better Understanding of the Art of War; I have, for that Reason, to this Geometry annexed Fortification, which has always been look'd upon as most honourable, as being the Business of a Hero, and is now in great Reputation. There are not many curious Things to be said concerning the Origin of Geometry, but there are abundance upon that of Fortification, and its Progress; which have been sufficiently mention'd by almost all those who treated of this Art: There-

fore to avoid being tiresome by too long a Preface, I shall content my self with saying something of Geometry.

The knowledge of the Distances of Things has been always found so necessary, that Men could never have liv'd without considering them, and marking them in their Memories, or by some exterior Sign: So that tho' 'tis pretended the *Egyptians* were the first Inventers of Geometry, because they were the first that were under any Necessity of preserving the Limits and Extents of their Lands and Possessions, either by Figures traced and delineated, or by Accounts and Memorials contain'd in Writing, the Inundations of their Lands by the overflowing of the *Nile*, effacing all other Marks; yet as there was the same Necessity before the inundated part of *Egypt* was inhabited, since there was no making any Division, without taking the Measures of the whole, and of each Part; nor any Building of Houses and Towns, without being appriz'd of their Figures, and regulating them by Measures; 'tis to be presum'd, that the Science of Measuring is as Old as the World. Man took from himself the Measures of all other Things; he did not borrow from others the Names of Fathoms, Cubits, Feet, Inches, he presently applied his Thumb, his Foot, his Hand, and his Arms to the Things he wou'd measure; and so he took from his own Body the Dimensions of all other Things, having no occasion for Books or Masters, and without burthening the Memory with Names or Figures. Thus the same necessity which forced Men to divide into Shares, make Partitions, and to use Measures which they borrow'd from themselves, obliged them to seek out a way of reducing Things proposed to an Equality, in order to come at their Shares; and this way of reducing Things to an Equality, could not be known without giving an

Account

Account of the divided Parcels, or the Meaſures which were neceſſary to be applied and reckon'd, in order to proceed in the Diviſions with Exactneſs: And this gave a Beginning to Geometry.

Vitruvius, to whom we owe all that is Beautiful in Architecture, ſays, that the *Grecians* by an Exceſs of Delicacy, in point of Building, did not only take the Parts of Humane Body for the common Standard of their Plans and Draughts, but even the Beauty of Human Bodies for the Model of that of Architecture: And by this nice Invention, they form'd the *Corinthian* Order, from the Beautiful Proportion of a well Shap'd Woman; the *Ionic* Order from the Shape of a handſom well-proportion'd Man, and the *Doric* Order from the Shape of a labouring Man, Strong and Robuſt. Upon the whole, Man was ſo much his own Idol, as to imagine himſelf the moſt Perfect of all created Beings, and ſo propos'd his own Body as the Model and Meaſure of all others.

Thus much for what relates to the Origin of Geometry; as for its Utility, we may ſay, That if the *Greeks* have been reckon'd more refin'd than other Nations, it is purely becauſe that by Means of this Science, they found the ſhorteſt, and moſt certain Road of becoming Reaſonable and Ingenious. *Socrates*, who was the ableſt of their Maſters, prov'd that all Men had Senſe or Ingenuity, by examining them in a certain Manner, and by turning their Wits more one way than another. We likewiſe know that he; and the other *Grecians* begun with Geometry; ſo that 'tis more than probable, 'twas by this Science their Minds were ſo far elevated above others. The *Grecians* at preſent are as unpoliſh'd as other People, and that undoubtedly from their having neglected the ſame Method of teaching their Children. So that now if here Perſons wou'd apply

 them-

themſelves more to Geometry, and begin earlier, we might hope their Faculties wou'd arrive to a higher Pitch, and improve to a much greater Perfection.

Again, tho' we ſhou'd ſuppoſe Geometry to be without this Advantage, and to leave the Minds of Men ſtill groveling in the Duſt; it would not, however, ceaſe to be uſeful in all ſorts of Profeſſions; it is even then more neceſſary for Men, who content themſelves with a moderate Condition, than for thoſe who have an Ambition to raiſe their Fortunes. There is no State of Life, that does not require the Uſe of all ſorts of Meaſures, it is the Soul of all Trades: All the Arts which tend to the ſupplying of Mankind with Food and Cloathing, ſubſiſt only by Meaſures of Lengths, Breadths, Circles and Squares. Thoſe whoſe Lot it is to drive a Trade in Proviſions, muſt know the Meaſures of dry and liquid Things, and their Reductions. Husbandmen underſtand Surveying, either thro' Neceſſity or Practice; Brewers underſtand Gauging; Maſons and Carpenters know how to lay out, to meaſure Angles, and Squares, and all the Dimenſions of a piece of Building. They who work in Wood, Iron or Stone, cannot be without the knowledge of thoſe Meaſures. But above all, thoſe who apply themſelves to the Liberal Arts, ought never to be without Rule and Compaſs, unleſs they will abandon their Deſigns, and be reckon'd as ſilly as thoſe who compoſe Diſcourſes without Reaſoning. Menſuration, if we may ſay ſo, is the Reaſon of all Arts, and of all Performances belonging to them; take away Geometry from any Arts, either Liberal or Mechanic, and you take away its Soul and Reaſon; that is, in plain Terms, you deſtroy it.

However allowable it may be in private Men to neglect theſe Sciences, Perſons of Quality, and

thoſe

those who have the Conduct of Societies and States, may by no Means be allow'd to do so. If Men were only attack'd by Hunger, Thirst, Cold, Heat, Wind, Rain, or Wild Beasts, they might live like Cynics, and chuse a Life like that of Beasts; eating the Fruits of the Earth, drinking meer Water, warming themselves in the Sun, and seek only Caves and Dens to defend them from the Inconveniences of the Air, or against the Insults of the Beasts. But when they make War upon each other, to preserve their Goods, their Liberty, and Lives; they must of necessity quit that Savage and Idle sort of Living; they must make Weapons, fortify themselves in Company with other Men, and associate together to avoid common Danger; they must oppose Force to Force, and Politicks to Politicks. This occasion'd the Building and Fortifying of Cities, in order to resist Strangers who might be dispos'd to make attempts on their Lives and Liberties.

Fortification therefore is the chief Art wherein Geometry is used, and is at present in such a degree of Perfection, that there is no likelyhood it will receive any essential Addition; because it seems impossible to find any Thing that is not already invented for the attacking of Places. Earth is made use of for Trenches, Banks, and Retrenchments; Water for Sluces and Inundations; and Fire is applied various ways below, by the contrivances of Mines; above, by Granadoes, Bombs and Carkasses, and directly by Canons, and other pieces of Ordnance. Methods of Defence have been invented against all these; and nothing is wanting but the perfecting them; and unless Mankind finds out some other Elements, there is not much probability that they can invent New Methods, either Offensive or Defensive.

I well enough know, 'tis not long ſince any very regular Fortifications have been made; and that, conſequently, Geometry was not ſo very neceſſary to Engineers of old, as 'tis to thoſe of the preſent time, who apply it with great Induſtry and Succeſs, to the improvement of Military Architecture. This ſhews there is nothing to be done without Geometry, eſpecially, at preſent, when War is now grown a neceſſary Art, which every Body is obliged to learn, becauſe of the unhappy Quarrels which are ſpread all over *Europe*.

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A TREATISE OF GEOMETRY.

GEOMETRY is a part of pure Mathematicks, which conſiders Magnitude, not ſo much in reſpect to it ſelf, as the relation it may have to another Magnitude of the ſame kind, in abſtracting it from all Matter or ſenſible Subject. It is divided into *Speculative* and *Practical*.

Speculative Geometry ſimply conſiders the Properties of Continu'd Quantity, that is to ſay, of Magnitude which has Extenſion, and whereof the Parts are united together, in reſpect to the Place that it occupies, and then it is called Continued permanent Quantity, as *Lines*, *Planes* and *Solids*: Or in reſpect to the Time wherein it ſubſiſts, and then it is called Continu'd ſucceſſive Quantity, as *Motion*.

Practical Geometry teaches to Meaſure and Divide continu'd Quantity according to its Extenſion, which may have only one Dimenſion, called *Length*, as *Line*: or elſe two Dimenſions, to wit, Length and Breadth, as *Plane*: or elſe again three Dimenſions, to wit, Length, Breadth and Depth, or Height, or Thickneſs, as *Body*.

It is of *Practical Geometry* that we intend to treat particularly in this place, and ſhall divide it into four parts; whereof, the Firſt, ſhall treat of *Geodeſy*, or the Diviſon of Lands; the Second, of *Longimetry*, or the Menſuration of acceſſible and inacceſſible Lines upon the Earth: The third of *Planimetry*, which is the Menſuration of Planes:

and the Fourth *Stereometry*, which is the Menſuration of Solids or Bodies.

DEFINITIONS.

I.

Plate I. Fig. 1.

A *Body* or *Solid*, is a Quantity extended in Length, Breadth and Depth, as ABCDEFG, which has the Figure of a Dye, the length whereof is AB, or FG, or DE; the Breadth, which is ſuppoſed to be leſs than the Length, is BC, or DG, or EF; and the Height, or Depth, or Thickneſs, is AF, or BG, or CD. It has alſo an upper part as DEFG, and an under, as ABC, which is term'd the Baſe, in regard of its height BG, which muſt be perpendicular to it for to be its true Height. It has beſides a fore-part, as ABGF, and a hinder Part, as CDE. Laſtly, it has Sides, as BCDG, AFE. But in terms of Geometry, they call the Lines which bound a Figure, the Sides of it, as the Length, Breadth and Depth: And that which encloſes a Space, and is bounded on all ſides, is generally called a *Figure*, whence it is eaſie to conclude that an Angle is not a Figure.

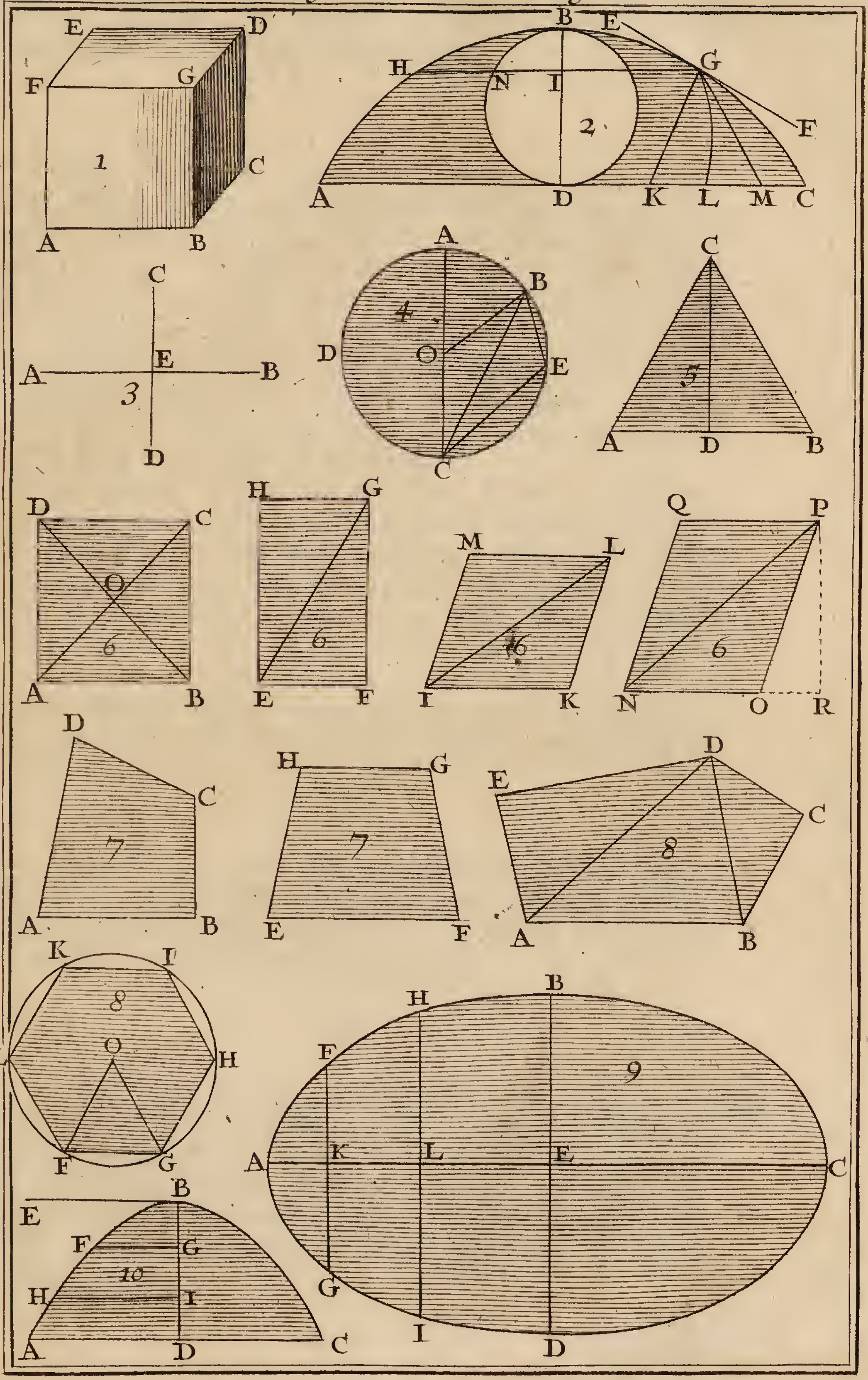
II.

A *Surface*, or *Superficies*, is the Extremity of a Body, which has two Dimenſions, to wit, Length and Breadth, without conſidering any Thickneſs, or Depth, as DEFG, the length whereof is DE, and the Breadth EF, which is always conceiv'd leſs than the Length. It is evident that a Body is compos'd of an infinite Number of Superficies, and that many Superficies being plac'd one upon the other, cannot produce but one Superficies; inſomuch that if one Superficies produce a Body, you muſt imagine it to move from one place to another.

III.

A *Line* is the Extremity of a Body, or Superficies, which has only one Dimenſion, to wit, Length, without conſidering either Breadth or Depth, as EF, or FG. It is evident that a Body and a Surface are compoſed of an infinite number of Lines, and that many Lines being plac'd one againſt the other, cannot produce but one Line; ſo that for one Line to generate a Surface, you muſt move it from one place to another: And as this Motion

Geometry Plate 1. Page 2.



Motion cannot be made but by a Line, therefore two Lines being multiply'd together produce a Surface, whereof those Lines represent the two Dimensions: and as the Surface likewise cannot be mov'd but by a Line, therefore the Product also of three Lines is a Solid, whereof those Lines are the three Dimensions. *Plate I. Fig.* 1.

IV.

A *Point* is the Extremity of a Body, a Surface, or a Line, which is conceiv'd as indivisible, or without Dimension, that is to say, to which there is not attributed either Length, Breadth or Depth, having consequently no Parts: as A or B. It is evident that a Line, as well as a Surface, and Solid, is compos'd of an infinite number of Points, and that many Points being plac'd one against the other, cannot produce but one Point. Therefore that one Point may generate a Line, it must be mov'd from one place to another.

Points cannot subsist separate from Lines, or Surfaces, or Bodies; nor Lines from Surfaces, or Solids; nor Surfaces from Solids: Yet Mathematicians do conceive them separate, when they seek the Measures of Lines, or of Surfaces, by considering a Line as in such a manner distinguished from a Surface; as when they measure the Length of a Road, without considering its breadth: and a Surface as actually separate from a Body, as when they measure the Superficies of a Wall without considering its Solidity.

V.

A *Right Line* is that, whereof all the Points are plac'd equally, as AB. When one says simply a Line, it is to be understood a Right Line. It is evident that there are no Right Lines of different Species.

VI.

A *Curved Line* is that, whereof all the Points are not equally placed, as ABC. It is evident that there are an infinite number of Curved Lines of different Species, and of different Kinds: And as it would be too tedious, and even useless to mention them all, we shall only speak in the following Treatise of those which may be proper to our Subject. *Fig.* 2.

VII.

Plate I. Fig. 2. The *Tangent* of a Curve Line, is a Line which meets the Curve in one Point only, without cutting it, that is to ſay, without entring into it, as EF, which meets the Curve ABC in the point G only, without entring into it.

VIII.

Parallel Lines are thoſe which being extended at pleaſure, are always equally diſtant from each other: as AC, GH.

IX.

Fig. 3. *Perpendicular Lines* are thoſe that cut one another in ſuch manner, that the one in reſpect to the other, doth not incline to one ſide more than the other. Thus it is ſaid that the Line AB is Perpendicular to the Line CD, and that reciprocally the Line CD is Perpendicular to the Line AB; becauſe that the Line AB in reſpect to the Line CD does not incline more towards C than towards D; and that in like manner the Line CD in reſpect to the line AB, does not incline more towards A than towards B.

X.

Fig. 2. The *Diameter* of a Curve Line of the firſt kind, is a ſtrait Line drawn within that Curve, which divides into two equal Parts all the right parallel Lines drawn within the ſame Curv'd Line. Thus it is ſaid that the line BD is the Diameter of the Curve ABC, becauſe it divides into two equal Parts at the Points D and I, the two Parallels AC, GH, which are called *Ordinates* to the Diameter BD, which is called the *Axis*, when it is perpendicular to its Ordinates.

XI.

The *Vertex* of a Curve is the Point where that Curve is cut by its Diameter, as B. It is evident that as a Curv'd Line may have an infinite number of different Diameters, it may alſo have an infinite number of different Vertexes, becauſe one may draw in it many parallel right Lines in divers manners, and make as many different Diameters paſs through their middle Points.

XII. The

XII.

The *Perpendicular of a Curv'd Line* is that which passing through the Point of Contact, is Perpendicular to the Tangent: as GK, which I suppose Perpendicular to the Tangent, EF, which touches the Curve ABC at the Point G. Plate I. Fig. 2.

XIII.

An *Angle* is an indefinite Space formed by the Inclination of two Lines that cut one another, which is call'd a *Rectilinear Angle*, when its two Lines are right, as IGK: a *Mixtilinear Angle*, when one of its two sides is right, and the other curv'd, as IGB: and a *Curvilinear Angle*, when the two Lines which form it are Curv'd, as LGC, which is term'd a *Spherical Angle*, when its two Lines are upon a Spherical Superficies; and every Angle, whose two lines are upon a Plane, is call'd a *Plane Angle*.

A Plane Mixtilinear and Curvilinear Angle are reduc'd to a Rectilinear, by two right Lines, that touch the Curv'd Lines of the Angle at the Point of Intersection, which is call'd the Angular Point, or Vertex. Thus the Mixtilinear Angle KGL is reduc'd to a Rectilinear KGM, by supposing that the right Line GM touches the curve GL at the Point G: and the Curvilinear Angle LGC is reduc'd to a Rectilinear MGF, by supposing that the two curv'd lines GL, GC, are touch'd at the Point G, by the right Lines GM, GF.

XIV.

A *Right Angle* is that, whose two Lines are Perpendicular to each other: as KGE, or KGF, supposing the Line KG is Perpendicular to the Line EF. Whence it is easie to conclude by the Definition of Perpendicular Lines, that all Right Angles are equal to one another, therefore when two Lines are Perpendicular to each other, they are said to cut one another at right Angles, as AB, CD, which intersecting at the Point E, make four right Angles, which are equal to one another. Fig. 2. Fig. 3.

XV.

Plate I. Fig. 2. An *Oblique Angle* is that which is made by the meeting of two oblique Lines, that is to ſay, of two Lines which are not Perpendicular, or which cut one another at unequal Angles. Or elſe it is that which is greater or leſs than a right Angle: and then it is called an *Acute Angle*, when it is leſs than a Right, as IGE, which is leſs than the right Angle KGE: and an *Obtuſe Angle*, when it is greater than a Right, as IGF, which is greater than the Right Angle KGF.

XVI.

The *Baſe of a Curve Line* is the laſt of the Ordinates which terminates its Diameter and Curve. Thus it is ſaid that the Baſe of the Curve ABC, is the right Line AC, which terminates the Diameter BD, which is here Perpendicular to the Baſe AC, and then this Perpendicular or Axis BD, is call'd the Height of the Curve ABC, in reſpect to the Baſe AC.

XVII.

A *Plain Surface*, which is ſimply call'd a Plane, is that whoſe Lines are all right, in whatſoever manner they are drawn, although one may draw upon it curv'd Lines, Fig. 1. yet all thoſe curv'd Lines will be equally plac'd, that is to ſay, that one will not be lower nor higher than another. Such is the Plane ABGF, or BCDG, or DEFG.

XVIII.

A *Curv'd Surface*, is that whoſe Lines are all Curv'd, it being impoſſible that there ſhould be in it any right Line; ſome being higher, or lower, than others: as the Surface of a Sphere, which is call'd a *Spherical Superficies*; which being conſider'd outwardly, is term'd a *Convex Surface*, and conſider'd inwardly is call'd a *Concave Surface*.

XIX.

A *Circle* is a plain Surface bounded by one curv'd Line only, which is called the *Circumference of the Circle*, Fig. 4. within which there is a Point call'd the *Centre of the Circle*, from whence all right Lines drawn to the Circumference,

call'd

call'd the *Radij of the Circle*, are equal to one another. *Plate I. Fig. 4.* Thus it is evident that ABCD is the Circumference of a Circle, whose Centre is O, and the Radij, or Semi-Diameters are the equal right Lines OA, OB, OC, *&c.*

XX.

A *Diameter* of a Circle is a right Line drawn at pleasure through the Centre of the Circle, and terminated at both ends by the Circumference of the same Circle: as AC, which divides the Circle and its Circumference into two equal parts, each of which is indifferently call'd a *Semi-circle*, the half whereof is consequently term'd the *Quadrant of a Circle.*

XXI.

The *Arc of a Circle* is a part of the Circumference, either less, or greater than a Semi-circle, or half of the Circumference: as the Arc AB, which is less than the Semi-circle ABC; or else the Arc ADCB, which is greater than the Semi-circle ADC.

Mathematicians have divided the Circumference of a Circle into 360 equal Parts, or small equal Arcs, which they have call'd *Degrees*: and each Degree into 60 other less equal Parts, call'd *Minutes*; Every one of which has been divided into 60 equal Parts call'd *Seconds*, and so on; that having been done chiefly to determine the Quantity of a Rectilinear, or Spherical Angle: For,

XXII.

The *Measure of an Angle* is an Arc of a Circle describ'd at pleasure from its Angular Point, and terminated by its two Lines. Thus it is said that the Measure of the Rectilinear Angle AOB is the Arc of a Circle AB; for as many Degrees and Minutes as this Arc AB will contain, of so many Degrees and Minutes will be the Angle AOB, which it measures.

XXIII.

The *Sector of a Circle* is the Part of a Circle terminated by two Radij, and by a part of the Circumference, less or greater than half of the same Circumference, which is called the Base of the Sector: As the Sector

Plate I. *Fig.* 4. BOCE, whose Base is the Arc BEC: Or the Sector BOCDA, whose Base is the Arc BADC.

XXIV.

The *Segment of a Circle* is a part of a Circle terminated by an Arc of a Circle, less or greater than half of the Circumference of the Circle, whereof it is a part, and by a right Line, which joins the Extremities of that Arc, and which is call'd a *Chord*: As the Segment BCE, which is less than the Semi-circle ACE; or the Segment BCDA, which is greater than the Semi-circle ACD.

XXV.

Fig. 5. A *Rectilinear Figure* is a Plane Surface terminated by Right Lines called Sides, which is named a *Triangle*, when it is bounded by three right Lines: a *Quadrilateral Figure*, or *Quadrangle*, when it is bounded by four right Lines: and lastly a *Polygone*, when it is terminated by more than four right lines.

XXVI.

A *Triangle* is therefore a Plane terminated by three right Lines, as ABC, which is called an *Equilateral Triangle*, when its three Sides are equal; an *Isosceles*, when it has only two Sides equal: and a *Scalenum*, when all its Sides are unequal. But when it has one Right Angle, it is called a *Right Angled Triangle*: and when all its Angles are Acute it is called an *Oxigone*: and lastly, when it has one Obtuse Angle, it is called an *Amblygone*.

XXVII.

The *Base of a Triangle* is the Side whereon is drawn a Perpendicular from its opposite Angle, which is call'd the Height of the Triangle, in respect to its Base. Thus it is evident, that the Base of the Triangle ABC is the Side AB, in regard to its Height or Perpendicular CD, which divides the Base AB into two parts AD, BD, call'd the *Segments of the Base*, even when the Perpendicular CD falls without it, which will happen when one of the two Angles at the Base AB, is obtuse. The greatest Side of a Right angled Triangle, to wit, that which is opposite to the Right Angle, is likewise term'd the Base: But that Side is more commonly call'd the *Hypotenuse*, and we

shall

ſhall always make uſe of that Term in what follows, as we have already done in the two foregoing Treatiſes.

XXVIII.

A *Quadrilateral Figure* is alſo a Plane terminated by four Right Lines, which is call'd a *Tetragon*, and more commonly a *Square*, when it has all its Sides equal, and its four Angles right, as ABCD: An *Oblong*, when it has all its Angles right, but not all its Sides equal, as EFGH: A *Rhombus*, and *Lozenge* in Terms of Blaſoning, when it has all its Sides equal, and its Angles Oblique, as IKLM: A *Rhomboides*, when it has its Angles oblique, and only its two oppoſite Sides equal, as NOPQ: And a *Trapezium*, when two oppoſite Sides are not equal, as ABCD, which we ſhall call a *Trapezoides*, when it has two oppoſite Sides parallel to one another, as EFGH, whoſe two oppoſite Sides, EF, GH, are parallel. *Plate I. Fig. 6. Fig. 7.*

XXIX.

A *Parellelogram* is a Figure of four ſides, whereof the two oppoſite are parallel to one another: As the Square, the Oblong, the Rhombus, and the Rhomboides. When the Angles of a Parallelogram are right, it is call'd a *Right angled Parallelogram*, or ſimply a *Rectangle*: As the Square ABCD, and the Oblong EFGH, where the four Angles are right. *Fig. 6.*

XXX.

The *Baſe of a Parallelogram* is the ſide upon which is drawn from one of the two oppoſite Angles a Perpendicular, which is call'd the Height of the Parallelogram, with reſpect to its Baſe. Thus it is evident that the Baſe of the Parallelogram NOPQ, is the Side NO, in reſpect to its Height or Perpendicular PR, which in this place falls without it, and it would have fallen within, if it had been drawn from the Angle Q.

XXXI.

A *Polygon* is a Figure of more than four Sides, as ABCDE; which is called a *Pentagon*, when it has five Sides; a *Hexagon*, when it has ſix; a *Heptagon*, when it has ſeven; an *Octagon*, when it has eight; an *Enneagon*, when it has nine; a *Decagon*, when it has ten; an *Endecagon*, *Fig. 8.*

cagon,

Plate I. Fig. 8. *cagon*, when it has eleven; and a *Dodecagon*, when it has twelve Sides.

XXXII.

When all the Angles of a Polygon are equal, it is called a *Regular Polygon*; as the Hexagon FGHIKL, whoſe Centre O is the ſame as the Centre of a Circle circumſcrib'd; and when all the Angles of a Polygon are not equal to one another, it is call'd an *Irregular Polygon*; as the Pentagon ABCDE.

XXXIII.

They call the *Angle of the Centre* that which is form'd at the Centre of a regular Polygon by two Right Lines drawn from the Centre of the Polygon through the two Extremities of one of its Sides, as FOG: And the Angle of the Polygon, that which is form'd by two Sides of a regular Polygon, as FGH.

XXXIV.

Fig. 6. A *Diagonal* is a Right Line drawn on the Plane of a Rectilinear Figure from one Angle to another. Thus it is evident that the Right Line AC, or BD, is the Diagonal of the Square ABCD; that the Right Line EG is the Diagonal of the Oblong EFGH; that the Right Line IL is the Diagonal of the Rhombus IKLM, and that the Right Line NP is the Diagonal of the Rhomboides NOPQ.

The Diagonal of a Parallelogram is moſt commonly call'd the *Diameter of the Parallelogram:* And the Point where the two Diameters of a Square Interſect, is call'd the Centre of the Square, as O. It is evident that a Polygon is divided by Diagonals drawn from the ſame Angle into as many Triangles as there are Sides, Fig. 8. except two. Thus it is plain, that the Pentagon ABCDE is divided into three Triangles by the Diagonals DA, DB.

XXXV.

A *Mathematical Oval*, commonly call'd an *Ellipſis*, is a Plane terminated only by one Curv'd Line, term'd the Fig. 9. *Circumference of the Ellipſis*, as ABCD, within which drawing ſo many Ordinates at pleaſure to any one Diameter AC, as FG, HI, the Squares of thoſe Ordinates FG, HI,

HI, or only of their halves FK, HL, which are commonly taken for the Ordinates in all Curve Lines *of the first Order*, are proportional to the Rectangles under the correspondent Parts of the same Diameter AC; that is to say, that the Square of the Ordinate FK is to the Rectangle under the correspondent Parts AK, KC, as the Square of the Ordinate HL, is to the Rectangle under the correspondent Parts AL, CL. *Plate I. Fig.* 9.

We have said in *Def.* 2. that a Curve Line may have an infinite number of different Diameters, and we will say here, that when those Diameters are not parallel to one another, the Point of Intersection is call'd the *Centre of the Curve Line*: So that the Centre of the Ellipsis will be the Point E, where the two Diameters AC, BD, divide each other into two equal Parts: And when they intersect one another at Right Angles, as in this place, the greater AC, which represents the Length of the Ellipsis, is call'd the *Great Axis*, and the lesser BD, which represents the Breadth of the Ellipsis, is call'd the *small Axis*.

XXXVI.

A *Parabola* is an indefinite Plane terminated by a Curve Line, call'd a *Parabolick Line*, and which is commonly taken for the Parabola it self, as ABC, within which, to any Diameter BD, drawing so many Ordinates at pleasure, as FG, HI, the Squares of those Ordinates FG, HI, are to one another as the correspondent Parts of the Diameter BG, BI; or else, which comes to the same thing, the Square of each Ordinate is equal to a Rectangle under the correspondent part of the Diameter, and a certain Line BE of a determin'd Magnitude, call'd the *Parameter of the Parabola*, that is to say, that the Square FG is equal to the Rectangle under the correspondent Part BG, and the Parameter BE, and likewise the Square HI is equal to the Rectangle under the correspondent Part BI, and the same Parameter BE. *Fig.* 10.

This Parabola is call'd the *Quadratic Parabola*, to distinguish it from the *Cubic Parabola*, where the Cube of an Ordinate, as FG, is equal to a Solid under the correspondent Part BG, and the Square of the Parameter BE: And from the *Biquadratic Parabola*, where the Biquadrate of an Ordinate, as FG, is equal to the Product under the correspondent Part BG, and the Cube of the Parameter BE: And so of other Parabolas *in infinitum*.

XXXVII.

Plate II. Fig. 14. An *Hyperbola* is an indefinite Plane Surface, terminated by a Curved Line, call'd the *Hyperbolic* Line, and which is commonly taken for an Hyperbola it self, as ABC, within which drawing to any Diameter BD, as many Ordinates as you please, as FG, HI, and producing the same Diameter outward towards E, to a certain distance BE, which is call'd a *Determin'd Diameter*, and by the Ancients, a *Transverse Diameter*; the Square of the Ordinate FG is to the correspondent Rectangle under the whole Line EG, and the Part BG, as the Square of the Ordinate HI, is to the correspondent Rectangle under the whole Line EI, and the Part BI.

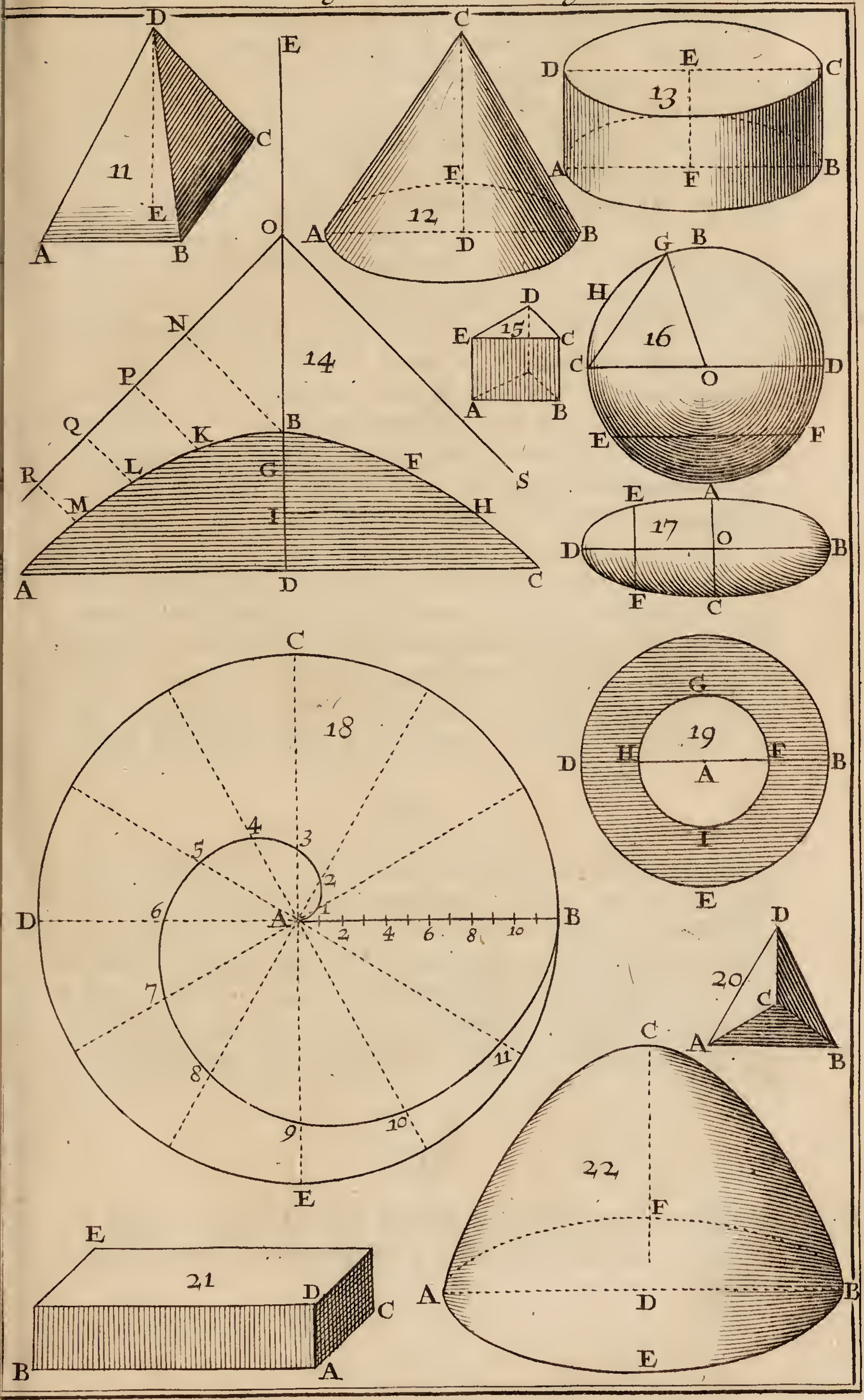
The Point O, in the middle of the Determin'd Diameter DE, is call'd the *Centre of the Hyperbola*, because it is to this Point O, that all the infinite Diameters of the Hyperbola run, when they are produced outwardly, every one of which has its determin'd Diameter; among those that which belongs to the Axis of the Hyperbola, may be call'd the *determinate Axis*, as BE, whereof the Extremity B is the Vertex of the Hyperbola ABCI, and the other Extremity E is the Vertex of another Hyperbola like the foregoing ABC, which having the same Line BE for its determinate Axis, is call'd the *Opposite Hyperbola*.

XXXVIII.

The *Asymptotes of an Hyperbola* are two indefinite right Lines drawn from the Centre of the Hyperbola, towards which they always approach without ever meeting it, as OR, OS, of which the Property is such, that if to one of the two Asymptotes, as OS, are drawn the Parallels BN, KP, LQ, MR, terminated by the other Asymptot OR, and the Hyperbola ABC, all the Rectangles ONB, OPK, OQL, ORM, are equal one to another.

XXXIX.

Plate I. Fig. 2. A *Cycloid* is a Curve Line made by the Motion of a Point of the Circumference of a Circle, which, being perpendicular to a Plane, turns along upon a Right Line of that Plane. As if along the right Line AC describ'd upon a Plane, one does imagine a Circle perpendicular to that Plane, to turn, as BND, so that the Point B of its Circumference being in A, comes to C, in which case the right Line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Line AC will be equal to the Circumference of that Circle which is call'd the *Generating Circle*; this Point B will describe by its Motion the Curve ABC, call'd a *Cycloid*, and also a *Roulette*. *Plate I. Fig. 2.*

XL.

A *Spiral*, or *Helice*, is a curve Line made by the Motion of a Point, which is mov'd equally upon a right Line, whilst that right Line is also mov'd equally upon the Circumference of a Circle round its Centre, whence the Spiral has its beginning, insomuch that when the Point shall have run through all its Line, beginning from the Centre of the Circle, that Line will have also compleated the whole Circumference of its Circle,

As if the Line AB indefinite towards B, is mov'd about the Centre A by an uniform Motion, by compleating in equal times equal parts of the Circumference BCDE, and that a Point is also mov'd from A towards B, by an uniform Motion, so that from the Radius AB in runs through Parts like to those which that Radius AB compleats of its Circumference, in which case that Point will arrive at B, when the Radius AB shall have compleated its whole Circumference; that same Point will describe by its compound Motion the Spiral A, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, B, which is call'd the *First Spiral*, to distinguish it from the *Second Spiral*, which you will have by imagining the prolong'd Line AB to move about the same Centre A through all the Points of the Circumference BCDE, whilst the Point continues to move in the same time from B, by an uniform Motion, and like to that of the Line AB, &c. *Plate II. Fig. 2.*

XLI.

The *Crown* is a Plane terminated by two Circumferences of Circles parallel to one another, that is to say, by the Circumferences of two Circles described upon a Plane from the same Centre, which for that reason are call'd *Concentric Circles*. As if from the Centre A, you describe the two Circumferences of a Circle BCDE, FGHI, those two Circumferences will enclose a Space which is call'd a *Crown*. *Fig. 19.*

XLII.

A *Zone* is a part of the surface of a Sphere, terminated by the Circumferences of two Circles of the same Sphere, which *Fig. 16.*

Plate II. Fig. 16. which are parallel to one another, that is to ſay, which have the ſame two Points for their Poles, which are two Points of the Surface of the Sphere, diametrically oppoſite, and equally diſtant from the Circumferences of the Circles, whereof they are the Poles. As if from the two Poles A, B, you deſcribe upon the Surface of the Sphere ADBC, the two parallel Circles CD, EF, they will encloſe a Space call'd a *Zone*.

XLIII.

A *Sphere* is a ſolid terminated only by one curved Surface, call'd a Sphærical Superficies, as ADBC, within which there is a Point as O, call'd the *Centre of the Sphere*, from whence all right Lines drawn to the Surface, as OC, OG, OD, call'd Radij, or Semidiameters of the Sphere, are equal to one another. The double of one of thoſe Radij, as CD, is term'd a Diameter of the Sphere, which is alſo call'd a *Globe*.

XLIV.

A *Segment of the Sphere*, call'd alſo a *Section* and *Portion of the Sphere*, is one of two unequal Parts of a Sphere cut in a Plane which does not paſs through its Centre, otherwiſe inſtead of a Portion of the Sphere, you will have the half of a Sphere, which is term'd a *Hemiſphere*.

As if you cut the Sphere ADBC, by the Plane CG, which does not paſs through its Centre O, you will have the Segment of the Sphere CGH, which is leſs than the Hemiſphere CDB: And the Segment of the Sphere CGDA, which is greater than the Hemiſphere CDA. Becauſe the Section of a Sphere and of a Plane is a Circle, as we have demonſtrated in our Spherical Trigonometry *L.* 3. *cap.* 1. *Theor.* 1. it is eaſy to judge that the Baſe of an Hemiſphere is a great Circle of the Sphere, to wit, a Circle whoſe Diameter is equal to that of the Sphere; and that the Baſe of a Section of the Sphere is a ſmall Circle of the Sphere, to wit, a Circle whoſe Diameter, as CG, is leſs than the Diameter CD of the ſame Sphere. By a *Circle of the Sphere* is underſtoood, that whoſe Circumference is found in the Surface of the ſame Sphere, that is the Section of the cutting Plane and of the ſurface of the Sphere, that Section being neceſſarily the Circumference of a Circle, whoſe Plane is the Section of the cutting Plane and of the Sphere.

XLV.

The *Angle of a Segment of the Sphere* is that which is form'd at the Centre of the Sphere by two Radij drawn to the Extremities of one of the Diameters of the Base of the Segment of the Sphere less than a Hemisphere. Thus it is said that the Angle of the Segment of the Sphere CGH, which is less than the Hemisphere CDB, is COG. *Plate II. Fig.* 16.

We will also call the *Angle of a Segment of a Circle*, that which is made at the Centre of the Circle by two Radij drawn to the Extremities of the Arc of a Segment less than a Semi-circle. Thus it is plain that the Angle of the Segment of the Circle CBE, which is less than the Semi-circle ACEB, is COB, which is likewise call'd the *Angle of the Sector of a Circle*. *Plate I. Fig.* 4.

But that is call'd *an Angle in the Segment of a Circle*, which has its Angular Point in the Arc of that Segment, and whose two Lines pass through the Extremities of the same Arc, or of the Chord of that Arc, term'd the Base of the Segment. Thus it is evident that the Angle CEB is in the Segment of the Circle BCE, whose Base is the Chord BC of the Arc BEC. That Angle BEC is also term'd the *Angle at the Circumference*, because the angular point E touches the Circumference of the Circle.

XLVI.

The *Sector of a Sphere* is a Solid terminated in a Point at the Centre of the Sphere, and having for its Base the Surface of a Segment of the Sphere: as COGH. It is evident that a Sector of the Sphere is necessarily less than a Hemisphære, as the foregoing COGH, or else greater than a Hemisphere, as COGDAC. *Plate II. Fig.* 16.

XLVII.

The *Angle of a Sector of the Sphere* is the same as that which belongs to the Segment of the Sphere which it comprehends. Thus it is plain that the Angle of the Sector of the Sphere COGH, which is less than the Hemisphere CDB, is the Angle COG of its Segment CGH.

XLVIII.

Plate II. Fig. 17. A *Spheroid* is a Solid generated by the entire Circumvolution of a Demi Ellipsis, about one of its two Axes, which in this case is called the *Axis of the Spheroid*; and when that Axis is equal to the greater Axis of the Ellipsis, that Solid is call'd an *Oblong Spheroid*, as ABCD; whose Axis is BD: And an *Oblate Spheroid*, when that Axis is equal to the lesser Axis of the Ellipsis, that is to say, when the Circumvolution is made about the lesser Axis of the Ellipsis.

The Point O, in the middle of the Axis BD, is call'd the *Centre of the Spheroid*, and the right Line AC, which cuts at right Angles at the same Centre O, the Axis DB, is called the *Diameter of the Spheroid*, to distinguish it from the Axis. Lastly, they call those *Similar Spheroids*, whose Axes are proportional to their Diameters: And a Segment of a Spheroid, one of the two unequal Parts of a Spheroid, which is cut off by a Plane which does not pass thro' its Centre, as EFD, or EFB.

XLIX.

Fig. 22. A *Paraboloid*, term'd also a *Parabolic Conoid*, is a Solid, which is produc'd by the entire Circumvolution of a Demi-Parabola about its Axis, which for that reason is call'd the *Axis* of the *Paraboloid*, which passes thro the Centre of its Base, which is a Circle. As ACBE, whose Axis is CD, which passes through the Centre D of its Base AEBF, which is a Circle, whose Diameter is AB.

If instead of a Parabola, you make an Hyperbola turn about its Axis, the Solid which will be produc'd by that Circumvolution, will be call'd an *Hyperbolic Conoid*, whose Axis will be the same as that of the Hyperbola, and whose Base will be likewise a Circle.

But if you move at the same time about the same Axis of the Hyperbola one of its two Asymptots, there will be produc'd by the entire Circumvolution another greater Solid, which we shall call an *Asymptotic Cone*, whose Base will be in like manner a Circle, whereof the Vertex will be at the Centre of the Hyperbola.

One may also call a *Spheroid* an *Elliptic Conoid*; because a *Conoid*, generally speaking, is a Solid, which is produced by the entire Circumvolution of a Conic-Section, that is to say, of the Section of a Cone by a Plane, about its Axis: And that the Ellipsis, as well as the Parabola and

and Hyperbola are Conic Sections. See the Treatise of Conic Sections, which we formerly publish'd, under the Title of *Lines of the Second Kind*. *Plate II. Fig.* 22.

As the Circle is likewise a Conic Section, and that the Sphere is produc'd by the entire Circumvolution of a Semi-circle about its Diameter, the Sphere may be also call'd a *Circular Conoid*: and all the Curv'd Lines that bound those four Conic Sections may be call'd *Conic Lines*, as being the Sections of a Plan and of a Conic Superficies, that is to say, of the Surface of a Cone.

L.

The *Cone* is a Solid ending in a Point, call'd the *Vertex of the Cone*, which is produc'd by the entire Circumvolution of a Triangle about one of its Sides, which therefore is call'd the *Axis of the Cone*, which passes through the Centre of its Base, which is a Circle. As if about the immovable Side CD, you imagine the Triangle CDB to be mov'd, that Triangle will describe the Cone ACBE, whose Axis is the immoveable Side CD, and the Side BC, which is call'd the *Side of the Cone*, will describe the Conic Superficies; and lastly, the other Side DB will describe the Circle AEBF, which serves for the Base of the Cone, and whose Centre is the immoveable Point D, and the Diameter AB double the Side DB.

When the Angle D, of the generating Triangle CDB is right, the Solid that is produc'd by its Motion, is call'd a *Right Cone*, because its Axis is Perpendicular to its Base: and also an *Isosceles Cone*, because all its Sides are equal. But when the Angle D of the same Triangle CDB is oblique, the Solid produc'd by its Circumvolution is call'd an *Inclin'd Cone*, because its Axis is inclin'd to its Base: and also a *Scalene Cone*, because it has not all its Sides equal.

They say that two Cones are *alike inclin'd*, when their Axes make equal Angles with their Bases: and that two Cones are *Similar*, when they are alike inclin'd, and that their Axes are proportional to the Diameters of their Bases.

LI.

The *Frustum of a Cone* is a Solid generated by the intire Revolution of a Trapezoid upon one of its two Sides, that are not Parallel, which on that account is call'd the *Axe* of the *Frustum*, and connects the Centers of the two opposite and Parallel *Bases*, that are two un- *Plate II. Fig.* 12.

Plate 2. *Fig.* 12. equal Circles describ'd by the two opposite Parallel and unequal Sides of the *Generating Trapezoid.*

Thus if you conceive the Trapezoid IKBC to revolve about its immovable Side IK, its opposite Sides IC, KB being parallel, you will have the Frustum ABCD, generated, whose Axe is the immoveable Side IK, and the two opposite and parallel Bases are the two Circles AEBF, DGCH, whose Centers are K, I, and Diameters AB, *Plate* 3. *Fig.* 23. CD. 'Tis evident, that the two Circles are describ'd by the motion of the two opposite and parallel Sides, IC, KB, and that the other moveable Side BC, which is the *Side of the Frustum*, describes by its motion the *Surface* of the Frustum.

LII.

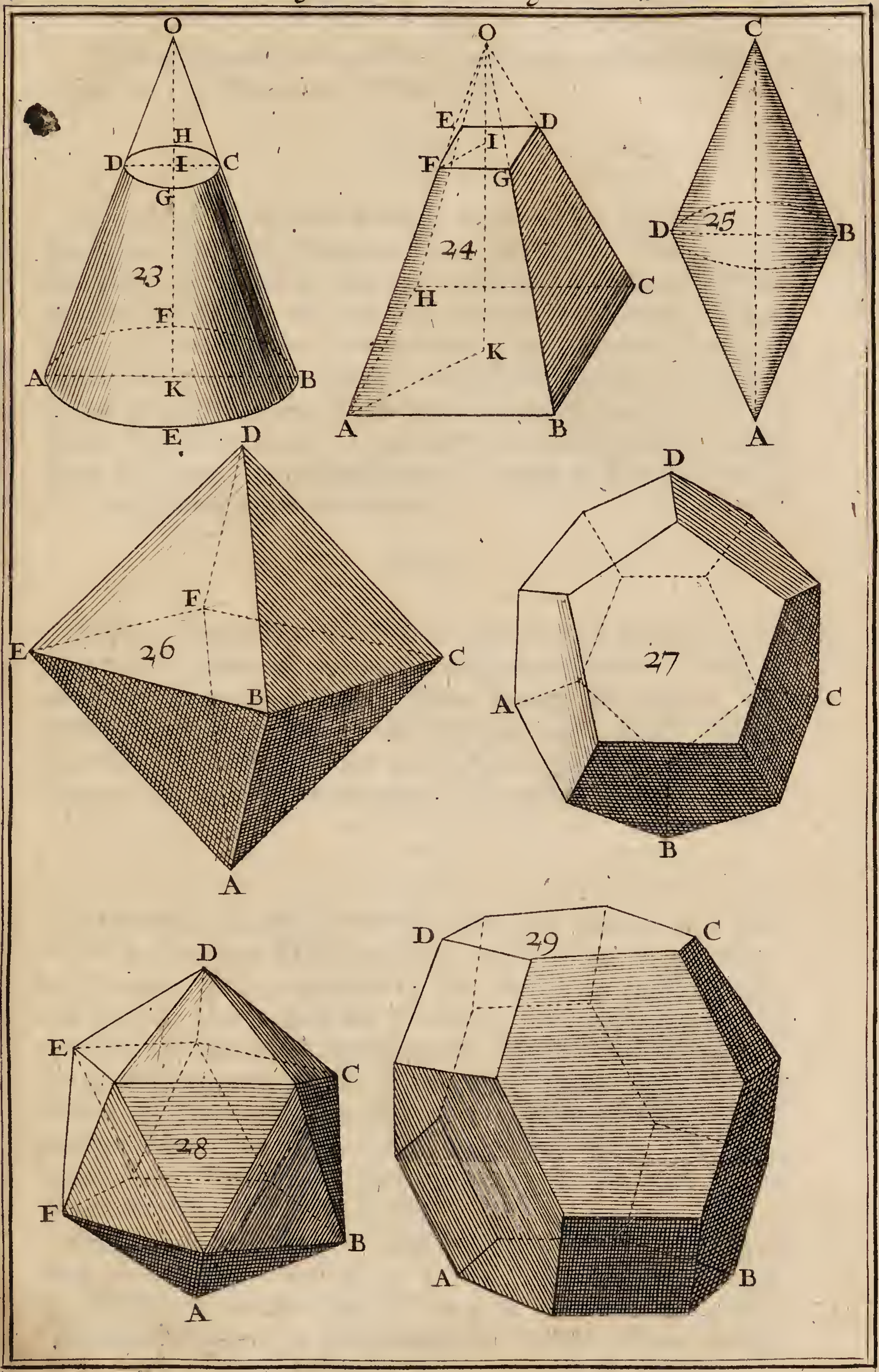
A *Cylinder* is a Solid generated by the intire Revolution of a Parallelogram about one of its Sides, which on that account is call'd the *Axe of the Cylinder*, and passes thro' the Centers of the two opposite and parallel Bases, which are two equal Circles describ'd by the two other opposite parallel and equal Sides of the generating Parallelogram.

Thus if you conceive the Parallelogram EFBC to revolve about the immovable Side EF of the Parallelogram EFBC, you will have the *Cylinder* or *Column* ABCD, *Plate* 2. *Fig.* 13. generated, whose Axe is the immoveable Side EF, and its opposite and parallel Side BC, call'd the *Side of the Cylinder*, will describe a *Cylindric Surface*; But the two other opposite equal and parallel Sides FB, EC will describe two equal Circles, that are the *Bases of the Cylinder*, and have for their Centers the two Points E, F, for their Diameters the two Lines AB, CD.

When the Angle E of the generating Parallelogram EFBC is a right one, *i. e.* when the Parallelogram is a Rectangle, the Solid describ'd by its motion, is call'd a *Right Cylinder*, because its Axe is Perpendicular to its two Bases: But when the Angles are Oblique, the Solid generated by its Revolution, is call'd an *Oblique Cylinder.*

Two Cylinders are *similarly inclin'd*, when their Axes make equal Angles with their Bases: And two Cylinders are *Similar*, when they are similarly inclin'd, and their Axes proportional to the Diameters of their Bases.

That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That Cylinder is call'd a *Cubic* one, whose Height is equal to the Diameter of the Base. *Plate* 2. *Fig.* 13.

LIII.

An Orb is a Spherical Body bounded by two Spherical Surfaces, the one Concave, and the other Convex: As the Body bounded by the two Spherical Surfaces, BCDE, which is Convex, and FGHI, which is Concave. Where you may see that the Orb is what remains, when you have taken out of a great Sphere, as BCDE, a less as FGHI. These two Spheres have the same Center A, but they might have been *Excentric*, i. e. their Centers might have been different, and then the Orb would not have been of an equal thickness every where. *Fig.* 19.

LIV.

A *Solid Angle* is an indefinite Space bounded by more than two Planes cutting one another in a Point. Thus D, which is comprehended by three triangular Planes, two of which may be seen in the Figure, *viz.* ADB, BDC. The *Vertex* of a Cone also is a solid Angle compos'd, as it were, of an infinite number of Triangular Planes. *Fig.* 11.

LV.

A *Pyramid* is a Body terminating in a Point, and contain'd under four Planes at least, which are all Triangular, excepting that opposite to the Point or solid Angle, and is call'd the *Base of the Pyramid*, and may have more than three Sides, the opposite solid Angle is call'd the *Vertex of the Pyramid*, all the right Lines drawn from thence to the Base, are call'd the *Sides of the Pyramid*, which is represented by the Figure ABCD, whose Base ABC is of four Sides, whose *Vertex* is the solid Angle D, and whose Sides are DA, DB, DC, &c.

A *Pyramid* also may be *Right* or *Oblique*. 'Tis a right one, if its *Axe*, which is a right Line drawn from its Vertex through the Center of the Base, which I suppose Regular, is Perpendicular to the Plane of the Base; 'tis an *Oblique* one, when its Axe is Oblique to the Plane of the Base, *i.e.* when in regard of that Plane it inclines more to one Side than it does to the other. When its Base is a Triangle, 'tis call'd a *Triangular Pyramid*, as ABCD, and is a *Tetraëdrum* when its four Triangles are equilateral and equal. *Fig.* 20.

LVI.

The *Frustum* of a Pyramid is what remains of a Pyramid, after a small one towards the Vertex, has been cut off by a Plane Parallel to its Base. It has two similar and parallel *Bases*, a *great* one, the same with that of the whole Pyramid, and a small one, the same with that of the Pyramid cut off, or the Section of the cutting Plane, and of the whole Pyramid.

Plate III. Fig. 24. Thus if you cut the Pyramid ABCO with the Plane FGDE, parallel to the Base ABCH, it takes off the little Pyramid FGDO, and leaves the Frustum ABCDEFG, whose great Base is the Plane ABCH, and the small one the Plane DEFG, parallel to the former ABCH.

LVII.

A *Prism* is a Solid bounded by more than four Planes, all Parallelograms, but the two opposite ones call'd the *Bases of the Prism*, which may be of any other Figure, but always similar, parallel and equal. If they are Triangles, the Solid is call'd a *Triangular Prism*, as ABCDE, if Parallelograms, then a *Parallelopiped*, and takes the Name of a *Rectangled Parallelopiped*, when its six containing Planes are Rectangles, as ABCDE; which we further call a *Cube* and *Hexaedrum*, if its six Planes are equal Squares, as a Die.

Plate II. Fig. 15. *Fig.* 21.

LVIII.

Plate III. Fig. 25. A *Solid Rhombus* is a Body composed of two Right Cones, whose Bases are equal and joyn'd together, consequently whose Axes make one right Line; as ABCD.

LIX.

Fig. 29. A *Polyëdrum* is a Solid terminated by several regular Rectilineal Figures, which are call'd the *Face of the Polyëdrum*, and an *Inscriptible* in a Sphere, that is to say, all its solid Angles may touch the Surface of a circumscrib'd Sphere, whose Centre consequently is the same with the Sphere's: As ABCD, bounded by fourteen Faces, namely, eight equal regular Hexagons, and six equal Squares.

A *Polyëdrum*, as well as a *Polygon*, may be *Regular* or *Irregular*; *Irregular*, when its Faces are not equal and similar, as in ABCD; *Regular*, when they are equal and similar,

ſimilar, and conſequently their ſolid Angles alſo equal. *Plate III.* But tho' there is an infinite number of *Regular* Polygons, becauſe the Circumference of a Circle may be divided into as many equal Parts, as you pleaſe, yet no more than the five *Regular Bodies* are taken notice of, *viz.* the *Tetraëdrum*, mention'd already; The *Cube* or *Hexaëdrum*, alſo ſpoken of. The *Octaëdrum* which is terminated by eight Triangles, equal and equilateral, as ABCDEF. The *Fig.* 26. *Dodecaëdrum* contain'd under twelve regular and equal *Pentagons*, ABCD: And the *Icoſaëdrum*, comprehended under twenty equal and equilateral Triangles, as ABCDEF. 27. 28.

LX.

A *Meaſure* is a *continued Quantity* uſed to *meaſure* another continued Homogeneal but greater Quantity, *i. e.* to know how many of theſe leſſer Meaſures a greater Quantity contains, and determine the Content, which is call'd the *Length*, when a Line; the *Area*, when a Surface; but the *Solidity*, when a Solid is meaſur'd.

This Meaſure is always a right Line, when it expreſſes the Length of a Line; a Rectangle, and ordinarily a *Square*, and then 'tis call'd *Square Meaſure*, when it expreſſes the Area of a Superficies: But a Rectangled Parallelopiped, which is ordinarily a Cube, and then 'tis call'd *Cubic Meaſure*, if it repreſent the Solidity of a Body.

Meaſures are different in different Places: But among the Mathematicians the ordinary Meaſure, that ſerves as the Foundation of all the reſt, is the *Foot.*

'Tis call'd the *Long Foot*, when conſider'd according to its Length, which happens when 'tis uſed to meaſure a greater Line: *Square Foot*, when conſider'd as a Square, each of whoſe Sides is a Foot long, and is us'd for meaſuring a greater Superficies, whoſe Area is expreſſed by ſmall Squares, with Sides each a Foot in length. The *Cubic Foot*, when conſider'd as a Cube, each of whoſe Sides is a Foot long, and is uſed in meaſuring a bigger Solid, whoſe Solidity is expreſſed by ſmall Cubes, each of whoſe Sides is a Foot long.

After the ſame manner an *Inch Square*, is a Square each of whoſe Sides is an *Inch Long*, which is the 12th part of a Foot long: A *Cubic Inch* is a Cube that has each of its Sides an Inch long. In like manner a *Yard ſquare*, is a Square each of whoſe Sides is a *Yard long*, or three Feet: And a *Cubic Yard*, is a Cube, each of whoſe Sides is alſo a Yard long. Thus a *Perch ſquare*, is a Square, each of

whose Sides is a *Perch long*: And a *Cubic Perch*, tho' 'tis not in use, is a Cube, each of whose Sides is a Perch long.

Measures are proportion'd to the Magnitudes, that are to be measur'd. Thus Workmen use small Measures, as the Foot, Inch, *&c.* in measuring Board, Glass, *&c.* Architects, the Foot, Yard, *&c.* to measure Building, *&c.*

Astronomers measure Distances in the Heavens, as the Distance of a Planet from the Earth by *Semidiameters of the Earth*; but Distances on the Earth by *Miles* and *Leagues*, as the Distance of one City from another. The Leagues are different not only in Length but in Names, which are all comprehended under the general Term of *Itinerary Measures*, and differ in Length in the same Kingdom, for there are *Great*, *Mean*, and *Small* Ones. The *Great League of* France is ordinarily 3000 Geometric Paces, in some Places 3500; The *Small League of France*, or *Common League of France* is 2400; The *Small League of France* 2000, the double of an *Italian Mile*, so call'd, because it contains 1000 Geometric Paces, or 8 Stadia. The *Stadium*, which is a Measure peculiar to the Greeks, contains 125 *Geometric Paces*; or 625 Foot, For a *Geometric Pace* is 5 Feet, the *Common* but 2 Foot and half.

A Geometric Pace used in a Pendulum, *i.e.* a Pendulum 5 Foot long, measur'd from the Center of Motion to the Center of the Ball, suspended by the Extremity of the Pendulum, makes 1252 single Vibrations in half an Hour, which may serve to recover the Length of a Geometric Pace, if it should ever be lost, or alter'd; for Experience tells us, that *the Squares of the Number of Vibrations of two Pendulums, are in equal times reciprocally proportional to the Lengths of the Pendulums*, namely by making a second Pendulum of a known length, and whose Spherical Weight is the same with that of the first Pendulum, and reckoning the number of the single Vibrations in half an Hours time. For if the Number is equal to that of the single Vibrations of the first Pendulum, its Length will be equal to that of the First, *i.e.* a Geometric Foot, which by Consequence will be known; or you may find this Length by the Means of this Proportion following.

As the Square of the Number of Vibrations of the first Pendulum,
To the Square of the Number of Vibrations of the second Pendulum.
So the Length of the second Pendulum;
To the Length of the first Pendulum.

The remaining Terms you will find explain'd when they occurr, or in the Elements of *Euclide*.

The

The FIRST PART.

CONCERNING

GEODESIE.

GEODESIE is a Part of Practical Geometry, that teaches how to divide Lands, or Fields, that contain Arable Grounds, Woods, &c. betwixt two or more Inheritors. Hence 'tis, that this Part is call'd the *Division of Lands*, and we ſhall divide it into Three Chapters; the *Firſt* ſhewing the method of dividing Triangles; the *Second*, Quadrilateral Figures; and the *Laſt*, any Piece of Ground having more than 4 Sides, *i. e.* the Method of dividing Polygons.

CHAP. I.

The Diviſion of Triangles.

WE begin with a Triangle, becauſe 'tis the firſt and moſt ſimple Rectilineal Figure, and the only one conſider'd in Practice: And tho' all the Figures of Lands are not Rectilineal, yet they may be conceiv'd as Rectilineal, when the difference is but ſmall: Or they may be reduc'd to Rectilineals, by dividing the Sides that are crooked into ſo many Parts, as that they may paſs for right Lines.

PROBLEM I.

To divide a Triangle into as many Parts as you pleaſe, by right Lines drawn from a given Angle.

TO divide the Triangle ABC, for inſtance, into 3 equal Parts, by right Lines drawn from the given Angle C, divide the oppoſite Side AB into 3 equal Parts

Plate IV. Fig. 4. at the Points, D, E, and draw from the given Angle A, thro' the Points of Division D, E, the right Lines CD, CE, and they will divide the Triangle propos'd ABC, into 3 parts equally. The Demonstration is evident from 38. 1.

SCHOLIUM.

If instead of dividing the Triangle propos'd ABC, into 3 equal Parts, you would have it done into Parts of a given Ratio, by Lines drawn from the given Angle C, 'tis evident by 1. 6. no more is to be done, but to divide the Opposite Side AB, in the given Ratio by the *9th. 6th.* or by 10. 6. and compleat the rest as before.

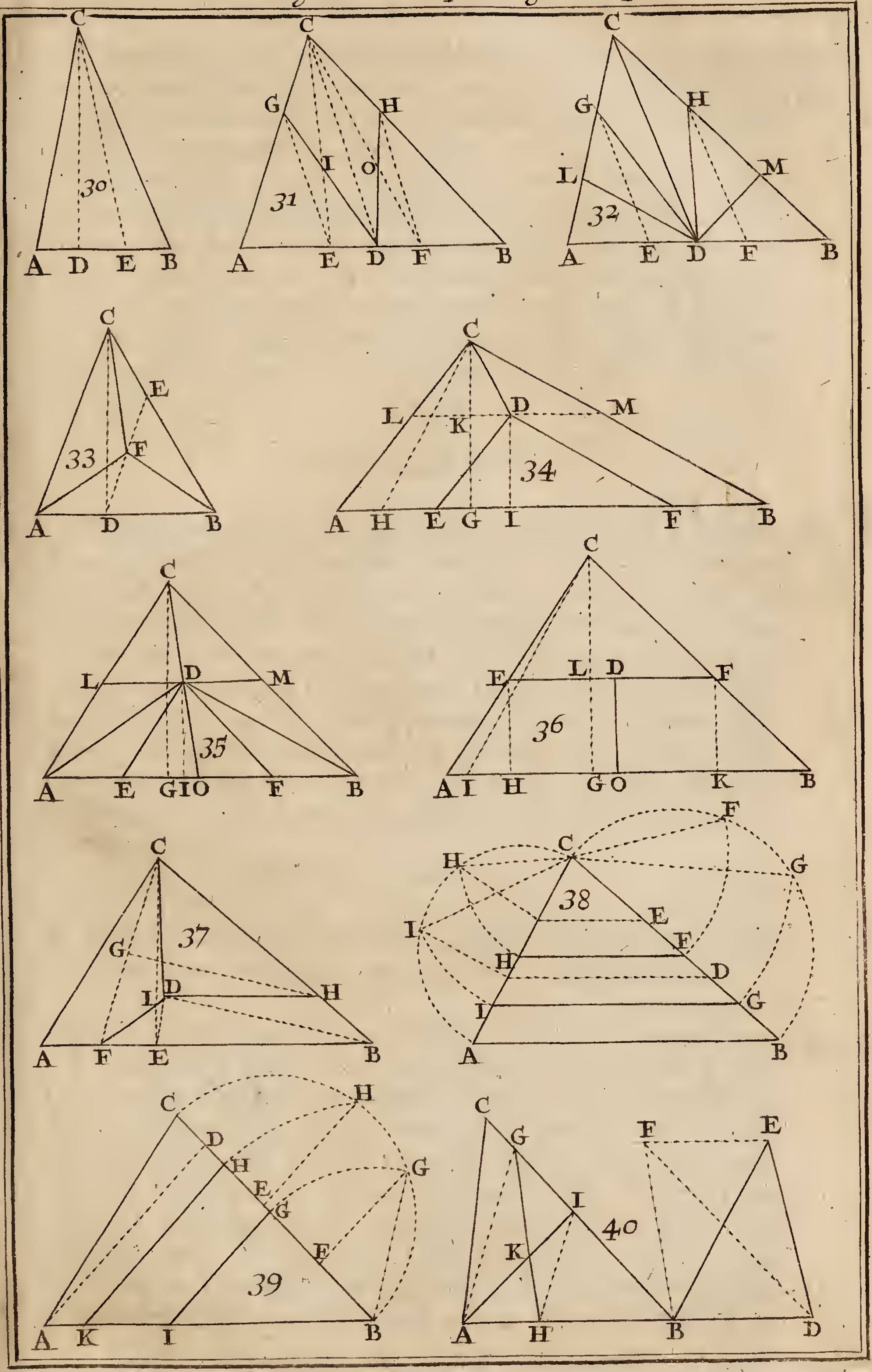
PROBLEM II.

To divide a Triangle into as many equal Parts as you please, by right Lines drawn from a given Point in one Side.

Fig. 31. TO divide the Triangle ABC, for instance, into 3 equal Parts, by right Lines drawn from the given Point D, in the Side AB; divide the Side AB into 3 equal Parts at the Points E and F, and having joined the right Line CD, draw to it thro' the Points of Division, E,F, the Parallels EG, FH, which will give in the Sides AC, BC, the Points G, H, thro' which and the given Point D, draw the right Lines DG, DH, and they will divide the Triangle propos'd into 3 equal Parts, so that each of the 2 Triangles ADG, BDH, will be equal to the third Part of the Triangle ABC propos'd, *i. e.* if you join the right Lines CE, CF, the Triangle ADG is equal to the Triangle AEC, which is the third part of the Triangle propos'd by 1. 6. and in like manner the Triangle BDH is equal to the Triangle BFC, which is also a third Part of ABC.

DEMONSTRATION.

Because the 2 Lines CD, EG, are parallel by *Construction*, the 2 Triangles GCE, GDE, having the same Line GE for their Base, will be equal, by 37. 1. consequently if from each you take away the common Triangle GIE, there will remain the Triangle GIC, equal to the Triangle DIE; and if to each of the 2 equal Triangles DIE, GIC, you add the Trapezium AEIG, you will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will have the Triangle AGD, equal to the Triangle ACE; by the same Method you may come to know that the Triangle BHD is equal to the Triangle BCF, because FH and CD are parallel. *Q. E. D.* *Plate* 4. *Fig.* 31.

Or which is all one, because the two Lines EG, CD are parallel *by Construction*, the two Triangles AGE, ACD, having the Angle at A common, will be Equiangular by 29. 1. and by 4. 6. the four Sides AG, AE, AC, AD will be proportional; consequently the two Triangles AGD, ACE will be equal by 15. 6. and thus also you may find that the two Triangles BHD, BCF are equal *Q. E. D.*

SCHOLIUM.

If the Point D given be in the middle of the Side AB, in which case the Triangle ABC will be divided into two equal Parts by the right Line CD, 38. 1. you may divide the Triangle ABC into six equal Parts, by dividing it first into three equal Parts, as before, and by dividing into two equal Parts each of the two Lines AG, BH, at the Points L, M, that you may draw the Lines DL, DM. Or by dividing each of the two Sides AC, BC, into three equal Parts at the Points L, G, M, H, and joyning the right Lines DL, DG, DH, DM, &c. *Fig.* 32.

PROBLEM III.

To divide a Triangle into three equal Parts by right Lines drawn from the three Angles of the Triangle propos'd.

To divide the Triangle ABC into three equal Parts by three right Lines drawn from the Angles A, B, C; having taken in one of the Sides of the Triangle, as AB, its third Part AD, draw thro' the Point D, the Line DE parallel to the adjacent Side, AC, and thro' the Point F, the middle of the Line DE, draw to the three Angles A, B, C, the right Lines FA, FB, FC, and they will divide the Triangle propos'd, ABC, into three equal Triangles AFB, AFC, BFC; so that each of them will be a third Part of the Triangle ABC. *Fig.* 33.

DEMONSTRATION.

Joyning the right Line CD, you know by 1. 6. that the Triangle ACD, is a third part of the Triangle ABC, because the Base AB is triple the Base AD, *by Construction*; and

Plate IV. Fig. 33. and conſequently the Triangle AFC, which is equal to the Triangle ADC, by 37. 1. is alſo a third part of the Triangle ADC. Conſequently the two other Triangles AFB, BFC taken together are equal to $\frac{2}{3}$ of the ſame Triangle ABC, and ſo each is a third part of the Triangle ABC, becauſe they are equal to one another, the two Triangle BFD, BFE being equal, by 38. 1. and alſo AFD, CFE being equal. *Q. E. D.*

PROBLEM IV.

To divide a given Triangle into three equal Parts, by two right Lines parallel to two Sides, and by a third Line drawn from the Angle of the ſame Sides.

Plate 41. *Fig.* 34. TO find a Point within a Triangle given ABC, from whence drawing two right Lines parallel to the two Sides AC, BC, and a third Line to the Angle C, the Triangle ABC, ſhall be divided into three equal Parts; having drawn from that Angle C, to the oppoſite Side AB, the Perpendicular CG, make the Angle GCH of thirty Degrees at the ſame Point C, with the Perpendicular CG, (which may be done Geometrically) that the Square of GH might be equal to a third Part of the Square CG, as will be evident to any one, that ſhall conſider, that the Angle G being right, and the Angle GCH of 30 Degrees, the Angle H is 60 Degrees, which is the Angle of an Equilateral Triangle, whoſe Side is CH, Perpendicular CG, and $\frac{1}{2}$ the Baſe is GH. Take therefore in the Perpendicular CG, the Line GK equal to the Line GH, and draw thro' the Point K, to the Side AB, the Parallel LM, whoſe middle Point D is the Point ſought; ſo that if from the Point D, you draw to the Angle C, the right Line CD, and to the two Sides AC, BC, the Parallels DE, DF, the Triangle propoſed will be found divided into three equal Parts.

DEMONSTRATION.

Becauſe the Triangle EDF is ſimilar to the Triangle ABC, for the two Sides DE, DF are parallel to AC, BC, *by Conſtruction.* And the Square of its height DI, or GK, or GH, is the third part of the Square of the height GC, of the Triangle ABC, 'tis eaſy to conclude by 19. 6. that the Triangle EDF is alſo a third of the Triangle ABC. From whence it follows, that the two Trapezoids ACDE, BCDF are together equal to $\frac{2}{3}$ of the

Triangle

Triangle ABC, and consequently each equal to a third of the same Triangle ABC, because the two Trapezoids are equal to one another, the two Triangles CDL, CDM being equal, by 38. 1. and the two Parallelograms also AEDL, BFDM, *by* 36, 1. *Q. E. D.*

SCHOLIUM.

If you make the height DI of the Triangle EDF equal to $\frac{1}{2}$ the height CG of the Triangle ABC, in which Case the Line AL will be $\frac{1}{2}$ the side AC, and BM, $\frac{1}{2}$ the side BC, the Triangle ABC will be found divided into four equal Parts, the two Triangles EDF, CLM, and the two Parallelograms AEDL, BFDM, and if you draw the two Diagonals AD, BD, and the right Line CDO, the Triangle propos'd ABC, will be found divided into eight equal Triangles. *Plate IV. Fig.* 35.

PROBLEM V.

To divide a given Triangle into three equal Parts, by two right Lines, the one Parallel, and the other Perpendicular to the same Side.

TO divide the Triangle ABC into three equal Parts by two right Lines, the one Parallel, and the other Perpendicular to the Side AB; draw to the Side AB, from its opposite Angle C, the Perpendicular CG, and make the Angle GCI, of thirty Degrees, at the Point C, with the Perpendicular CG, as in the preceeding Problem, where we took notice, that the Square of the Line GI is a third part of the Square of the Line CG: Wherefore, if upon this Line CG, you take a part CL equal to the Line GI, and draw a Line EF thro' the Point L parallel to the Side AB, you will have the Triangle CEF equal to a third part of the Triangle ACB proposed, to which it is Similar, as we also knew in the preceding Problem. Lastly, having drawn from the two Points, E, F, the two Lines EH, FK perpendicular to the Side AB, take in the Side AB, the Line AO equal to a quarter of the Sum of once BK, twice HK, thrice AH, and erect from the Point O, upon AB the Perpendicular OD, which with the Line EF parallel to AB, will divide the Triangle ABC into three equal Parts, the Triangle CEF, and the two Trapezoids AODE, BODF. *Fig.* 36.

DEMONSTRATION.

We know already that the Triangle CEF is a third of the Triangle ABC, from whence 'tis easie to conclude, that the Trapezoid ABFE, is equal to $\frac{2}{3}$ of the same Triangle ABC, and since 'tis divided into two equal parts by the Perpendicular DO, as we have demonstrated in *Prob.* 13. *Chap.* 2. it follows, that the two equal Trapezoids AODE, BODF, are each a third of ABC; consequently the Triangle ABC, is divided into three equal Parts by the Line EF parallel to the Side AB, and by the Line DO Perpendicular to the same Side BED. *Q. E. D.*

PROBLEM VI.

To divide a given Triangle into three equal Parts, by three right Lines drawn from a Point given within the Triangle.

Plate 4. *Fig.* 37. TO divide the Triangle ABC into three equal Parts, by three right Lines drawn from the Point D within the Triangle, take in one of the Sides, as AB, its third Part AE, and having joyn'd the right Line DE, draw to it thro' the opposite Angle C, the parallel CF, which must be bisected in the Point G, thro' which you must draw the Line GH parallel to the Line DB. Lastly, draw the three Lines DC, DF, DH, and they will divide the Triangle ABC into three equal Parts, the Triangle CDH, and the two Trapeziums ACDF, BFDH; so that each of the three Planes will be a third of the Triangle ABC.

DEMONSTRATION.

If you joyn the right Line CE, you may consider, that since the Line AE is a third part of the Line AB *by Construction*; the Triangle ACE also is a third Part of the Triangle ABC, *by* 1. 6. and you may find, as in *Prob.* 2. that the Trapezium ACDF is equal to the Triangle ACE, and consequently to a third of the Triangle ABC. Whence it follows, that the Trapezium CDFB is equal to $\frac{2}{3}$ of the same Triangle ABC; and because it is divided into two equal Parts by the right Line DH, as we shall demonstrate in *Prob.* 11. *Chap.* 2. it follows, that its $\frac{1}{2}$ the Triangle CDH, or Trapezium BFDH is also a third of the Triangle ABC. *Q. E. D.*

PRO-

PROBLEM VII.

To divide a Triangle into as many equal Parts as you please, by right Lines parallel to a given Side.

TO divide the Triangle ABC, for Example, into three equal Parts, by right Lines parallel to the Side AB. Divide any one of the Sides AC, BC, for instance BC, into three equal Parts at the Points D, E, and take in the Side BC, a part CF, a mean proportional betwixt the Side BC and its third CE, in like manner CG a mean proportional betwixt the same Side BC and its $\frac{2}{3}$ CD, draw thro' the two Points, F, G, the parallels FH, GI to the Side AB, which will divide the Triangle propos'd ABC, into three equal Parts, which are the Triangle CHF, the two Trapezoids AIGB, IHFG, so that each of the three Planes will be a third part of the Triangle ABC. *Plate IV. Fig. 38.*

DEMONSTRATION.

Because the Triangles HFC, ABC, are equiangular AB, BH being parallel *by Construction*, they will be to one another, as the Squares of their Homologous Sides CF, BC, *by* 19. 6. and because the Square CF is to the Square BC, as CE is to its Triple BC, *by Corol.* 20. 6. by reason of the three Proportionals BC, CF, CE *by Construction*, it follows that the Triangle HFC is a third of the Triangle ABC, and you may demonstrate after the same manner, that the Triangle IGC is equal to two thirds of the same Triangle ABC, whence 'tis easy to conclude, that each of the Trapezoids IGFH, ABGI, is $\frac{1}{3}$ of ABC. *Q. E. D.*

PROBLEM VIII.

To divide a Triangle into as many equal Parts as you please, by right Lines perpendicular to a Side given.

TO divide the Triangle ABC, for Example, into three equal Parts, by Lines perpendicular to the Side BC. Draw to that Side BC, from its opposite Angle A, the Perpendicular AD, and divide any one of its two Segments BD, CD, for instance BD, into three equal Parts, at the Points E, F. Take in the same Side BC, the Part BG a mean proportional betwixt the Side BC and a third Part, BF, of the Segment BD, and the Part BH, a mean Proportional between the same Side BC *Fig. 39.*

and

and $\frac{2}{3}$ BE of the Segment BD, and draw from the two Points G, H, upon the Side BC, the Perpendiculars GI, HK, and they will divide the Triangle propoſed ABC, into three equal Parts, the Triangle BIG, the Trapezoid KIGH, and the Trapezium AKHC; ſo that each of theſe three Planes will be equal to a third of ABC.

DEMONSTRATION.

Becauſe the Triangles ABD, IBG are equiangular, by reaſon of the two Parallels AD, IG *by Conſtruction*, the Ratio of the two Lines BD, AD will be equal to that of the two BG, IG *by* 4. 6. Wherefore if you give the common height BC, to the two firſt Terms BD, AD, and the common height BG, to the two laſt BG, IG, you know *by* 1. 6. that the Rectangle under BC, BD is to the Rectangle under BC, AD, as the Square BG, or the Rectangle under BC, BF, that is, equal to it by 17. 6. by reaſon of the three Proportionals BC, BG, BF, *by Conſtruction*, is to the Rectangle under BG, IG; and as the firſt Antecedent of that Proportion, namely, the Rectangle under BC, BD is triple of the ſecond Antecedent, *i. e.* the Rectangle under BC, BF, *by* 1. 6. becauſe they have the ſame height BC, and the Baſe of the firſt, BD, is triple the Baſe of the ſecond BF, *by Conſtr.* the firſt Conſequent, namely the Rectangle under BC, AD, or by 41. 1. the double of the Triangle ABC, will be alſo triple of the ſecond Conſequent, *i. e.* of the Rectangle BG, IG, or the double of the Triangle BGI. Whence it is eaſy to conclude, that the Triangle BGI, is a third of the Triangle ABC; and after the ſame manner you may find that the Triangle BHK is equal to two thirds of the ſame Triangle ABC, and conſequently that each of the two Trapeziums KIGH, AKHC, is a third of the Triangle ABC. *Q. E. D.*

Plate 4. *Fig.* 39.

SCHOLIUM.

You may divide after the ſame manner the Triangle ABC into equal Parts, by right Lines, making a given Angle with one Side; as if the Side be BC, one may make at the Point A, the Angle BAD equal to the given One, and having found as before the two Points G, H, one may draw thro' thoſe two Points G, H, the right Lines GI, HK, parallel to the Line AD, &c.

PRO-

PROBLEM IX.

To cut off from a Triangle, another equal to a given One.

TO take from the Triangle ABC, a Triangle towards B, that ſhall be equal to a given Triangle BDE, make at the Point D the Angle BDF equal to the Angle ABC, by the Line, to be terminated in F, by the right Line EF, parallel to the Side BD, and take in the Side BC, the Part BG equal to the Line BF, and in the Side AB, the Part BH equal to the Line BD, joyn the right Line GH, which will cut off the Triangle BGH equal to the given one BDE. *Plate* 4. *Fig.* 40.

DEMONSTRATION.

Becauſe the two Sides BD, DF of the Triangle BFD are equal to the two Sides BH, BG of the Triangle BGH, and the contain'd Angle BDF equal to the contain'd Angle HBC, *by Conſtr.* it follows *by* 4. 1. that the Triangle BGH is equal to the Triangle BFD, and conſequently to the Triangle BED, becauſe by the 37. 1. theſe two Triangles BFD, BED are equal, by reaſon of the two Parallels EF, BD. *Q. E. D.*

SCHOLIUM.

This Problem may be reſolved otherwiſe, for you may find within the Triangle propoſed ABC, a Triangle different from BGH, yet equal to the given one BED; by drawing thro' the Point H, the right Line HI, parallel to the Line AG, and joyning the right Line AI, which will cut off the Triangle AIB, equal to the Triangle BGH, and conſequently to the given one BED.

DEMONSTRATION.

Becauſe the two Lines AG, HI are parallel *by Conſtr.* the two Triangles GIA, GHA will be equal *by* 37. 1. wherefore if from each you take away the common Triangle AKG, there will remain the two equal Triangles AKH, GKI, which being added to the Trapezium BIKH, you may ſee that the Triangle ABI is equal to the Triangle BGH. *Q. E. D.*

Or having let fall from the Vertices C, E, upon the Baſes AB, BD, the Perpendiculars CG, EF, find a fourth proportional BH to the three Lines CG, EF, BD, and joyn

Plate V. Fig. 41. joyn the right Line CH, which will cut off the Triangle CHB equal to the given one BDE.

DEMONSTRATION.

Because the four Lines CG, EF, BD, BH are proportional *by Constr.* the Rectangle of the Extremes CG, BH, will be equal, *by* 16. 1. to the Rectangle of the Means EF, BD: Wherefore the halves of these two Rectangles will be equal, that is, *by* 41. 1. the Triangles CHB, BDE. *Q. E. D.*

Or take in the Perpendicular CG, the Part GI equal to the perpendicular EF, and draw thro' the Point I the right Line IK parallel to the Side AB, in which take the Part BL, equal to the Base BD, and joyn the right Line KL, and it will cut off the Triangle LKB equal to the given one BDE, because the two Triangles have equal Bases and Heights.

Fig. 42. Or having found the Line BH proportional to the three Lines CG, EF, BD, and the Line CL being at discretion, draw thro' the Point H, the Parallel HK, to that Line CL, and joyn the right Line KL, and it will cut off the Triangle LKB, equal to BCH, consequently to the given one BDE.

DEMONSTRATION.

Because the two Lines CL, KH are parallel *by Constr.* the two Triangles CKL, CHL will be equal, *by* 38. 1. Wherefore if from each you take the Triangle CIL, there will remain the two equal Triangles CIK, HIL, which being added separately to the Trapezium BHIK, you will find the Triangle BKL is equal to the Triangle BCH, which is equal to the given one BDE. *Q. E. D.*

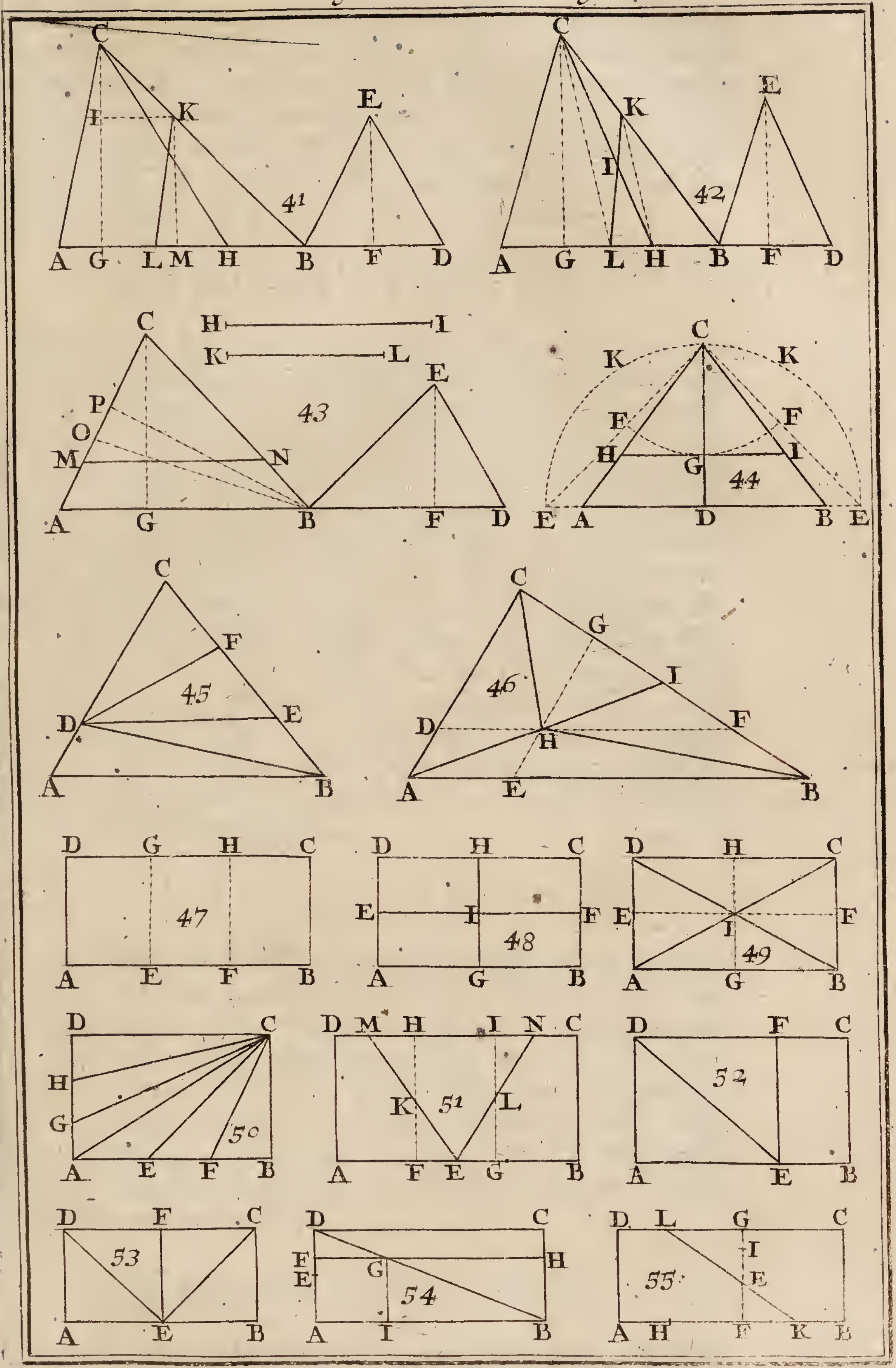
This Problem may also be resolved several other ways; for one might make the Side KL perpendicular to the Side KB, by the help of the Triangle BCH, equal to the given one BDE, as has been taught in *Probl.* 8. or parallel to the Side AC, as has been shewn in *Probl.* 7. and I shall farther shew another way in

PROBLEM X.

To cut off from a given Triangle another equal to a given one, by a right Line parallel to a given Side.

Fig. 43. To cut off from the Triangle ABC, a Triangle equal to a given one BDE, by a Line parallel to a Side AB.

Geometry Plate 5. Page 32.



AB. Find a mean proportional HI, between the Side AB and its perpendicular CG, in like manner a mean proportional KL between the Side BD and its perpendicular EF, and a fourth proportional CM to the three Lines HI, KL, AC, to find the Point M, thro' which draw MN parallel to the Side AB, and that ſhall cut off the Triangle CMN equal to the given One BDE.

DEMONSTRATION.

Becauſe the four Lines HI, KL, AC, CM, are proportional *by Conſtr.* their Squares will be alſo proportional *by* 22. 6. Wherefore if you ſubſtitute in the place of the Square HI, the Rectangle under AB, CG equal to it, *by* 17. 6. by reaſon of the three Proportionals AB, HI, CG *by Conſtr.* and in the place of the Square KL the Rectangle under BD, EF, which is alſo equal to it, by reaſon of the three Proportionals BD, KL, EF, and laſtly in the room of the two other Squares AC, CM, ſubſtitute the ſimilar Triangles ABC, MNC, that are in the ſame Ratio *by* 19. 6. and you will find, that the Rectangle of AB, CG, or, *by* 41. 1. the double of the Triangle ABC, is to the Rectangle of BD, EF, or the double of the Triangle BDE, as the Triangle ABC is to the Triangle CMN. Whence 'tis eaſy to conclude, that the Triangle CMN is equal to the Triangle BDE.

SCHOLIUM.

The Solution of this Problem may be abridg'd, by paſſing over the two mean Proportionals HI, KL, as you ſhall ſee. Having let fall from the Angle B, upon the oppoſite Side AC, the Perpendicular BP, find a fourth Proportional CO, to the three Lines BP, BD, EF, and a mean Proportional CM between the Lines AC, CO, and draw, as above, thro' the Point M, the Parallel MN, to the Side AB, and that will cut off the Triangle CMN equal to the given one BDE.

DEMONSTRATION.

By joining the right Line BO, you know as in the preceeding Problem, that the Triangle BOC is equal to the given One BDE,, and becauſe the ſimilar Triangles ABC, MNC are to one another, as the Squares of their Homologous Sides AC, CM *by* 19. 6. or as the Lines AC, CO by *Coroll*, 20. 6. by reaſon of the three Proportionals AC,

CM, CO, by *Constr.* or as the Triangle ABC, is to the Triangle BOC, by 1. 6. it follows that the Triangle MNC is equal to the Triangle BOC, consequently to the Triangle BDE. *Q. E. D.*

PROBLEM XI.

To divide an Isosceles Triangle into four equal Parts, by two right Lines perpendicular to each other.

TO divide the Triangle ABC, whose two Sides AC, BC, are equal, into four equal Parts, by two right Lines cutting one another at right Angles. Divide, first of all, the Base AB, into two equal Parts, in the Point D, and draw the right Line CD, which will divide the Triangle ABC into two equal Rectangled Triangles CDA, CDB. After that describe from the Point D, thro' the Point C, a quarter of a Circle CKE, and join the Cord CE, which Bisect at the Point F, thro' which describe from the Point C, the Arc FG, which will give in the Perpendicular CD, the Point G, thro' which draw HI parallel to the Base AB, and that together with its Perpendicular CD, will divide the Triangle propos'd ABC, into four equal Parts, two right-angled Triangles CGH, CGI, and the two Trapezoids ADGH, BDGI. Fig. 44.

DEMONSTRATION.

Because the two Sides CD, DE of the Rectangled Triangle CDE, are equal *by Constr.* the Square CE, being equal to the two equal Squares CD, DE, by 47. 1. is double of the Square CD · and because CF is half CE *by Constr.* one may know that the Square of CE is Quadruple the Square CF or CG by 10. 6. and double the Square of CD is double the Square CG; wherefore by 19. 6. The Triangle of DC will be also double the similar Triangle HGC: And in like manner the Triangle CDE will be double the Triangle CGI. Whence 'tis easy to conclude, that the two Right-angled Triangles CGH, CGI, and the two Trapezoids ADGH, BDGI, are equal and so the Triangle propos'd ABC is found divided into four equal Parts by the two Perpendiculars CD, HI. *Q. E. D.*

PRO-

PROBLEM XII.

To find a Point in a given Side of a Triangle, from whence the Triangle may be divided into as many equal Parts as you please.

TO find in the Side AC of the Triangle ABC, a Point, from whence you may divide the Triangle ABC into four equal Parts: Take the Line AD equal to a fourth Part of the Side AC, and the Point D, will be the Point sought: For join the right Line BD, and the Triangle ADC will be a quarter of the Triangle ABC, by 1. 6. Wherefore the Triangle BDC will be equal to three fourths of the same Triangle ABC. Then you have no more to do, but to divide the Triangle BDC into three equal Parts *by Probl.* 1. namely, by dividing the Side BC into three equal Parts, at the Points E, F, and joining the right Lines DE, DF, and the Triangle propos'd will be found divided into four equal Parts by the three Lines DB, DE, DF. *Plate V. Fig.* 45.

SCHOLIUM.

If you would have the Point you inquire after, within the Triangle ABC; having taken as before, the Line AD equal to a fourth Part of the Side AC, and in like manner the Line AE, equal to a fourth part of the Side AB, because 'tis propos'd to divide the Triangle ABC into four equal Parts, draw DF, EG thro' the Points D, E parallel to the two Sides AB, AC, whose Point of Section H will be the Point sought: So that if you join the right Lines HA, HB, HC, each of the two Triangles AHB, AHC, will be a fourth of ABC propos'd, and consequently the Triangle BHC will be half, so that only divide it into two equal Parts, by the right Line HI, that bisects the Side BC in I, and the Problem is solved. *Fig.* 46.

CHAPTER II.

The Division of Quadrilaterals.

QUadrilaterals may be easily divided by one that understands the Division of Triangles, tho' the Division of Triangles depends in several Cases upon the

Division of Quadrilaterals, and principally of Trapezoids, at least when you would divide a Triangle into more than two equal Parts, as you may observe in several Problems of the preceding Chapter.

PROBLEM I.

To divide a Parallelogram into as many equal Parts as you please, by Lines parallel to a given Side.

Plate V. Fig. 47. TO divide the Parallelogram ABCD, for Example, into three equal Parts, by Lines parallel to a given Side AD, draw the Parallels EG, FH, thro' the Points E, F, and they will divide the Parallelogram propos'd ABCD into three equal Parallelograms, as is evident by 36. 1.

PROBLEM II.

To divide the Parallelogram into four equal Parts by two right Lines parallel to two Sides.

Fig. 48. TO divide the Parallelogram ABCD into four equal Parts by two right Lines parallel to the two Sides AB, AD. Bisect the two opposite Sides AD, BC, in the Points E, F, which join by the right Line EF; in like manner bisect the two opposite Sides AB, CD, in the Points G, H, which join by the right Line GH, and that with the preceding will divide the Parallelogram proposed ABCD, into four equal Parallelograms. The Demonstration is too evident to be enlarged upon.

SCHOLIUM.

Fig. 49. You may easily divide the Parallelogram ABCD, into four equal Parts, that shall be four Isosceles Triangles by the two Diagonals AC, BD that divide them into four equal Triangles, as you may see, by drawing thro' the Center I, the Lines EF, GH parallel to the two Sides AB, AD. Where 'tis evident, they divide the Parallogram proposed HBCD, into four little equal Parallelograms, and that each of these Parallelograms is divided into two equal Parts by the Diagonals by 34. 1. You see also, that by this Method the Parallelogram proposed ABCD is divided into eight equal Parts, that are eight equal Triangles having their common Vertex in the Center I. Which makes it clear, that from the Center I, you may

may divide a Parallelogram in to any evenly even Number of equal Parts. By an *evenly even* Number, I mean such an one, as can be divided exactly by 4.

PROBLEM III.

To divide a Parallelogram into any even Number of equal Parts, by right Lines drawn from a given Angle.

TO divide the Parallelogram ABCD, for instance, into six equal Parts, by right Lines drawn from the Angle C. Draw thro' that Angle C, the Diagonal AC, which by 31. 4. will divide the Parallelogram ABCD, into two equal Triangles ACB, ACD: Then you have no more to do, but to divide by *Probl.* 1. *Chap.* 1. each of the two equal Triangles into three equal Parts, by dividing the Sides AB, AD each into three equal Parts, at the Points E, F, G, H, and joining the right Lines DE, DF, DG, DH, which solves the Problem. *Plate V. Fig.* 50.

SCHOLIUM.

'Tis evident, that if you leave out the Diagonal AC, and the two Lines CF, CH, the Parallelogram ABCD will be found divided into three equal Parts, by the two Lines CE, CG, that divide the given Angle C. But you may do it also, from any other Point given in one Side, as may be seen in the following Problem.

PROBLEM IV.

To divide a Parallelogram into three equal Parts, by two right Lines drawn from a given Point in a Side.

TO divide the Parallelogram ABCD, into three equal Parts, by two right Lines drawn from the Point E given in the Side AB. Divide the Side AB, into three equal Parts, at the Points F, G, thro' which draw FH, GI parallel to the other Side AD, which bisect in the Points K, L, and thro' them draw from the given Point E, the right Lines EM, EN, and they will divide the Parallelogram ABCD into three equal Parts, the Triangle MEN, and the two Trapezoids AEMD, BENC, so that each of these three Planes shall be equal to a third Part of the Parallelogram propos'd, ABCD. *Fig.* 51.

DEMONSTRATION.

Plate V. Fig. 51. Becauſe the two Triangles EFK, MHK are equiangular, and have one Side equal to another ſimilarly poſited, namely, the Side KF equal to the Side KH, *by Conſtr.* theſe two Triangles will be alſo equal, by 26. 1. Wherefore if you add ſeparately the Pentagon AFKMD, you will find that the Trapezoid AEMD is equal to the Parallelogram AFHD, *i. e.* by 1. 6. to a third of the Parallelogram ABCD; you may demonſtrate the ſame way, that the Trapezoid BENC is equal to the Parallelogram BCIG, or to a third of the Parallelogram ABCD; whence 'tis eaſy to conclude, that the Triangle MEN is alſo equal to a third of the ſame Parallelogram. *Q. E. D.*

SCHOLIUM.

Fig. 52. 'Tis evident, that when the Line BE is a third part of the Side AB, 'tis only drawing the Line EF, parallel to the Side BC, and you will have the Parallelogram EBCF, equal to a third of the propos'd one ABCD, by 1. 6. whence it follows, that the Parallelogram AEFD is two thirds of it, wherefore if you draw the Diagonal ED, which by 34. 1. will divide it into two equal Parts, the Parallelogram propos d ABCD will be found divided into three equal Parts, by the two Line EF, ED.

Fig. 53. 'Tis evident alſo, that when the given Point is preciſely in the middle of the Side AB, you may eaſily divide the Parallelogram propos'd ABCD, into four equal Parts, by drawing as before, EF parallel to the Side AD, or BC, to obtain the two equal Parallelograms AEFD, EBCF, which you may divide into equal Parts by the Diagonals ED, EC, &c.

PROBLEM V.

To divide a Parallelogram into four equal Parts, by right Lines drawn from a Point within a Parallelogram.

Fig. 54. TO divide the Parallelogram ABCD, into four equal Parts, by drawing right Lines from a Point, which we ſhall find in the Diagonal BD. Biſect the Side AD in E, and find a mean proportional AF, between the Side AD, and its half DE, to draw from the Point F, a parallel FGH, to the Side DB, which will give in the Hiagonal, the Point G, from whence drawing GI, parallel to

to the Side AD, the Parallelogram ABCD will be found *Plate V.* divided into four equal Parts, the Triangles BIG, BHG, *Fig.* 65. and the two Trapezoids AIGD, DGHC; so that each of these four Planes will be a quarter of the Parallelogram ABCD.

DEMONSTRATION.

Because *by Const.* the three Lines AE, AF, AD, are proportional, the Square of the mean AF, or GI, its equal, will be, *by* 17. 6. equal to the Rectangle of the Extreams AE, AD, that is to say, *by* 1. 6 to double the Square AD, because the Height AD is double the Base AE, *by Const.* and because *by* 19. 6. the Triangle ABD is to its Similar one IBG, as the Square of AD, is to the square of IG, half the Square AD; the Triangle BIG also will be half the Triangle ABD, half the Parallelogram ABCD, *by* 34. 1. whence it follows, that the Trapezoid AIGD, and the Triangle GIB, also its equal GHB, are a quarter of the Parallelogram ABCD, and consequently the Trapezoid GHCD is also a quarter of the same Parallelogram ABCD. *Which was to be demonstrated.*

PROBLEM. VI.

To divide a Parallelogram into two equal Parts by a right Line drawn from a given Point within it.

TO divide into two equal Parts the Parallelogram *Fig.* 55. ABCD by drawing a right Line thro' the Point E given within it: Draw thro' the point E the right Line FG parallel to the Side AD, or BC, and having made FH equal to FB, and EI equal to EF, find a fourth Proportional FK, to the three Lines, GI, IE, AH, to obtain the point K, thro' which and the given point E, draw the right Line KL, which will divide the Parallelogram ABCD propos'd into two equal Parts, because it cuts off the two equal Lines BK, DL from each Side, as we shall demonstrate.

DEMONSTRATION.

Because the four Lines GI, IE, AH, FK, are proportional *by Const.* you will find *by compounding*, that the four GE, IE, AH+FK, FK, are proportional: And if instead of the two first terms GE, IE, or EF; you put the two IG, FK, that are in the same Ratio, *by* 4. 6.

 by

by reaſon of the Similitude of the Equiangular Triangles GEL, FEK, you will know that the Ratio of the two Lines LG, FK, is equal to that of theſe two, AH+FK, FK, and conſequently the Line LG, or DG—DL, or AF—DL is equal to AH+FK, or AF—FH+FK, or AF—BK, becauſe FB is equal to FH, *by Conſt.* and the two Trapezoids AKLD, BKLC, having all their Angles and all their Sides equal reſpectively, are equal to one another, and ſo the Line KL divides the Parallelogram propos'd ABCD into two equal Parts. *Q. E. D.*

PROBLEM VII.

To divide a Trapezoid into as many equal Parts, as you pleaſe.

Plate VI. Fig. 56.

TO divide the Trapezoid ABCD for inſtance into three equal Parts. Divide each of the two parallel Sides AB, CD, into three equal Parts at the Points E, F, G, H, and join the right Lines EG, FH, and they will divide the Trapezoid propos'd ABCD into three Trapezoids ACGD, EFHG, FBCH, that are equal, as may be known by drawing the Diagonals AG, EH, FC, by which you will ſee, that the Trapezoids are made up of Triangles equal to one another *by* 38. 1.

LEMMA.

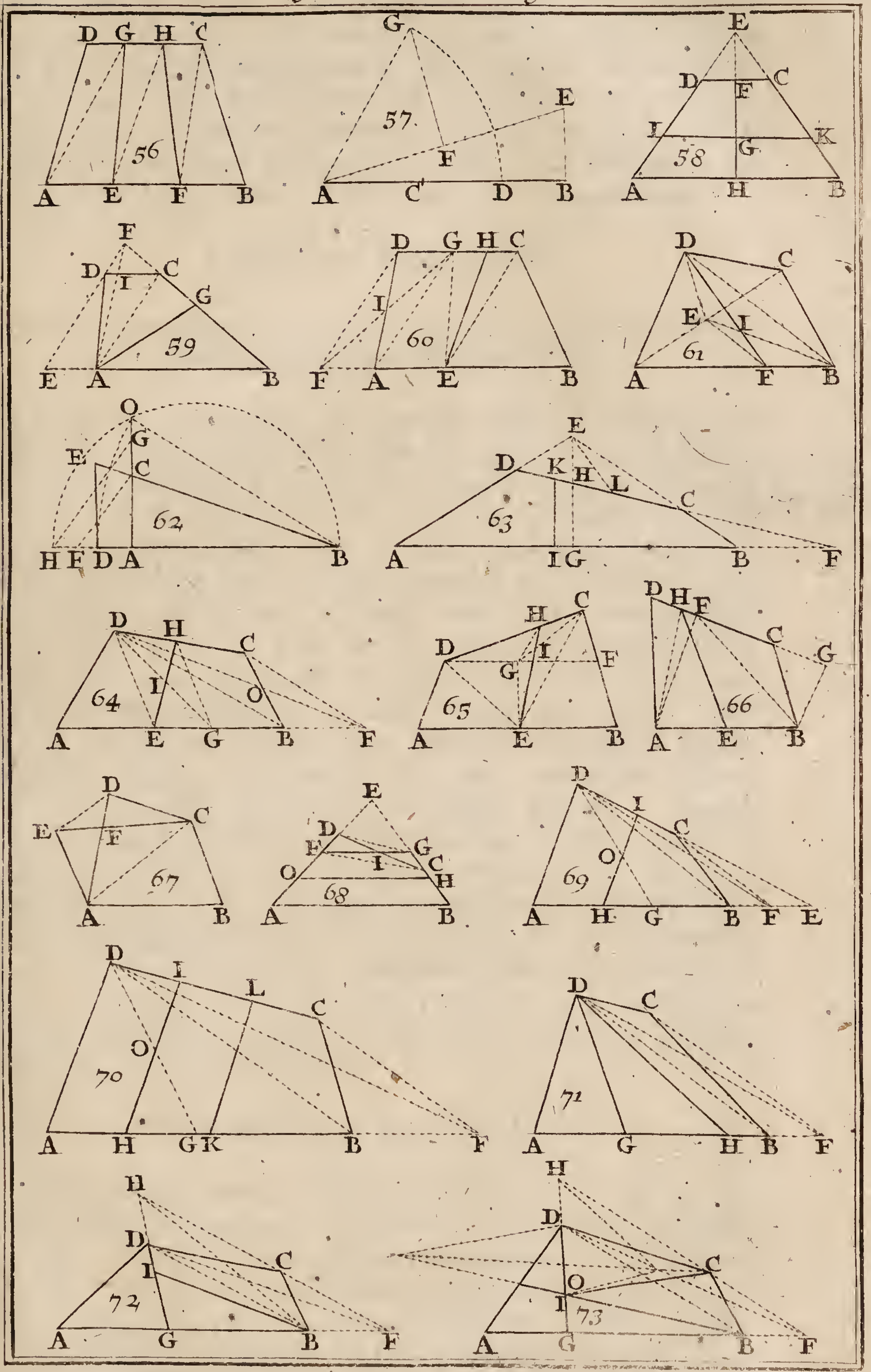
The Line AB being cut in C, to cut it againſt the Point D, between B and C, ſo as that the three Squares AC, AD, AB, may be in Arithmetic Proportion.

Fig. 57.

TO find the Point D between B and C, ſo as that the three Squares AC, AD, AB, may be in *Arithmetic Proportion*, that is to ſay, that the ſum of the two Extreams, AB, AC, be double the Mean AD; having erected the Perpendicular BE equal to AC, upon AB, and biſected the Line AE, at the Point F, draw from the Point F, the Line FG perpendicular and equal to the Line AF, half of AE, and make AD equal to AG, and the three Squares AC, AD, AB, will be in *Arithmetic Proportion.*

DEMONSTRATION.

Becauſe the Square AE is *by* 47. 1. equal to the ſum of the Squares AB, BE, or AC, the Square AF of half AE,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AE, will be *by* 20. 6. a quarter of the ſum of the two Squares AB, AC: and becauſe the Square AG or AD is *by* 47. 1. double the Square of AF, becauſe 'tis equal to the ſum of the two equals AF, FG, the ſame Square AD will be half the Sum of the ſquares AC, AD, and thus the three ſquares AC, AD, AB will be in Arithmetic Proportion. *Which was to be demonſtrated.*

PROBLEM VIII.

To divide an Iſoſceles Trapezoid into four equal Parts by two Lines Perpendicular to one another.

I Call that an Iſoſceles Trapezoid, whoſe two Sides that are not Parallel, are equal to one another, as ABCD, whoſe two Sides AD, BC are ſuppoſed equal, but not parallel. To divide it into four equal Parts, by two Lines Perpendicular to one another, produce the two equal Sides AD, BC, till they interſect one another in the Point E, and having biſected one of the two parallel Sides AB, CD, as AB, in H, draw the right Line EH, that will biſect alſo the other Side CD in F, and will be Perpendicular to each of the two Sides, AB, CD, by reaſon of the two Iſoſceles Triangles ABE, CDE, which it divides alſo into two equal Parts, as well as the Trapezium propoſed ABCD. Laſtly cut the Line EH, *by the preceding Lemma* which is already cut in F, at the Point G, ſo as that the three ſquares EF, EG, EH be in Arithmetic Proportion, draw through the Point G the right Line IK perpendicular to the Line FH, and theſe two Perpendiculars IK, FH, will divide the Trapeziod propos'd ABCD into four equal Parts, which are the four Trapezoids AHGI, BHGK, IGFD, KGFC. *Plate 6. Fig 58.*

DEMONSTRATION.

Becauſe the three ſquares EF, EG, EH are in Arithmetic Proportion, *by Conſtr.* 'tis eaſie to conclude, *by* 19. 6. that the three Iſoſceles Triangles ABE, IKE, DCE, are alſo in Arithmetic Proportion, that is to ſay, the Sum of the two Triangles ABE, CDE, is double the Triangle IKE; wherefore if from each Side you take the double of the Triangle CDE, you will find that the Trapezoid ABCD is double the Trazoid IKCD, and conſequently the Trapezoid IKCD is equal to the Trapezoid ABKI; and ſince each of the

the two Trapezoids is divided into two equal Parts by the Line FH, *by Problem* 7. It follows that the four Trapezoids AHGI, BHGK, IGFD, HGFC, are equal to one another. *Which was to be demonstrated.*

PROBLEM IX.

To divide a Trapezoid into two equal Parts, by a right Line drawn from one of its Angles.

Plate. 6. *Fig.* 59. TO divide the Trapezoid ABCD into two equal Parts by a right Line drawn from a given Angle A, produce the Side adjacent AB, parallel to the other Side CD to E, that the Line AE may be equal to the other Side CD, and draw from the Point E, to the Point D, the right Line ED, which being produced will meet the other Side also CB produced in some Point, for Instance F; afterwards having bisected the Line BF, in the Point G, draw the right Line AG, and it will divide the Trapezoid propos'd ABCD, into two equal Parts, so that the Triangle ABG will be half the Trapezoid.

DEMONSTRATION.

If you join the Diagonal AC, and the right Line AF, you know *by* 33. 1. that the Diagonal AC is parallel to the Line EF, by reason of the two Parallel and equal Lines AE, DC, *by Constr. and by* 31. 1, that the Triangles AFC, and ADC, are equal: Wherefore if from each you take the Triangle AIC, where will remain the Triangle AID equal to the common Triangle FIC, and if you add each of the two Triangles to the Trapezium ABCI, you will see that the Trapezoid ABCD is equal to the Triangle ABF; and because *by* 6. 1. the Triangle ABG is half the Triangle ABF, the Base BG being half the Base BF, *by Construct.* it will be also half the Trapezoid ABCD propos'd. *Which was to be demonstrated.*

PROBLEM X.

To divide a Trapezoid into two equal Parts, by a right Line drawn from a given Point in the Base.

Fig. 60. I Call *the Base of a Trapezoid*, one of the two Parallel Sides, as AB, of the Trapezoid ABCD; if therefore you have a Point E given in that Side AB, from whence you

you are to divide the Trapezoid ABCD into two equal Parts, having taken in the Base AB, the Line EF, equal to the Line EB, draw through the Point A, the right Line AG parallel to the right Line FD, and having bisected the Line CG, in the Point H, draw the right Line EH, and that will divide the Trapezoid proposed ABCD, into two equal Parts; the two Trapezoids AEHD, BEHC. *Plate* 6. *Fig.* 60.

DEMONSTRATION.

If you join the Lines EG, FG, EC, you will find *by* 37. 1. that, because the Lines AG, FD are parallel *by Constr.* The two Triangles ADG, AFG are also equal, wherefore if from each, you take away the common Triangle AIG, there will remain the Triangle AIF equal to the Triangle DIG, each of which being separately added to the Trapezium AIGE will show you, that the Trapezoid ADGE is equal to the Triangle EGF, and consequently to the Triangle ECB its equal *by* 38, because the two Bases EB, EF are equal *by Constr.* and the two Triangles GEH, CEH are also equal *by* 38. 1. by reason of their equal Bases, GH, CH, *by Constr.* 'Tis easie to conclude, that the whole Trapezoid AEHD, is equal to the whole Trapezoid BEHC. *Which was to be demonstrated.*

PROBLEM XI.

To divide a Trapezium into two equal Parts, by a right Line drawn from a given Angle.

TO divide the Trapezium ABCD into two equal Parts, by a right Line drawn from the Angle D, Bisect the Diagonal AC, opposite to the given Angle D, at the Point E, through which draw EF parallel to the other Diagonal BD, and join the right Line DF, and that will divide the Trapezium propos'd ABCD into two equal Parts, the Triangle ADF, and the Trapezium BCDF. *Fig.* 61.

DEMONSTRATION.

Because the two Lines EA, EC are equal *by Constr.* the two Triangles EDA, EDC will be equal, as well as those two EBA, EBC, *by* 32. 1. which makes the two Trapeziums ADEB, CDEB equal also; and because the Trapezium ADEB is equal to the Triangle ADF,

and

and the Trapezium CDEF equal to the Trapezium BCDF, the Triangles DIE, BIF, being equal, as may be seen by taking away the Triangle DIB from the two Triangles DEB, DFB, which are equal, *by* 37. 1. by reason of the Parallels BD, EF *by Constr.* it follows, that the Triangle ADF is equal to the Trapezium BCDF. *Which was to be demonstrated*; See *Prob.* 19.

SCHOLIUM.

Plate 6. *Fig.* 61. You see by this Figure, you may easily divide a Trapezium into two equal Parts, by two right Lines drawn from the two opposite Angles given; as if the two Angles B, D, were given, bisect at the Point E, the Diagonal AC, that passes thro' the two other Angles A and C, and join the the right Lines EB, ED, and they will divide the Trapezium propos'd ABCD into two equal Parts, *viz.* the two Trapeziums ABED, BCDE, because *by* 38. 1. the two Triangles EBA, EBC, are equal, as well as those two EDA, EDC.

LEMMA.

Having a Triangle ABC right-angled in A, to find in the side AB produced, the Point D, from whence drawing to the other side AC, the Parallel DE, terminated in E, by the Hypotenuse BC produc'd; the Trapezium ACED, may be equal to the Square of the given Line AF.

Fig. 62. If you find a fourth Proportional AH, to the Lines, AC, AF, and AG double AF, and a mean proportional BD, between AB, and BH the Sum of these two AB, AH, you will find the Point D sought for: So that if from the Point D you draw to the Side AC, the Parallel DE meeting the Hypotenuse BC produc'd in E, the Trapezoid ACED will be equal to the square of double the Line given AF.

DEMONSTRATION.

Because the Rectangled Triangles ABC, DBE are equiangular, AC, DE being Parallel, *by Constr.* they are to one another, as the squares of their Homologous Sides AB, DB, *by* 19. 6. wherefore if instead of the square DB, you put the Rectangle under AB, BH, that is equal to it, *by* 17. 6. by reason of the three Proportionals AB, BD, BH, *by Constr.* you will find the Triangle

angle ABC is to the Triangle DBE, as the ſquare AB, is to the Rectangle under AB, BH, or *by* 1. 6. as the Baſe AB, to the Baſe BH, by reaſon of the common Height AB, and *by Converſion* the Triangle ABC is to the Trapezium ACED, as AB, is to AH; and if to the two laſt terms AB, AH, conſider'd as Baſes, you give the common Height AC, you know *by* 1. 6. that the Triangle ABC is to the Trapezoid ACED, as the Rectangle under AB, AC to the Rectangle under AH, AC, that is to ſay, to double the ſquare AF, by reaſon of the four Proportionals, AC, AF, AF, AH, *by Conſtruction* Where you ſee that the firſt Antecedent, namely the Triangle ABC being *by* 41. 1. half the ſecond Antecedent, which is the Rectangle under AB, AC; alſo the firſt conſequent, namely the Trapezoid ACED, is equal to half the ſecond conſequent, which is double the ſquare AF, and conſequently is equal to the ſquare AF. *Which was to be Demonſtrated.* See *Problem* 19.

PROBLEM XII.

To divide a Trapezium, that has two equal Angles, into two equal Parts by a Line Perpendicular to a Side between the two given Angles.

TO divide into two equal Parts the Trapezium ABCD, whoſe two Angles A and B are equal, by a right Line Perpendicular to the interjacent Side AB, produce AB, DC, till they meet in F, alſo AD, BC till they interſect in E, and then the Triangle AEB will be an Iſoſceles, *by* 6. 1. becauſe the Angles A, B, are equal *by Supp.* Wherefore if you draw from the Angle E, the Perpendicular EG, to the oppoſite Side AB, the two Rectangled Triangles AGE, BGE, will be equal. Whence 'tis eaſie to conclude, that the Trapezium AGHD is greater than the Trapezium BGHC, becauſe the Triangle DEH is leſs than the Triangle CEH, by the whole Triangle CEL, ſuppoſing the Baſe HL equal to the Baſe HD, and conſequently the Triangle HEL equal to the Triangle HED, *by* 38. 1. Wherefore, if you reduce half the Triangle CEL into a ſquare, and make the Trapezium GHKI equal to the ſquare *by preced. Lemm.* The Perpendicular IK will divide the Trapezium propos'd ABCD into two equal Parts. *Which was to be done.* *Fig.* 63.

PROB-

PROBLEM XIII.

To divide a Trapezium into two equal Parts, by a right Line Perpendicular to two Parallel Sides.

Plate 4. *Fig.* 36.

TO divide the Trapezoid ABEF into two equal Parts, by a right Line Perpendicular to the two Parallel Sides AB, EF. Let fall from the two Points E, F, the Lines EH, EK, perpendicular to the Base AB, and take in the Base AB, the part AO equal to a quarter of the Sum of once BK, twice HK, and thrice AH, to obtain the Point C, from whence erect the Perpendicular OD, to AB, and that will divide the Trapezoid propos'd ABFE into two equal Parts, so that the Area of the Trapezoid AODE will be precisely half that of the Trapezoid ABFE.

DEMONSTRATION.

Put *a* for AH, *b* for HK, *c* for BK, and *p* for the common Height DO, or HE, or KF, and you will have $\frac{1}{4}c+\frac{1}{2}b+\frac{1}{4}a$ for AO, $\frac{1}{4}b+\frac{1}{4}c-\frac{1}{4}a$ for HO, and the Area of the Triangle AHE will be $\frac{1}{2}ap$, that of the Rectangle HODE, will be $\frac{1}{4}cp+\frac{1}{2}bp-\frac{1}{4}ap$, that of the Rectangle HKFE will be bp, that of the Triangle BKF, $\frac{1}{2}cp$, so that the Area of the Trapeziod ABFE, will be $\frac{1}{2}ap+bp+\frac{1}{2}cp$, and that of the Trapezoid AODE, will be $\frac{1}{2}ap+\frac{1}{2}bp+\frac{1}{4}cp$, which is exactly half that of the Trapezoid ABFE. *Which was to be Demonstrated.*

PROB-

PROBLEM XIV.

To divide a Trapezoid into two equal Parts, by a right Line drawn from a Point given in a Side.

TO divide into two equal Parts the Trapezium ABCD, by a right Line drawn from the Point E, given in the Side AB. Join the right Lines DE, DB, and draw to the Diagonal DB thro' the Point C, the Parallel CF, which cuts the Side AB produced in the Point F, through which and the Point D, draw the right Line DF, which will make the Triangle ADF equal to the Trapezium proposed ABCD, by reason of the two equal Triangles BOF, COD, as you will know by taking away the two Triangles DCB, DFB, which are equal *by* 37. 1. because BD, CF are parallel *by Constr.* and the Triangle BOD common. Wherefore if you bisect the Base AF, in the Point G, and draw the right Line DG, the Triangle ADG will be *by* 1. 6. half the Triangle ADF, or of the Trapezium ABCD. Lastly, draw through the Point G, the right Line GH, parallel to the right Line DE, and join the Line EH, and that will divide the Trapezium ABCD proposed into two equal Parts, the two Trapeziums AEHD, BEHC, so that the first of these Planes, namely AEHD, will be half the Trapezium ABCD, or equal to the Triangle ADG. *Plate* 6. *Fig.* 64.

DEMONSTRATION.

Because the two Lines DE, GH are parallel, *by Constr.* the two Triangle GDH, GEH will be equal, *by* 38. 1. wherefore, if from each you take the common Triangle GHI, there will remain the Triangle DIH, equal to the Triangle GIE, each of which being added separately to the Triangles AEID, you will have the Trapezium AEHD equal to the Triangle ADG, and consequently to half the Trapezium proposed ABCD. *Which was to be demonstrated.*

SCHOLIUM.

There will be no need of reducing the Trapezium proposed ABCD into a Triangle, when the Point E is given in the middle of the Side AB, because you may divide it

Plate VI. it another way into two equal Parts, by a Line drawn from that middle Point E, as you shall see immediately.

Fig. 65. Having join'd the right Line CE, and drawn from the Angle D, the Line DF parallel to the Side AB, bisect that Line DF in the Point G, thro' which draw GH parallel to the Line CE, and that will give the Point H in the Side CD, thro' which and the Point given E, draw the right Line EH, and it will divide the Trapezium proposed ABCD into two equal Parts, the two Trapeziums AEHD, BEHC.

DEMONSTRATION.

Because the two Trapezoids AEGD, BEGF are equal *by Probl.* 7. and the two Triangles GDC, GCF, are also equal by 38. 1. it follows, that the Pentagon AEGCD is equal to the Trapezium BEGC: And because the two Triangles EIG, CIH, are also equal, as may be known by taking away the two Triangles EGC, EHC, which are equal by 38. 1. by reason of the two Parallels CE, GH, and the common Triangle EIC; it follows that the Trapezium AEHD is equal to the Trapezium BEHC; *Which was to be demonstrated.*

Fig. 66. Or draw from the two Extremities A, B, of the Side AB, the right Lines AF, BG perpendicular to the Side CD, and find a fourth proportional DH, to the Sum of two Perpendiculars AF, BG, the Perpendicular BG and Side CD, to obtain the Point H in the Side CD, thro' which and the Point given E, draw the right Line EH, and it will divide the Trapezium proposed ABCD into two equal Parts, which are the Trapeziums, AEHD, BEHC.

DEMONSTRATION.

Because the four Lines AF+BG, BG, CD, DH, are proportional *by Constr.*. You know *by Conversion*, that the four AF, BG, CH, DH are also proportional, and by 16. 6. the Rectangle of the two Extremes AF, DH, or by 41. 1. the double of the Triangle AHD, is equal to the Rectangle of the Means BG, CH, or the double of the Triangle BHC, and consequently the two Triangles AHD, BHC, are equal, to which if you add the two Triangles AHE, BHE, which are also equal by 38. 1. by reason of the two equal Bases AE, BE, *by Supp.* you may find that the two Trapeziums AEHD

AEHD, BEHD, BEH are alſo equal. *Which was to be Demonſtrated.*

Or again, if you will reduce the Trapezium propoſed ABCD into a Triangle, you may find in the Side CD produced, which is oppoſite to the Point given E, the Baſe of the Triangle, whoſe Vertex ſhall be at the Point given E, by drawing from the Point A a parallel to the Line ED, and in like manner from the Point B, a Parallel to the Line EC, and by biſecting that Baſe at the Point H, &c. *Plate VI. Fig.* 65.

LEMMA.

To reduce a Trapezium into a Trapezoid.

TO reduce the Trapezium ABCD into a Trapezoid. Draw from the Point A to the Side BC the parallel AE, and from the Point D, to the Diagonal AC, another parallel DE, and thro' the Point E, where the two Parallels AE, DE interſect, draw to the Point C, the right Line CE, and the Trapezoid ABCE ſhall be equal to the Trapezium propoſed ABCD.

DEMONSTRATION.

Becauſe the two Lines AC, DE are parallel, *by Conſtr.* the two Triangles ADC, AEC are equal, by 38. 1. Wherefore if from each you take away, what is common AFC, there will remain the Triangle EFA equal to the Triangle CFD, and if you add each of theſe two equal Triangles to the Trapezium ABCF, you may find that the Trapezium propos'd ABCD is equal to the Trapezoid ABCE. *Which was to be Demonſtrated.*

SCHOLIUM.

Becauſe here the two Sides are changed, the Reſolution tho' good, as you may ſee by the Demonſtration, yet is not ſo proper for our Deſign: Wherefore I ſhall here give another without changing a Side.

To turn the Trapezium ABCD, therefore, into a Trapezoid without changing the Side AB, or Poſition of the Lines AD, BC. Produce the Sides AD, BC, till they meet in a point, as E, and cut the Side AE in F, ſo as that the Side BE may be to the Side AE, as the Rectangle of the Lines EC, ED, to the ſquare EF, to gain the point F, which is eaſily done, then drawing *Fig.* 68.

Plate VI. Fig. 68. through F, a Line FG parallel to the Side AB, you will have the Trapezoid ABGF equal to the Trapezium propos'd ABCD.

DEMONSTRATION.

Join the Lines FC, DG, and you will soon see that the Triangle ECD is equal to the Triangle EGF, and consequently the Triangle CIG, equal to the Triangle DIF, after this manner.

Because the Line BE is to the Line AE, as the Rectangle of the Line EC, ED, is to the square EF, *by Constr.* If in the place of the two first Terms BE, AE, you put the two EG, EF, that are in the same ratio *by* 46. by reason of the two Parallels AB, FG, *by Constr.* You know that the Ratio of the Lines EG, EF, is equal to that of the Rectangle under the Lines EC, ED, to the square EF, and giving to the two first Terms EG, EF, the Line EF for a common Height, you may find *by* 1. 6. that the Ratio of the Rectangle under the Lines EF, EG, to the square EF, is the same with that of the Rectangle under the Lines EC, ED to the same square EF, and consequently the Rectangle of the Lines EF, EG, is equal to the Rectangle of the Lines EC, ED. Wherefore *by* 16. 6. the four Lines, EC, EF, EG, ED are proportional, and by 15. 6. the two Triangles ECD, EFG, will be equal; from whence taking the common Trapezium EGID, there will remain the Triangle CIG, equal to the Triangle DIF, and if to each of the two equal Triangles you add separately the Pentagon ABCIF, you will find that the Trapezium proposed ABCD is equal to the Trapezoid ABGF. *Which was to be Demonstraed.*

PROBLEM. XV.

To divide a Trapezoid into two equal Parts by a right Line parallel to its parallel Sides.

Fig. 68. TO divide the Trapezoid ABGF into two equal Parts, by a right Line parallel to the two parallel Sides AB, GF. Produce the two other Sides AF, BG, till they meet in a Point, for instance E, and cut the Side AE, in O, so as that the square of EO may be equal to half the Sum of the two squares AE, EF, to obtain

tain the Point O, through which draw, to the Baſe AB, the Parallel OH, and that will divide the Trapezoid proposed ABGF, into two equal Parts. *Plate VI. Fig.* 68.

DEMONSTRATION.

Becauſe the Sum of the two Squares AE, EF, is double the Square EO, *by Conſtr.* the three ſquares AE, EO, EF, will be in Arithmetic Proportion, as well as the three ſimilar Triangles ABE, OHE, FGE, *by* 19. 6. Wherefore the exceſs of the firſt above the ſecond, namely the Trapezoid ABHO, will be equal to the exceſs of the ſecond above the third, or to the Trapezoid OHGF. *Which was to be Demonſtrated.*

COROLLARY.

From this Problem may be drawn the method of dividing a Trapezium, as ABCD, into two equal Parts, by a right Line parallel to a Side, for inſtance AB, namely by deſcribing *by the preced. Lem.* on the Side AB, the Trapezoid ABGF equal to the Trapezium propoſed ABCD, and compleating the reſt as has been taught. But this Diviſion may be made much eaſier another way, as we ſhall ſhew in the following

PROBLEM XVI.

To divide a Trapezium into two equal Parts, by a right Line parallel to a given Side.

TO divide into two equal Parts the Trapezium ABCD, by a right Line parallel to a Side AD; produce the other two Sides AB, CD, till they meet in a Point, for inſtance E, and having drawn from the Angle C, the right Line CF parallel to the Diagonal BD, biſect the Line AF, in the Point G, and find a mean proportional EH between the two Lines AE, EG, to obtain the Point H, thro' which drawing the Line HI, parallel to the Side AD, it will divide the Trapezium propoſed ABCD into two equal Parts. *Fig.* 69.

DEMONSTRATION.

By joyning the right Lines DF, DG, you will find as in *Prob.* 14. that the Triangle ADF is equal to the Trapezium ABCD, and *by* 1. 6. that the Triangle ADG is

Plate VI. Fig. 69. is half of the Triangle ADF, or of the Trapezium ABCD, becauſe the Baſe AG is half the Baſe AF *by Conſtr.* and becauſe the three Lines AE, EH, EG, are proportional *by Conſtr.* you may find by *Corol.* 20. 6. that the firſt AE, is to the third EG, as the Square of the firſt AE, to the Square of the ſecond EH, or by 19. 6. as the Triangle AED, to its ſimilar one HEI, by reaſon of the two Parallels AD, HI, *by Conſtr.* and as the Ratio of the ſame two Lines AE, EG is alſo equal to that of the two Triangles AED, GED, it follows that the Triangle AED is to the Triangle GED, as the ſame Triangle AED is to the Triangle HEI, and conſequently the two Triangles GED, EHI, are equal: Wherefore if from each you take the common Triangle BCE, there will remain the Trapezium BCDG equal to the Trapezium BCHI, and if from each of theſe two equal Planes, you take away the common Pentagon BCIOG, there will remain the Triangle DOI, equal to the Triangle GOH, each of which being added to the Trapezium ADOH, you may ſee that the Trapezium AHID is equal to the Triangle ADG, or to half the Trapezium propoſed ABCD, which is thus divided into two equal Parts, by a right Line HI. *Which was to be demonſtrated.*

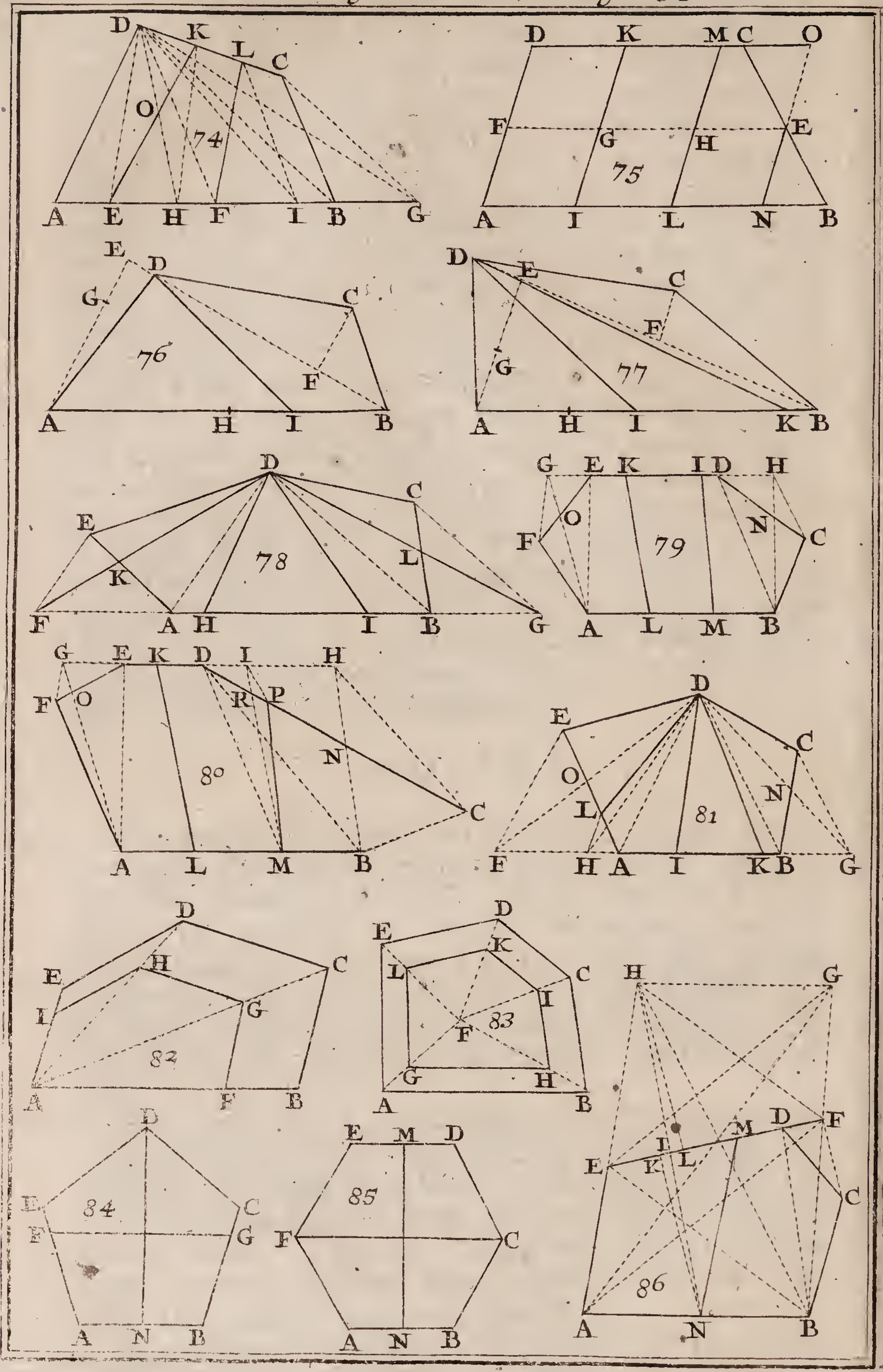
SCHOLIUM.

Fig. 70. You may Demonſtrate after the ſame manner, that if you make the Line AG equal to a third part of the Line AF, the Trapezium AHID, will be equal to a third part of the propoſed one ABCD. Wherefore if you divide BCIH, into two equal Parts by a Line KL parallel to the Side HI, the Trapezium propoſed ABCD, will be found divided into three equal Parts by the two right Lines HI, KL, parallel to one another and to the Side AD.

Fig. 71. 'Tis evident by 38. 1. that becauſe the Triangle ADF, is equal to the Trapezium ABCD, if you divide the Baſe AF into three equal Parts, at the Points G, H, and from the Angle D, you draw the two Lines DG, DH, to the Points G, H, the Trapezium ABCD, will be found divided into three equal Parts by the two Lines DG, DH, drawn from the Angle D.

Fig. 72. 'Tis evident alſo, that if you produce the Lines DG, CF, till they cut one another in a Point, for inſtance H, and then biſect the whole Line GH, with the right Line BI, the Trapezium ABCD will be found divided into three equal Parts, by the two Lines DG, BI, drawn from the

Geometry Plate 7. Page 53.



the two Angles D,B,by reaſon of the Triangle ADG equal to a third part of the Triangle ADF, or of the Trapezium ABCD, and of the Triangle GBI, equal to half the Triangle GBH, or Trapezium GBCD, which is equal to two Thirds of the propoſed ABCD, &c.

Laſtly, 'Tis evident, that if you take the Line AG equal to a fourth part of the Line AF, and the Line GI, equal to a fourth of the Line GH, and divide the Trapezium BCDI into two equal parts by the right Line CO, the Trapezium propos'd ABCD, will be found divided into four equal Parts by the three Lines BI, CO, DG, drawn from the three Angles B, C, D. *Plate VI. Fig.* 73.

PROBLEM XVII.

To divide a Trapezium into three equal Parts, by right Lines drawn from two given Points in a Side.

TO divide the Trapezium ABCD into three equal Parts by two right Lines drawn from the Points E, F, given in the Side AB. Draw from the oppoſite Angle C, a Parallel CG to the Diagonal BD, and thro' the Point G, where it cuts the Side AB, draw to the other oppoſite Angle D, the right Line DG, that ſhall make the Triangle ADG equal to the Trapezium ABCD, by *Probl.* 14. Wherefore if you divide the Baſe AG into three equal Parts at the Points H, I, and draw the Lines DH, DI, you will find by 1. 6. that each of the three Triangles ADH, HDI, IDG, is a third of the Triangle ADG, or of the Trapezium propoſed ABCD. Draw thro' the Point H, the right Line HK parallel to the Line DE, and through the Point I, the right Line IL, parallel to the Line DF, and join the right Lines EK, FL, which will divide the Trapezium propoſed ABCD into three equal Parts, which are the three Trapeziums AK, EL, FC, ſo that each of theſe three Planes are equal to a third of the Trapezium ABCD. *Plate VII. Fig.* 74.

DEMONSTRATION.

Becauſe the two Lines DE, HK, are parallel *by Conſtr.* the two Triangles HDK, HEK are equal, by 38. 1. Wherefore if from each you take away the common Triangle HOK, there will remain the Triangle DOK equal to the Triangle EOH, and if to each of theſe two equal Triangles you add the Trapezium AEOD, you will ſee,

that the Trapezium AEKD is equal to the Triangle ADH that is, to a third of the Trapezium ABCD. After the ſame manner it may be demonſtrated, that the Trapezium AFLD is equal to the correſponding Triangle ADI, that is to ſay, to two thirds of the Trapezium ABCD. Whence 'tis eaſie to conclude, that each of the two Trapezium's EL, FC, is a third of the ſame Trapezium ABCD. *Which was to be Demonſtrated.*

PROBLEM XVIII.

To divide a Trapezoid into as many equal Parts as you pleaſe, by Lines parallel to one of the two Sides, that are not parallel.

Plate VII. Fig. 75. TO divide the Trapezoid ABCD, for inſtance into three equal Parts, by Lines parallel to the Side AD. Biſect the oppoſite ſide BC, in the point E, and draw thro' that point E, the Line EF, parallel to the Baſe AB, which divide into three equal Parts at the points G, H, thro' which and the point E, draw to the ſide AD, the three parallels IK, LM, NO, and they will divide the Trapezoid propos'd ABCD into three equal Parts, which are the two Parallelograms AK, IM, and the Trapezoid LC, ſo that each of theſe three Planes will be a third of the Trapezoid ABCD.

DEMONSTRATION.

Becauſe the two Triangles BEN, CEO, are equiangular, and have a ſide EB, equal to a ſide EC, by *Conſtr.* they will be equal *by* 26. 1. Whence 'tis eaſie to conclude, that the Trapezoid BCML is equal to the Parallelogram MLNO, and the whole Trapezoid ABCD, equal to the whole Parallelogram ANOD: and as each of theſe three Parallelograms AK, IM, LO, is a third of the whole Parallelogram, they will alſo be a third of the Trapezoid propos'd ABCD. *Which was to be Demonſtrated.*

PROBLEM XIX.

To divide a Trapezium into two Parts of a given Ratio.

Fig. 76. TO divide the Trapezium ABCD into two Parts, whoſe Ratio let be equal to that of the two Lines AH, HB. Having let fall from the two Angles A, C, the

the right Lines AE, CF, perpendicular to the Diagonal BD, find a fourth proportional AG, to the ſum of the two given Lines AH, HB, to the ſum of the two Perpendiculars AE, CF, and to the right Line AH; and another fourth proportional AI, to the three Lines AE, AG, AB, to gain the point I, thro' which and the Point D, draw the right Line DI, and it will divide the Trapezium propos'd ABCD, into two Parts ADI, BCDI, whoſe Ratio is equal to that of the two given Lines AH, HB. *Plate VII. Fig.* 76.

DEMONSTRATION.

Becauſe *by* 1. 6. the Triangle ADI, is to the Triangle ADB, as the Baſe AI, to the Baſe AB, or *by Conſtr.* as AG, to AE: and for the ſame reaſon becauſe the Triangle ADB is to the Triangle CDB, as the Height AE, is to the height CF, and *by Compoſition* the Trapezium ABCD, is to the Triangle ADB, as the ſum of the Heights AE, CF, is to the Height AE, the Trapezium ABCD, will be to the Triangle ADI, as the ſum of the Heights AE, CF, to the Line AG, or *by Conſtr.* as AB, to AH, and *by Diviſion* the Trapezium BCDI, will be to the Triangle ADI, as BH, to AH. *Which was to be Demonſtrated.*

COROLLARY.

A Method different from that in *Probl. II.* may be drawn from this Problem, for dividing into two equal Parts a Trapezium by a right Line drawn from a given Angle, for inſtance D, by biſecting the oppoſite ſide AB in the point H, and compleating the reſt, as has been ſhewn.

If you would divide the Trapezium ABCD into three equal Parts otherwiſe than by the Scholium of *Probl.* 16. make AH equal to a third of the oppoſite ſide AB, to obtain the Triangle ADI equal to a third of the Trapezium ABCD; and after the ſame manner the Trapezium BCDI into two equal Parts by the right Line DK, &c. *Fig.* 77.

PROBLEM. XX.

To cut off from a given Trapezium a Figure equal to a given Figure.

Plate VII. Fig. 76. REduce into a Triangle the given Figure, and the Trapezium propos'd ABCD into a Triangle, and then cut off from it a Triangle ADI equal to the propos'd one, and the Problem is solved.

Or find the Ratio of the Trapezium propos'd ABCD to the given Figure, which is very easie to be done, then by the help of the preceding Problem divide the Trapezium propos'd ABCD, in that Ratio.

CHAP. III.

The Division of Polygones.

THE Division of Polygones will be made easie by that of Triangles and Quadrilaterals. Because a Polygone may easily be reduc'd into a Quadrilateral or Triangle, by what shall be said in this

LEMMA.

To reduce a Polygone propos'd into a Triangle.

Fig. 78. TO reduce the Pentagon ABCDE into a Triangle, so that one of the Angles of the Triangle shall be for instance in D. Draw from the point D, to the point A, the Diagonal AD, and draw a Parallel to it EF, thro' the point E, which will meet the Side AB in the point F, thro' which and the point given D, draw the right Line DF, and it shall make the Quadrilateral BCDF equal to the Pentagon propos'd ABCDE, because the Triangles AKF, DKE are equal, as may be seen by taking away the Triangle AKD, from each of the two Triangles AED, AFD, that are equal, *by* 37, 1. AD and EF being parallel. All that remains is to reduce the Trapezium BCDF into a Triangle, which may be done by drawing a Parallel CG, as nigh as may be to the Diagonal BD, thro' the Angle C, and joining the right Line DG. For then, the two Triangles CLD, BLG, being equal, you will have the Triangle FDG equal to the

the Quadrilateral BCDF, and consequently to the Pentagon propos'd ABCDE. *Which was to be done.*

From hence 'tis plain you may easily reduce what Polygon you please, into a Triangle, because you may at any time reduce it into a Figure that has one Side less, and that next Figure again into another, that has one side less, and so on till it becomes a Triangle. Thus in this Example, we have made a Figure of four Sides BCDF, out of one of five, ABCDE, and one of three or a Triangle FDG, out of that.

PROBLEM I.

To divide a given Polygon into three equal Parts, by two right Lines drawn from a given Angle.

TO divide the Pentagon ABCDE, into three equal Parts, by two Lines drawn from a given Angle D. Reduce by the help of the preceding Lemma, the Polygon proposed ABCDE, into a Triangle FDG, having its Vertex the given Point D, and having divided the Base FG into three equal Parts, at the Points H, I, draw from the Point given D, through the two Points H, I, the right Lines DH, DI, and they will divide the Pentagon proposed into three equal Parts, the Triangle HDI, and the two Trapeziums AEDH, BCDI, so that each of these three Planes will be equal to a third part of the Polygon proposed ABCDE. *Plate VII. Fig.* 78.

DEMONSTRATION.

'Tis evident, *by* 1. 6. that each of these Triangles FDH, HDI, IDG is a third of the Triangle FDG, for their Bases FH, HI, IG, are each a third part of the Base FG, *by Constr.* Wherefore each of them will be a third also of the Polygon propos'd ABCDE ; and as the Trapezium AEDH, is equal to the Triangle FDH, because the Triangle AKF is equal to the Triangle DKE, as may be seen in the preceding Lemma, and in like manner the Trapezium BCDI, is equal to the Triangle IDG, the Triangle BLG being equal to the Triangle CLD, hence it follows, that the Polygon proposed ABCDE is divided into three equal Parts, by the two Lines DH, DI. *Which was to be Demonstrated.*

SCHOLIUM.

Plate VII. Fig. 79. When a Polygon has two Sides parallel, as the Hexagon ABCDEF, whose two Sides AB, DE, are parallel, you may divide it into three equal Parts, and them too, if you please, by reducing it into a Quadrilateral ABHG, which will be a Trapezoid, which consequently you may divide *by Prob.* 7. *Chap.* 2. namely, by dividing each of the two parallel Sides AB, GH, into three equal Parts, at the Points L, M, I, K, and by joining the right Lines LK, MI, which will divide the Hexagon proposed into three equal Parts, the Trapezoid LMIK, and the two Pentagons, ALKEF, BCDIM, that are equal to the two Trapezoids ALKG, BMIH, the Triangle AOF, being equal to the Triangle GOE, and the Triangle BNC to the Triangle DNH, *&c.*

Fig. 80. If one of the two Points K, I, for instance I, fall without the Side DE, the Trapezoid LMIK, tho' equal to a third part of the Hexagon ABCDEF cannot be taken for one of the three Parts sought for. In this Case a Line IP must be drawn parallel to the Line MD, thro' the Point I, which will give in the Side CD, the Point P, through which and the Point M, draw the right Line MP, and instead of the Trapezoid LMIK, take the Pentagon LMPDK, equal to it, because the Triangles MRP, DRI, are equal as may be seen, by taking away the Triangle DRM, from the two Triangles DIM, DPM, that are equal also *by* 37. 1. MD, PI, being parallel *by Construct.*

PROBLEM II.

To divide a given Polygon into as many equal Parts as you please, by right Lines drawn from a given Angle.

Fig. 81. TO divide the Pentagon ABCDE, for instance into four equal Parts, by right Lines drawn from the given Angle D. Reduce the Polygon into the Triangle FDG, whose Vertex is at the Point given D, and having divided the Base FG, into four equal Parts, at the Points H, I, K, draw the right Lines DH, DI, DK, and they will make four Triangles FDH, HDI, IDK, KDG, each of which will be *by* 1. 6. a fourth part of the Triangle FDG, and consequently of the Pentagon propos'd ABCDE ; but seeing the Point H falls without the Side AB, you must change the Triangle HDI, into the Trape-

Trapezium ALDI, which is found by drawing the right Line HI parallel to the Line AD, &c.

PROBLEM III.

To divide a Polygon into two equal Parts, by right Lines parallel to its Sides.

TO divide the Pentagon, for instance, ABCDE, into two equal Parts, by right Lines parallel to the three Sides BC, CD, DE. Reduce the Pentagon into Triangles, by the Diagonals AC, AD, and divide *by Prob.* 1. *Chap.* 2. One of those Triangles, as ABC, into two equal Parts by the right Lines FG, parallel to the Side BC, so that the Triangle AFG, may be equal to the Trapezium FBCG, and then if you draw through the Point G, the right Line GH parallel to the Side CD, and thro' the Point H, the right Line HI, parallel to the Side DE, you may know *by* 19. 6. that the Triangle AGH, is also equal to the Trapezium GCDH, and in like manner the Triangle AHI, to the Trapezium HDEI, and then the Problem is done. *Plate* 7. *Fig.* 82.

SCHOLIUM.

Instead of taking the Angle A, to reduce the Polygon proposed into Triangles, you might have taken any other Point at pleasure within the Figure, as F, and then you need only look upon the Figure to comprehend the rest, and conceive that by this method, you may divide the Polygon proposed into as many equal Parts as you please, and that the same method may be apply'd to Figures of three and four Sides. *Fig.* 83.

PROBLEM IV.

To divide a given Polygon into two equal Parts, by a right Line drawn from the middle of one of its Sides.

TO divide into two equal Parts the Pentagon ABCDE, by a right Line drawn from the middle Point N, of the Side AB. First reduce the Polygon proposed into a Triangle having the same Side, and the same Angle A, which may be done by the Principles of the foregoing *Lemma* as you shall see. *Fig.* 86.

Having

Plate VII. Fig. 86: Having drawn from the Angle C, a Line CF parallel to the Diagonal BD, meeting the Side DE, produced, in F, draw the right Line BF, and the Pentagon ABCDE, w be found changed into the Quadrilateral Figure ABF which may be reduc'd into a Triangle, by drawing thro' the Point E, a Line EG, parallel to the Diagonal AF, meeting the Side BF produc'd in G, and by joining the right Line AG, for the Triangle AGB, will be equal to the Trapezium propos'd ABCDE: But when there is not the same Angle A, draw through the Point F, the Line FH parallel to the Diagonal BE, or which is as well, through the Point G, a Line GH, parallel to the Side AB, meeting the Side AE produc'd in the Point H, through which draw to the Point B, the right Line HB, and the Triangle AHB, shall be the Triangle sought, and *by* 1. 6. shall be divided into two equal Parts by the right Line HN, cutting the Side DE, in the Point I.

This preparation being made, draw from the Point H, the right Line HL perpendicular to the Side DE, and from the Point N, the right Line NK, perpendicular to the same Side DE, and find a fourth proportional IM to the three Lines NK, HL, IE, for joining the right Line MN, that will divide the Pentagon proposed into two equal Parts, so that the Trapezium AEMN shall be half that Polygon.

DEMONSTRATION.

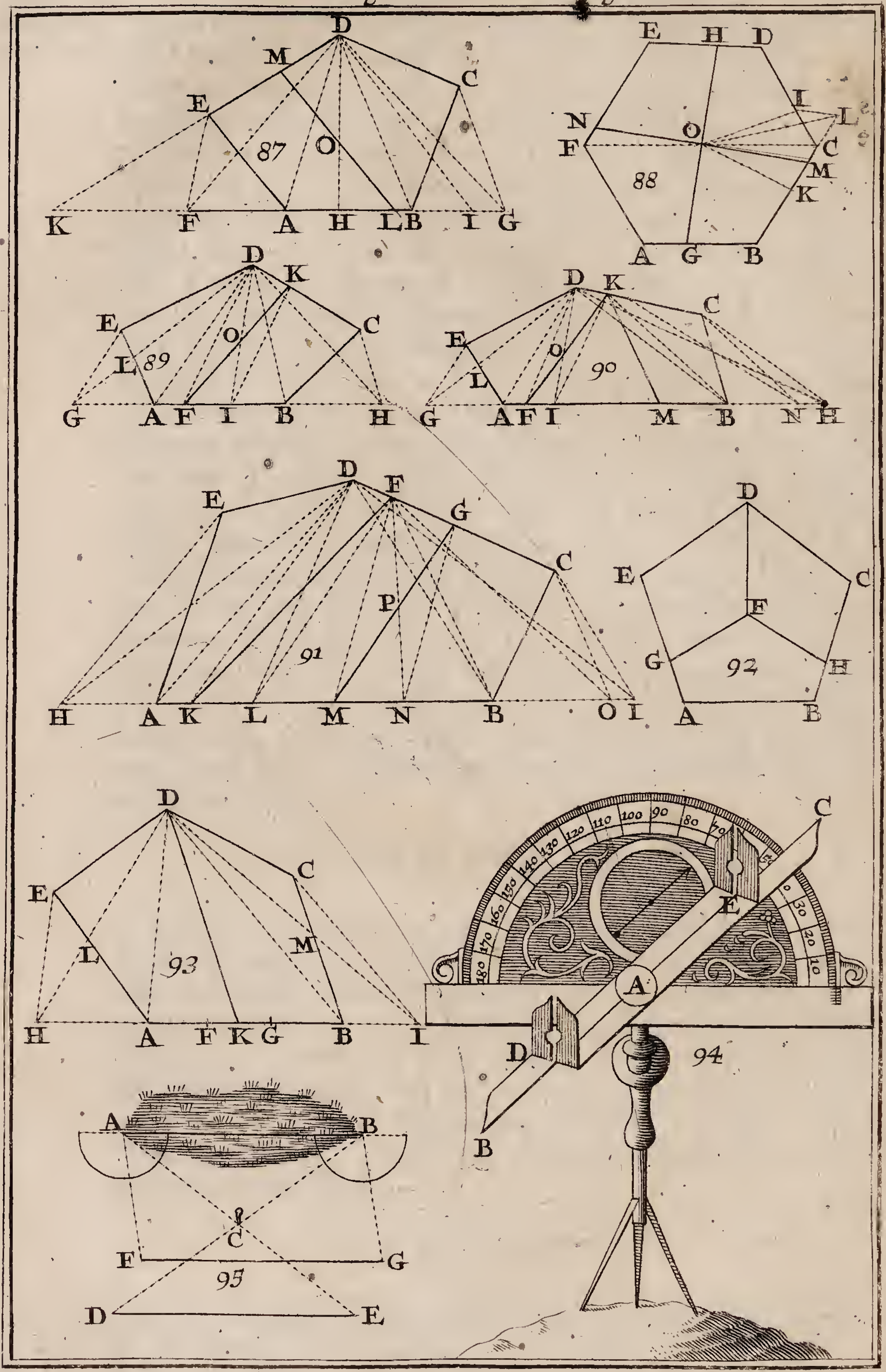
Because the four Lines NK, HL, IE, IM, are proportional *by Construct.* the Rectangle of the extreams NK, IM, is equal to the Rectangle of the means HL, IE, *by* 16. 6. Wherefore the halves of these Rectangles which are *by* 41. 1. the two Triangles IMN, IEH, are equal, adding therefore to each the common Trapezium AEIN, you will see that the Trapezium AEMN, is equal to the Triangle AHN, or half the Pentagon ABCDE. *Which was to be demonstrated.*

SCHOLIUM.

This Problem may easily be resolved, when the given Polygon is regular; for if it be an Uneven one, that is, made up of an uneven Number of Sides, as the Pentagon ABCDE, you may only draw from the Point given N, to the opposite Angle D, the right Line DN, and it will divide the Pentagon propos'd into two equal Parts, each made up of an equal Number

Fig. 84.

of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of Angles and Sides equal to one another. But if the given Polygon be an *Even one*, that is to say, made up of an even number of Sides, as the Hexagon ABCDEF, 'tis evident you have no more to do, but to draw from the middle Point N of the Side AB, thro' the middle M of the opposite and parallel Side DE, the right Line MN, &c.

PROBLEM V.

To divide a given Polygon into two equal Parts, by a right Line drawn parallel to a given Side.

TO divide the Pentagon ABCDE, into two equal Parts, by a right Line parallel to a Side AE; reduce the given Polygon into a Triangle FDG, which may be done here, by the two Lines EF, CG, parallel to the two Diagonals DA, DB, and having bisected the Base FG, in the Point H, join the right Line DH, to gain the the Triangle FDH, equal to half the Triangle FDG, *by* 1. 6. or the Polygon proposed ABCDE. Draw from the Point D, the right Line DI, parallel to the given Side AE, and having produced the Sides AB, DE, till they intersect one another, as at K, find a mean proportional KL between the Lines KH, KI, and draw thro' the Point L, a parallel LM, to the given Side AE, and that will divide the Pentagon proposed into two equal Parts, the Trapezium ALME, and the Pentagon BCDML, so that one of these two Planes, as the Trapezium ALME, will be half the Pentagon ABCDE. *Plate* 8. *Fig.* 87.

DEMONSTRATION.

Because the two Triangles KDI, KML, are equiangular, the Bases DI, ML, being parallel, *by Constr.* The Triangle KML, will be to the Triangle KDI, as the square of the Side KL, to the square of the homologous Side KI, *by* 19. 6. or as the Line KH, to the Line KI, *by Coroll.* 20. 6. KH, KL, KI being three Proportionals *by Constr.* or as the Triangle KDH to the Triangle KDI. Whence it follows, that the Triangle KML is equal to the Triangle KDH, and consequently the Triangle DOM equal to the Triangle LOH, and the Trapezium ALME equal to the Trapezium AHDE, or the Triangle FDH, that is to say, to half the Polygon proposed ABCDE. *Which was to be Demonstrated.*

SCHO-

SCHOLIUM.

Plate VII. *Fig.* 85. 'Tis evident that, when the Polygon propoſed ſhall be a regular one, compoſed of an even Number of Sides, as the Hexagon ABCDEF, you may divide it into two equal Parts by a Line parallel to the Side AB, by drawing through the two Angles Diametrically oppoſite, and equaly diſtant from the Side given AB, namely F, C, the right Line FC, which being parallel to the Side AB, and perpendicular to the Line MN, and paſſing thro' the middle Points M, N, of the two oppoſite and parallel Sides DE, AB, divides alſo into two equal Parts the Hexagon ABCDE, which by this method is found divided into four equal Parts, by the two Perpendiculars MN, FC.

'Twill be eaſie alſo to divide a regular Polygon, made up of an uneven number of Sides, into four equal Parts,
Fig. 84. by two Lines Perpendicular to one another, as the Pentagon ABCDE, namely by dividing it firſt into two equal Parts by the Line DN, perpendicular to the Side AB, as has been ſhewn in *Prob.* 4. and again into two equal Parts by the Line FG parallel to the ſame Side AB, as ſhall be ſhewn, &c.

PROBLEM VI.

To divide a Polygon into two equal Parts, by a right Line drawn from a given Point in a Side.

Plate 8. *Fig.* 89. TO divide into two equal Parts the Pentagon ABCDE, by a right Line drawn from the Point F, given in the Side AB. After having reduced it into the Triangle GDH, by the Lines EG, CH, parallel to the two Diagonals DA, DB, divide the Baſe GH into two equal Parts, in the Point I, and draw through the Point I, a parallel IK, to the Line DF, to gain the Point K, in the Side CD, through which and the Point given F, draw the right Line FK, and it will divide the Polygon propoſed ABCDE, into two equal Parts, the Pentagon AFKDE, and the Trapezium BCKF, ſo that each of the two Planes will be half the Pentagon ABCDE.

DEMONSTRATION.

Becauſe the Baſe GI, of the Triangle GDI, is half the Baſe GH, of the Triangle GDH, the Triangle GDI, will

will be by 1.6. half the Triangle GDH, or Pentagon ABCDE, and becauſe the Pentagon AFKDE, is equal to the Triangle GDI, the Line EG, being parallel to the Diagonal DA, *by Conſtr.* and making the Triangles GLA, DLE, equal to one another; and the Line IK parallel to the Diagonal DF, making the Triangles FOI, DOK equal; whence it follows, that the ſame Pentagon AFKDE, is alſo half the propoſed one ABCDE. *Which was to be demonſtrated.*

SCHOLIUM.

'Tis evident, that to divide the Polygon propoſed ABCDE, into three equal Parts, you muſt take the Baſe GI, equal to a third part of the Baſe GH, for ſo the Pentagon AFKDE, will be a third of the propoſed ABCDE, and conſequently the Trapezium BCKF will be two thirds: Wherefore if you reduce the Trapezium BCKF into the Triangle FKN, by the Line CN parallel to the Diagonal KB, and divide the Baſe FN into two equal Parts in the Point M, joining the right Line KM, the Polygon propoſed ABCDE, will be found divided into three equal Parts by the two Lines KF, KM, for the Line KM divides the Trapezium BCKF, into two equal Parts, ſeeing by 1. 6. the Triangle FKM is equal to half the Triangle FKN equal to the Trapezium BCKF. *Plate VIII. Fig.* 90.

'Tis evident alſo that to divide into two equal Parts, a regular Polygon of an even number of Sides, for Inſtance, the Hexagone ABCDEF, by a right Line drawn from the Point G, given in the Side AB, you are only to take in the oppoſite and parallel Side DE, the Part DH equal to the Part AG, and join the right Line GH, and that will divide the Hexagon propoſed ABCDEF, into two equal Parts, the two Pentagons AGHEF, BGHDC, which you may divide into two equal Parts, by the help of *Probl.* 4. by a right Line drawn from the middle O of the common Side GH, which middle O is the Center of the Hexagon, and thus you have the Polygon divided into four equal Parts by right Lines drawn from the Center. But this Diviſion may be much eaſier after this manner. *Fig.* 88.

To divide into two equal Parts the Pentagon BGHDC, take in the Sides BC, CD, the two Lines BK, CI, each equal to the Line AG, or DH, or which is as well, the two Parts CK, DI, equal each of them to the Part BG or EH, and draw the Lines OI, OK, and they

they will divide the Pentagon into three equal Parts: Wherefore to divide it into two equal Parts, 'tis only dividing into two equal Parts that of the middle, that is the Trapezium KOIC, by reducing it into a Triangle KOL, by the Line IL, parallel to the Diagonal OC, and by dividing the Base KL into two equal Parts, in the Point M, to join the right Line OM, which being produced towards N, will divide each of the two Pentagons AGHEF, BGHDC, into two equal Parts, and thus the Hexagon proposed will be found divided into four equal Parts by right Lines that cut one another in the Center O.

PROBLEM VII.

To divide a Polygon into three equal Parts, by two right Lines drawn from two given Points in a Side.

Plate VIII. Fig. 91. TO divide into three equal Parts the Pentagon ABCDE, by two right Lines drawn from the two Points K, M, given in the Side AB. Reduce it into the Triangle HDI, by the two Lines EH, CI, parallel to the two Diagonals DA, DB, and having taken the Line HL equal to the third part of the Base HI, draw thro' the Point L, the Line LF, parallel to the Line DK, and thro' the Point F to the given Point K, the Line KF, which will cut off the Pentagon AKFDE, equal to the Triangle HDL, or the third Part of the Polygon proposed ABCDE, as has been demonstrated in *Probl.* 6. There remains therefore only the Trapezium BKFC, to be divided into two equal Parts, by a right Line drawn from the other given Point, M, which may be done after the same manner, as you shall see presently. Having reduced the Trapezium BKFC into the Triangle KFO, by the Line CO, parallel to the Diagonal BD, divide the Base KO, into two equal Parts at the Point N, and having drawn thro' the Point N the right Line NG parallel to the Line FM, draw the right Line MG, and it shall cut off the Trapezium KFGM equal to the Triangle KFN, or half the Trapezium KFCB, the Triangle FPG being equal to the Triangle MPN. Whence it follows, that each of the two Trapeziums KFGM, MGCB, is a third part of the Polygon proposed; which is found divided into three equal Parts by the Lines KF, MG, drawn from the two given Points K, M; *Which was to be done.*

PRO-

PROBLEM VIII.

To divide a regular Pentagon into three equal Parts, by as many right Lines drawn from its Center.

TO divide into three equal Parts the regular Pentagon ABCDE, by three right Lines drawn from the Center F. Take on each of the two Sides BC, AE, their third Parts AG, BH, and draw the right Lines FD, FG, FH, and they will divide the Pentagon proposed into three equal Parts, as may be seen by dividing all the Sides of the Pentagon into three equal Parts, and drawing from the Center F thro' all the Points of Division, as also thro' all the Angles of the Pentagon, as many right Lines, and they will form fifteen little Triangles by 38. 1. of which each of three Planes terminated by the three right Lines FD, FG, FH, will comprehend five. *Plate VIII. Fig. 92.*

PROBLEM IX.

To divide a Polygon into two Parts of a given Ratio, by a right Line drawn from a given Angle.

TO divide the Pentagon ABCDE into two Parts, whose Ratio is to be equal to that of the two given Lines AF, FG, by a right Line drawn from a given Angle D. Reduce the Polygon proposed into the Triangle HDI, whose Vertex let it be at the Angle given D, and having cut the Base in K, so that the four Lines AG, AF, HI, HK may be proportional, draw the right Line DK, and it will divide the Polygon proposed into two equal Parts AKDE, BKDC, proportional to the two given Lines AF, FG. *Fig. 93.*

DEMONSTRATION.

Because the four Lines AG, AF, HI, HK, are proportional, *by Constr.* you may know *by dividing* that the four Lines FG, AF, KI, KH, are also proportional: Wherefore if you place the two Triangles KDI, KDH, in the room of the two last KI, KH, that are in the same Ratio, by 1. 6. or which is as well, place the two Trapeziums BKDC, AKDE in the room of the two Triangles, which are their equals, BMI being equal to the Triangle CMD, and the Triangle ALH being equal to the Triangle DLE, you may know that the Ratio of the two Lines FG, AF is equal to that of the two Trapeziums BKDC, AKDE. *Which was to be demonstrated.*

The SECOND PART.

OF

LONGIMETRY.

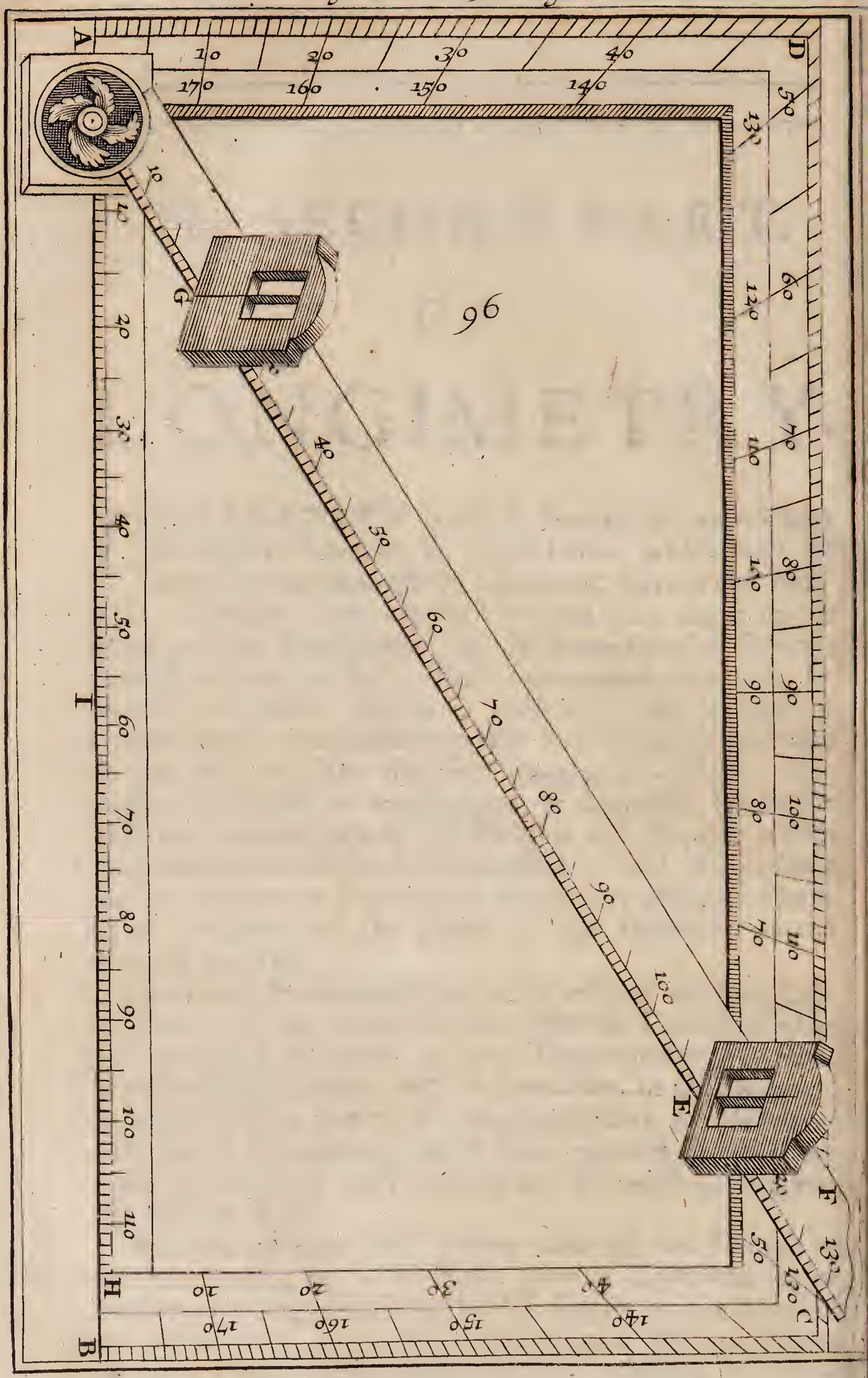
LONGIMETRY is so call'd, because it teaches how to measure Lengths, or right Lines, which may be *Accessible* and *Inaccessible*, *Horizontal*, *Vertical* and *Inclined*: *Accessible* ones are such as you may come up to at one of their Extremeties, as the Breadth of a River: *Inaccessible* ones are such, whose extremeties can only be seen at a distance, such as is ordinarily the distance of two Bastions: *Horizontal* ones are such as lie on the Plane of the Horizon, like that you imagine in a Plain: *Vertical ones* are such as are elevated or depressed below the Horizon perpendicularly, as Heights and Depths, which may also be accessible and inaccessible: And the *Inclined ones* are such as are slope or are elevated at oblique and unequal Angles upon the Plane of the Horizon, as the slope of an Hill.

Lines may be measured two ways, with or without Calculation. If you would measure them by Calculation, the best method I can think of is by Trigonometry, to measure the Visual Angles with a *Semicircle*, or *Graphometer*, of which I have spoken in the *Introduction to Mathematics*, and is so common, that I have no need to speak any more about it. I shall content my self with giving you the Figure of it.

Plate 8. *Fig.* 94. Wherein the Line BC, passing through the Center of the Instrument, A, is call'd the *Line of Sight*, and serves to shew the Degrees of the Visual Angles on the Limb of the Instrument, and to which the slits of the two Sights D, E, answer, being elevated on the *Alidade*, or movable Rule BC, thro' which you look upon distant Objects in measuring the Quantity of the Visual Angles.

Or

Geometry Plate 9. Page 67.



Or you may as well uſe the *Univerſal Inſtrument*, where the Degrees of a Semicircle are alſo marked. But when you would meaſure a Line without Calculation, you have nothing to do, but to find the uſual Angles, which can indeed never be found exactly; hence in working by the Angles, there is always ſome ſmall failure, and 'tis dangerous to miſs much, when the Viſual Angles are very acute or obtuſe. In this Caſe 'tis beſt to uſe the *Univerſal Inſtrument*, which is a Rectangular Frame of Metal or any other ſolid Matter, as ABCD, in which you may place or remove, as you pleaſe, one or more pieces of ſtiff Paper to work upon, by drawing with a Point right Lines repreſenting the Viſual Rays along the Line of Sight EO of the Alidad FO, that is moveable not only upon its Center O, but alſo along the Line OH, which I ſhall call the *Line of Direction*, under which there are two Sights like theſe two E, G, of the Alidad EO, whoſe Line of Sight contains a certain Number of equal Parts, and ſimilar to thoſe of the Line of Direction OH; which repreſent Feet, Yards, &c. But having already publiſhed a particular Treatiſe touching the Conſtruction and uſe of this Inſtrument. I ſhall only ſay that this Inſtrument contains on its three Sides AD, CD, BC, the Degrees of a Semicircle, whoſe Center I is about the middle of the Line of Direction OH, on which the Center O of the Alidad, advances when the Viſual Angles are to be meaſur'd. *Plate IX. Fig. 96.*

PROBLEM I.

To meaſure an Horizontal Line acceſſible at both its extremities.

THO' the Horizontal Line AB be acceſſible at each of its extremities A and B, yet I will ſuppoſe, you can't run all along the Side of it, for ſome Precipice perhaps between the extremities A and B, or ſome other impediment, as ſome Water between them, or whatever it be that hinders the actual meaſuring it with a Cord or Chain. In this Caſe you may find the Length after this manner. *Plate VIII. Fig. 95.*

Having fixed in the Ground a Stick in a commodious Point as C, from whence you may meaſure the two Lines CA, CB, produce the two Lines CA, CB, to D and E, till the Line CD be equal to one of the two CA, CB, for inſtance the Line CA, and the other Line CE to the other Line CB, and meaſure the Length

Plate VIII. Fig. 95. of the Line DE, which *by* 4. 1. is equal to the propoſed one AB.

Or, which is as well, having fixed a Stick at the Point F, taken at diſcretion on the Ground, meaſure with a Chain the length of the Line AF, and with a Graphometer the Quantity of the Angle BAF, and make at the Point B, with the ſame Graphometer the Angle ABG, the Supplement of the Angle BAF, by the Line BG, which *by* 28. 1. is parallel to the Line AF; wherefore if you make it equal to the Line AF, and meaſure the Length of the Line FG, you will have the Length of the propoſed one AB; *by* 33. 1.

PROBLEM II.

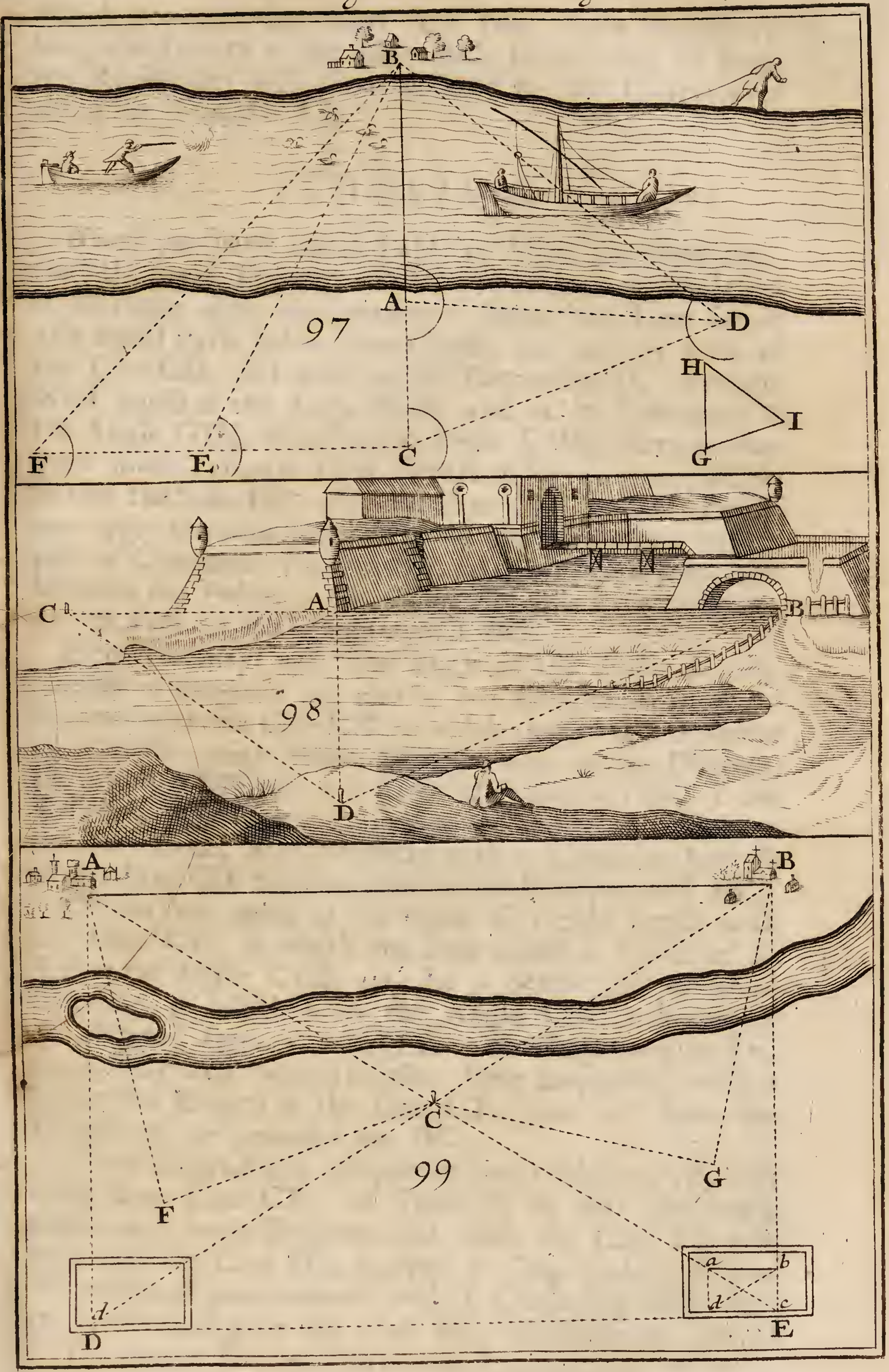
To meaſure an Horizontal Line Acceſſible only at one extremity.

Plate X. Fig. 97. TO meaſure the Line AB, that I ſuppoſe acceſſible at A. Fix in a right Line a Stick at a commodious Point of the Ground, for inſtance C, and another at ſome other commodious Place, as D, ſo that the Diſtance CD, of the two Points C, D, called *the Points of Station*, may be of a conſiderable Length, in regard of the Line BC, that is to be meaſured, for if that Line CD be ſmall in reſpect to the Line ſought, AB, or BC, the Angle BDC will become ſo big, that but a ſmall miſtake committed in the meaſuring, may cauſe a conſiderable one in the meaſure of the Line BC.

Having therefore meaſured with a Chain the Line CD, which we will ſuppoſe 125 Feet, and with a Graphometer or ſome other way each of the Angles BCD, BDC, their Sum ſubſtracted from 180 Degrees will give the Angle CBD, *by* 32. 1. As if the Angle C be 73 Degrees, and the Angle D 58 Degrees, the third Angle GBD will be 49 Degrees. Wherefore in the Triangle CBC, knowing the Angles and the Side CD, you may find by Trigonometry the Side BC, by making this Proportion.

As the Sine of the Angle B	75471
to its oppoſite Side CD :	125
So is the Sine of the Angle D	84805
to its oppoſite Side BC	140

Which


B
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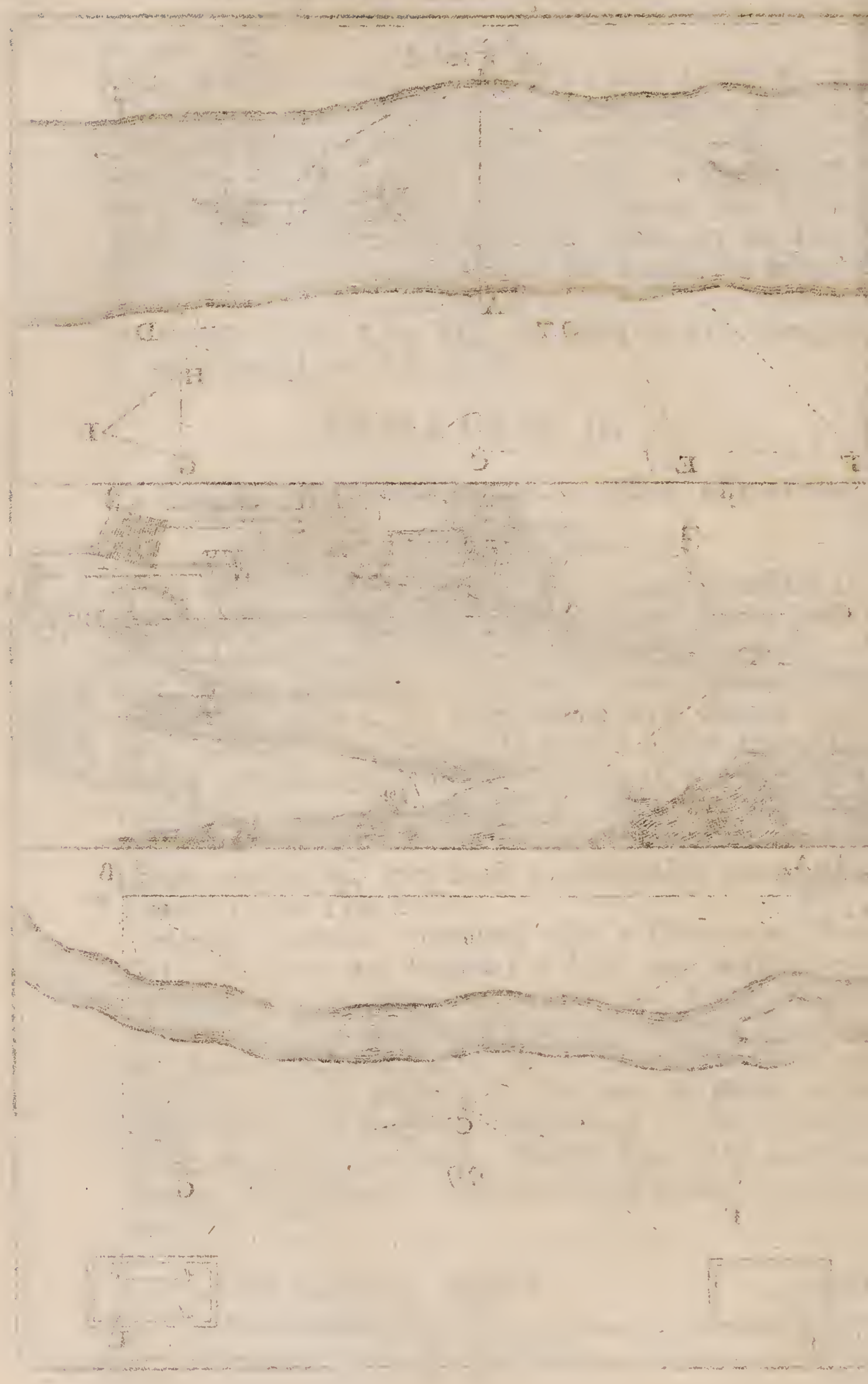


Which you will find about 140 Feet, from whence taking the Length of the Line AC, for instance, 52 Feet, the Remainder will give 82 Feet for the Quantity of the Line propos'd AB. *Plate X. Fig.* 97.

SCHOLIUM.

When you have not a Table of Sines, you may protract the great Triangle BCD, and make one similar to it on Paper after this manner. Draw the Line GI of 125 equal Parts taken from a Scale, for the 125 Feet of the Line CD, and make at its Extremity G, an Angle IGH equal to the Angle BCD, and at its Extremity I, the Angle GIH, equal to the Angle CDB, that you may get a small Triangle GHI similar to the great one BCD, so that the Side GH will represent the Line BC, and if you apply the same Side GH, to the same Scale with a pair of Compasses, the number of equal Parts contained between the Points of the Compasses will give the number of Feet in the Line propos'd BC.

Since you may make the Angle at the Point C any number of Degrees you please, you may make it of 60 Degrees with the Line CD, in which chooseing a Point as D, that the Angle BDC likewise may be 60 Degrees, the Angle CBD also will be 60 Degrees, and *by* 6. 1. the Triangle BCD will be equilateral, and then you have no more to do, but to measure with a Chain the Length of the Line CD, to obtain that of the Line propos'd BC.

Or you may make at the Point C a right Angle with the Line CF, in which you may choose a Point, as F, so that the Angle CFB may be a Semi-right or of 45 Degrees, in which Case the Angle CBF will also be 45 Degrees, *by* 32. 1. and *by* 6. 1. the Right-angled Triangle BCF will be an Isoscoles, then measuring with a Chain the Length of the Line CF, you will have the Length of the proposed one BC.

If the Ground will not permit you to approach F, seek in the same Line CF, the Point E, so that the Angle CEB may be 60 Degrees, and then the Line EB will be double the Line EC, *by Prop.* 5. *Chap.* 3. *L.* 1. *Trigon.* Wherefore if you measure with a Chain the Line EC, its double will give the Line EB, whose Square lessen'd by the Square EC will leave the Square BC; *by* 47. 1. and the Square Root of that remainder will consequently give the Line sought BC.

Plate X. Fig. 97. *Lastly*, if it be not commodious to make at the Point C, with the Line BC, an Angle of 60 or 90 Degrees, make one of what degrees you please, suppose 73, so that the Angle BCD is 73 Degrees, then seek in the Line CD, a Point as D, that the Angle CDB may be 53° 30′ namely the complement of half the preceeding BCD, for 'tis easie to demonstrate, that in this Case the Line CD will be equal to the purposed one BC, and then measuring with a Chain the Line CD, you will have the Line BC requir'd.

But the Line propos'd AB may be found immediately, making at the extremity A, the Angle BAD of any magnitude at pleasure, with the Line AD of any length, and measuring the Angle ADB, to make, in the Triangle ADB, where the Angles and Side CD are known, this Proportion

As the Sine of the Angle B,
to the opposite Side AD;
So is the Sine of the Angle D,
to its opposite Side AB.

PROBLEM III.

To measure an inaccessible Horizontal Line.

Fig. 98. TO measure the inaccessible Line AB, erect two Sticks at the two Points C, D, taken at Discretion, so that if possible, one of the Points, as C, may be in a right Line with that which is to be measur'd AB, finding with a Chain the Line CD, which should be of a reasonable Length and proportion'd to the Line AB that is to be measur'd, and by a Graphometer the Angles C, CDA, ADB; substract from 180 Degrees the Sum of the two Angles C, and CDA, to get the Angle CAD, and from thence the Angle ADB, to get the Angle B. Then in the Triangle CAD, knowing the Angles and the Side CD, you may find the Side AD, by this Proportion,

As the Sine of the Angle A,
to its opposite Side CD;
So is the Sine of the Angle C,
to its opposite Side AD.

And in the Triangle ADB knowing besides the Angles the Side AD, you may find the Line sought AB, by making this Proportion, *As*

As the Sine of the Angle B,
To its opposite Side AD;
So is the Sine of the Angle D,
To its opposite Side AB.

But if the Point of Station C be not in a right Line, with the Line to be measured AB, produce each of the two Lines AC, BC, to E and D, till the two Lines CD, CE be each of a known Length, and measure the two Angles ADB, AEB, which substracted from the Angle ACB, will give the Angles CAD, CBE, by 32. 1. and in the Triangle ADC, you may find the Side AC by this proportion, *Plate X. Fig. 99.*

As the Sine of the Angle A,
To its opposite Side CD;
So is the Sine of the Angle D,
To its opposite Side AC.

In like manner in the Triangle BCE, you may find the Side BC, by this Proportion,

As the Sine of the Angle B,
To its opposite Side CE;
So is the Sine of the Angle E,
To its opposite Side BC.

Then in the Triangle ACB, you know the two Sides AC, BC, and the Angle contain'd ACB, which is enough to find the two others A and B, and the third Side or Line AB sought.

SCHOLIUM.

There is no need of Trigonometry, if, in the Lines AC, BC, produced, you take the two Points D, E, so as that each of the two Angles ADB, AEB, be half the Angle ACB, because in this case the Line sought AB will be equal to the Line DE, whose Length may readily be found by a Chain, as may easily be demonstrated.

If the Ground will not permit you to produce the Lines AC, BC, make at the Point C, with the Line AC, an Angle of what bigness you please with the Line CF of an arbitrary Length, and measure the Angle F, then in the Triangle ACF, where besides the Angles, the Side CF is known, you may find the Side AC. In like manner

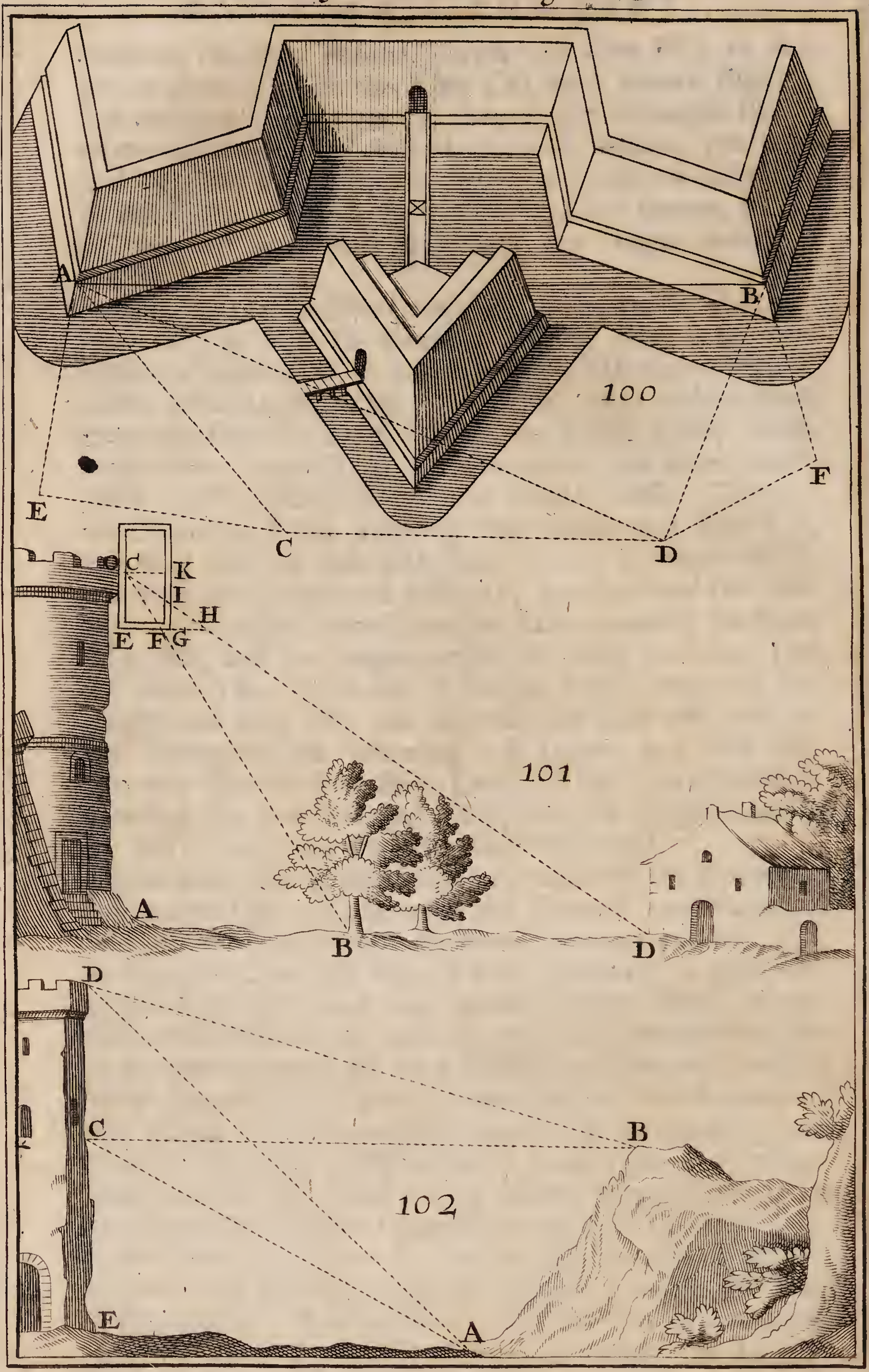
make at the ſame Point C, with the Line BC, an Angle at pleaſure with the Line CG of a known Bigneſs, and meaſure the Angle G, then in the Triangle BCG, where the Angles and Sides CG being known, you may find the Side BC, after which you may find the Side AB, in the Triangle ABC, having three things known, namely, the two Sides AC, BC, and the Angle contain'd ACB.

The ſame inacceſſible Line AB, may be meaſured otherways ſtill, as ſhall be ſhewn. Having choſen the two Points of Station, D, E, whoſe Diſtance DE may be found, meaſure the Angles ADB, EDB, AEB, AED, and ſubſtract from 180 Degrees the Sum of three ADB, BDE, AED, to get the Angle DAE, in like manner the three Angle AEB, AED, BDE, to get the Angle DBE. Then in the Triangle ADE, knowing the Angles and Side DE, you may find the Side AD, and in the Triangle DEB, knowing the Angles and Side DE, you may find the Side BD. And laſtly in the Triangle ADB knowing the Sides AD, BD, and the Angle contain'd ADB, you may find the Side AB: Or in the Triangle DBE, knowing the Angles and Side DE, you may find the Side AE, and in the Triangle DBE, knowing the Angles, and Side DE, you may find the Side BE: Laſtly, in the Triangle ABE, knowing the two Sides AE, BE, and the Angle contained AEB, you may find the third Side AB.

This Line may be meaſured very eaſily by the Univerſal Inſtrument thus. Take upon the Ground two Points of Station, at a conſiderable Diſtance from one another, and ſo near the Line AB, that is to be meaſured, as that the viſual Rays may croſs one another on the Plane of the Univerſal Inſtrument, as D, E, whoſe Diſtance DE ought to be exactly meaſured by a Chain, or ſome other way: let us ſuppoſe it 150 Yards. Having fix'd the Center of the Alidad and Univerſal Inſtrument in an advantagious Point of the Line of Direction as in *d*, apply the Inſtrument ſo that the Point *d* may anſwer perpendicularly to the Point D, and the Line of Direction to the Line DE, which may eaſily done by looking downwards thro' the Sights; and having turn'd the Alidad towards the two Extremities A, B, of the Line AB, that is to be meaſured draw on the Surface of the Inſtrument along the Line of Sight the two viſual Rays *d*A, *d*B.

Then, becauſe we ſuppoſed the Line DE to be 150 Yards, advance the Center of the moveable Rule or Alidad 150 equal Parts of the Line of Direction from *d* to *e*, and apply again the Univerſal Inſtrument, ſo as that the

Point


A
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D
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102
E
A

Point *e* may correſpond perpendicularly with the Point E, and the Line of Direction *de* to the Line DE, that you may turn as before the Alidad towards the Extremities A, B of the Line AB, that is to be meaſured, and draw after the ſame manner on the Surface of the Inſtrument along the Line of Sight, the two Lines or viſual Rays *ea*, *eb*, whoſe Diſtance *ab* being taken upon the Line of Direction will give, by the number of equal Parts, it contains, the number of Yards in the Line propoſed AB.

At the ſame time you may find the length of the Lines DA, DB, EA, EB, by applying the Alidad upon the Lines *da*, *db*, *ea*, *eb*, by this means you will find on the Diviſions of the Line of Sight the number of Yards, thoſe Lines contain. Thus you ſee that by the help of the Univerſal Inſtrument you may meaſure five Lines at once with all poſſible exactneſs.

It may ſometimes happen, that the two Extremities of the Line to be meaſured AB, can't be ſeen from the ſame Point of Station; as here, becauſe of a Ravelin between, you can only ſee the Extremity A, from the Point C, and the Extremity B, from the Point D. In this Caſe having meaſured with a Semicircle the Angles ACD, CDB, and the Line CD with a Chain, make at pleaſure at the Point C, an Angle ACE, with the Line CE, of any given bigneſs, that by meaſuring the Angle E, there may be known in the Triangle AEC, the Side AC, and in the Triangle ACD, the Side AD, and angle CDA, which taken from the Angle CDB, will leave the Angle ADB. Laſtly at the Point D, make the Angle BDF at Diſcretion, comprehended by the Line BD, and the Line DF of an arbitrary length, that by meaſuring the Angle F, you may find in the Triangle BDF, the Side BD, and then in the Triangle ADB, the Line ſought AB, by Calculation. *Plate XI. Fig.* 100.

PROBLEM IV.

To meaſure an Horizontal Line from above.

TO meaſure from the Point C the Horizontal Line AB, whoſe extremity A anſwers perpendicularly to the Point C, meaſure with a Semicircle the Angle ACB, and by the help of a Plummet let down from C, the height AC, that you may have in the Triangle ACB, Right-angled in A, beſides the Angles known, the Side AC, which with the viſual Angle ABC, will ſerve to find the Line AB, by making this Proportion, *Fig.* 101.

As

As the Radius
To the Tangent of the viſual Angle ACB;
So is the Height AC,
To the Line AB.

Plate XI. Fig. 101. But if you would meaſure that Line with the Univerſal Inſtrument, place the Center of the Alidad at the Point O diſtant from the Point E, as many equal Parts of the Line of Direction, as the Height AC contains Feet. Then apply the Point O, to the Point C, ſo as that the Line of Sight EO may be perpendicular, and the Inſtrument remaining in this Situation, turn the Alidad toward the Extremity of the Line to be meaſured AB, and draw on the Surface of the Inſtrument along the Line of Sight the right Line OF, which will determine the Line EF, whoſe Length apply'd to the Line of Direction will give, in the number of equal Parts it contains, the number of Feet in the Line propoſed AB.

SCHOLIUM.

Sometimes the viſual Ray cuts the Side EG, without the Inſtrument, as in the Point H here, beyond the Point G, for the meaſure of the Line AD. In this Caſe, becauſe you can't apply the Line EH to the Line of Direction, to find the number of equal Parts it contains, becauſe you have not the Point of Interſection H, you may find the number by the Golden Rule thus.

Draw in your Imagination from the Point O, the Line OK, parallel to the Side EG, and ſubſtracting the Number of equal Parts in the Line GI, from that in the Line GK, or EG, you will have the Number of equal Parts in the Line IK; and becauſe we know that of the Line CK, or EG, applying its Length to the Line of Direction in the ſimilar Triangles OKI, OFH, reaſon thus, if the Number of equal Parts in the Line IK, give ſo much for the Number of equal Parts in the Line CK, how much will the Number of equal Parts in the Line CE give? And you will find the number of equal Parts in the Line EH, or the Number of Feet in the Line ſought AD.

We have ſuppoſed that the Extremity A of the Line to be meaſured, anſwers perpendicularly to the given Point C.

But

But now we will ſuppoſe that it does not, as if you were to meaſure by Trigonometry the Line BD. Firſt then meaſure with a Semi-circle the viſual Angle ACD, to find by the help of that the whole Line AD, by a proportion Similar to the former, which gives us the Line AB, which taken from the Line AD gives the Line BD, which may be found immediately by drawing from the two Proportions, that are to be made to find the Lines AB, AD, one ſingle Proportion as the following one, *Plate XI. Fig.* 101.

As the Radius
To the difference of the Tangent of the Viſual Angles ACB, ACD,
So is the Height AC,
To the Line BD.

The Demonſtration of this Proportion will be evident to any one that conſiders the ſide AC common to the two right Angled Triangles CAB, CAD, being taken for the Radius, the two Lines AB, AD, are Tangents of the viſual Angles ACB, ACD, and conſequently the Line BD is the difference of the Tangents.

PROBLEM V.

To meaſure from above an inclin'd Line.

TO meaſure the inclin'd Line AB without going out of the Tower DE, to which I ſuppoſe it directly oppoſite, ſo as that the two Lines AB, DE, may be in the the ſame Plane; chooſe two advantageous Places, as the two Windows C, D, to make your Stations at, and meaſure with a String their diſtance CD, that ought to be as great as poſſible, and with a Semi-circle the Angles ECA, ECB, EDA, EDB, which being thus known, the Angles CAD, and CBD, will be alſo known, as alſo the Angles ACB, ADB; becauſe the Angle CAD, is equal to the difference of ECA, and EDA, *by* 32. 1. and the Angle CBD to the difference of ECB, and EDB, and the Angle ACB is equal to the difference of ECA, ECB, and in like manner the Angle ADB to the difference of EDA, EDB. This being ſuppos'd, you may find in the Triangle ACD, the ſide AC, and in the Triangle BCD, the ſide BC, and laſtly in the Triangle ACB the ſide ſought AB. *Fig.* 102.

SCHO-

SCHOLIUM.

The Univerſal Inſtrument may alſo ſerve very well to meaſure the Line AB, which is ſo eaſily conceiv'd by what has been ſaid above, that 'tis needleſs to give a particular Example, I ſhall only ſay that, to make the viſual Rays cut one another on the Inſtrument, the diſtance CD of the Stations ought not to be too ſmall compared with the Line to be meaſured AB.

It ſeems needleſs alſo to tell you, that you may after the ſame manner meaſure from an eminence the Length of a Roof, the Breadth of a Meadow, the Height of a Caſtle, a Houſe and any other Line, whoſe Extremities ſhall be in a right Line with the Height ED.

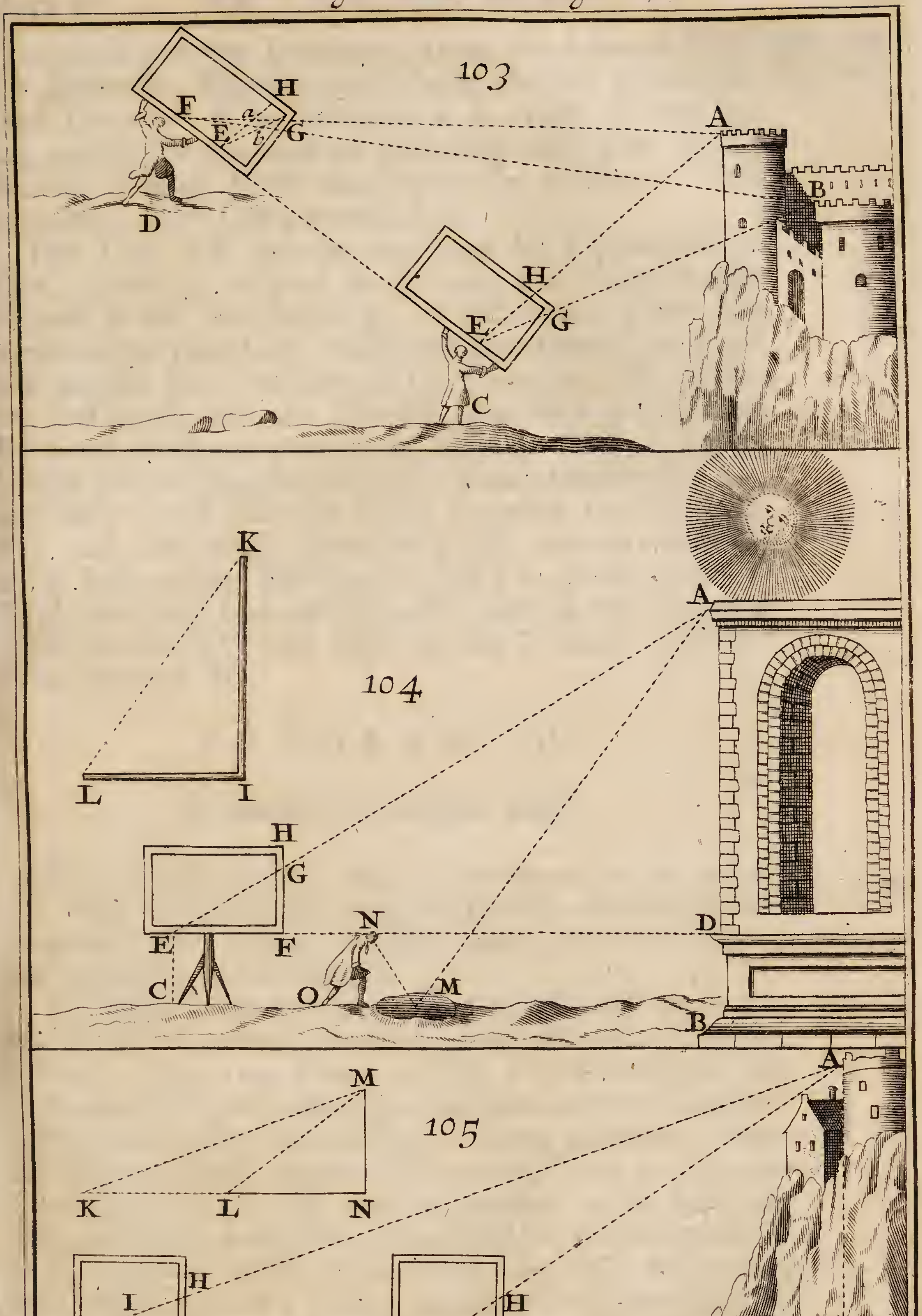
PROBLEM VI.

To meaſure on the Ground below a Line parallel to, and elevated above the Horizon.

Plate XII. Fig. 103. THE Horizontal Line AB, that repreſents in this caſe the diſtance of the two Windows, may be meaſured very eaſily by the help of the Univerſal Inſtrument after this manner.

Having choſen on the Ground the two Points of Station C, D, which ſhould be as nigh as may be in a Line parallel to the Line AB propos'd, which is eaſe enough in this Example. Place in the firſt Station C, the Univerſal Inſtrument, ſo as that its Line of Direction may be parallel to the Line CD, which may be eaſily done, by erecting at the points C, D, two Sticks perpendicularly of the ſame Height, as of three or four Feet, that is to ſay of an Height equal to that of the Foot or *Staff* that ſuſtains the Inſtrument, and while you view thro' the lower Sights, the Stick erected at D, the Inſtrument being at C, and the Alidad at the point E, anſwering perpendicularly to the point C, look thro' the Sights of the Alidad at the Extremities A, B, of the Line AB that is to be meaſured, and draw along the Line of Sight on the Surface of the Inſtrument the two Lines EG, EH. Then make a ſecond Station at D, when the Inſtrument being placed as before, ſo as that when the Alidad ſhall be advanced from E to F, as many equal Parts as the Line CD contains Feet, the point F anſwers perpendicularly to the point D, view after the ſame manner the ſame Extremities AB, and draw on

that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that Surface of the Instrument along the Line of Sight the Lines F*a*, F*b*, that cross there, the two first drawn from the point E to the points *a*, *b*, whose distance being applied to the Line of Direction will give by the number of equal Parts thus found, the number of Feet contain'd in the Line propos'd AB. *Plate XII. Fig. 103.*

The Line AB may be measured by Trigonometry, thus, measuring with a Semi-circle the visual Angles formed at the two Points E, F, and with a Chain the length of the Line CD, which will give that of its equal and parallel EF: and then in the Triangle EBF, knowing the Angles and the Side EF, you may find the side BF; and in like manner in the Triangle EAF, knowing besides the Angles, the side EF, you may find the side AE, and lastly in the Triangle BAE, knowing the sides AE, BE, and the Angle contain'd AEB, you may find the third Side, or the Line sought AB; or in the Triangle EBF, you may find the Side BF, and in the Triangle EAF the side AF, and lastly in the Triangle ABF the Line propos'd AB.

PROBLEM VII.

To measure an accessible Height.

AN accessible Height may be measured several ways, when the Ground is level and smooth: But I shall only mention such as may be useful, easily put in Practice, and agreeable to the Height. *Fig. 104.*

To measure the accessible Height AB, first with the Universal Instrument, choose the point of Station C as far distant, as you can, from the Base B of the Height AB to be measured, for a Reason, that the course of the Operation will easily let you see. Having therefore placed the Universal Instrument at the point C, so as that the Line of Direction EF, may be parallel to the Horizon BC, and that, when the Alidad shall be advanced from F to E, as many equal Parts as the Line BC, which you may measure with a Chain, contains Feet, the point E may answer perpendicularly to the point C: and having turned the Alidad towards the top A of the Height to be measured AB, which is done by viewing the top A thro' the Sights of the Alidad, make a point G, whence the Alidad cuts the side FA, that I suppose perpendicular to the other side EF, and then the Part FG being measured on the Line of Direction EF, the number of equal Parts so found shall be the number of Feet in the Height of the

Plate XII. Fig. 101. the point A above the point E, that is to say, the Height AD, continuing the Line EF parallel to the Horizon to D; wherefore if to this Height AD thus found, you add the Height BD, or CE, you will have the Height sought AB.

If you would find the Height AB by Trigonometry, measure with a Semi-circle the visual Angle AED, and with a Chain the Line BC equal to the Line DE, that in the Triangle ADE right Angled at D, you may find first the side AD by this Proportion,

As the Radius,
To the Tangent of the visual Angles AED,
So is the Side DE
To the Height AD.

When the Height AB is not very great, you may measure it with exactness enough by its shadow, suppose BM.

For if you mark at the same time the Shadow IL, of the Stick IK, erected perpendicularly on the Horizon, the two Shadows IL, BM, being bounded by the two Rays of the Sun KL, AM, that may pass for Parallels, make the two Right Angled Triangles KIL, ABM Similar, and then you may find the Height AB, by making this Proportion,

As the Shadow of the Staff, IL,
To its Heigbt IK,
So is the Shadow BM,
To the Height AB.

But to find this Height with more exactness, you must take with a Semi-circle, or some other way, the Height of the Sun, which will give the Angle AMB, which together with the Shadow BM being known, you may find by Trigonometry the Height AB, by making this Proportion,

As the Radius,
To the Tangent of the Sun's Height;
So is the Length of the Shadow BM,
To the Height sought AB.

When the Height AB is but small, you may measure it pretty exactly by reflection thus: Place a small piece of Glass in an Horizontal Situation, but as that is diffi-

cult

cult to do, and an insensible error here may cause a considerable one in the measure of the Height AB, instead of a Glass you had better use Water, that always retains an Horizontal Situation. Placing Water therefore, for instance at M, retire till being at O, and your Eye in N of the *Ray of Reflexion* MN, you see the top A by reflexion in the Water, and then, the *Angle of Reflexion* OMN, being equal to the *Angle of Incidence* AMB, the two Right Angled Triangles ABM, MON, will be Similar, wherefore you may find the Height AB, by measuring with exactness the distances MB, MO, and Height of the Eye ON, and by making this Proportion, *Plate XII. Fig.* 104.

As the Distance MO,
To the Distance MB;
So is the Height NO,
To the Height AB.

PROBLEM VIII.

To measure an inaccessible Height.

AN inaccessible Height may be measured, just as if it were an accessible one, when the Base can be seen, because the distance of the Base from the Point of Station may be found by *Probl.* 2. and the rest may be done by what has been already Taught. But when the Base is invisible, two Points of Station must be taken in a right Line with the Height to be measur'd, that is to say, so as that the Line of the two Stations, and that you would measure, may be in the same Plane, as you shall see.

To measure therefore the inaccessible Height AB, first by help of the Universal Instrument, having fixed the Center of the Alidad at an advantageous Point of the Line of Direction EF, as at E, and having chosen in the Field two Points of Station on a level and in a right Line with the Base B of the Height AB to be measured, which may be easily done, tho' the Base B be not visible, as CD, which ought to be distant from one another more or less, as the Height AB is greater or less; erect perpendicularly the Universal Instrument, so as that the Point E may answer perpendicularly to the Point of Station C, and the Line of Direction EF may be parallel to the Horizon CD, and the Alidad being turned toward the Top A, draw on the Surface of the Instrument along the Line of Sight, the right Line EH. Then advance *Fig.* 105.

Plate XII. Fig. 105. advance the Center of the Alidad from E to F, as many equal Parts as the Line CD contains Feet or Yards, and having erected the Universal Instrument perpendicularly, so that the Point F answer perpendicularly to the Point of Station D, and the Line of Direction EF, be as before, parallel to the Horizon CD, turn the Alidad also towards the Top A, and draw in like manner on the Surface of the Instrument, along the Line of Sight the right Line FA, which cuts the first EH in the Point I, whose Distance from the Line of Conduct EF, found by describing on the Point I, an Arc of a Circle touching the Line of Direction EF, being measured on the Line of Direction will give, by the number of equal Parts it contains, the number of Feet or Yards in the height of the Point A, above the Line of Direction EF, that is to say, in the Height AG, to which adding three or four Feet, for the Line BG, or DF, or CE, which is commonly about thus much, if you will have the Height proposed AB.

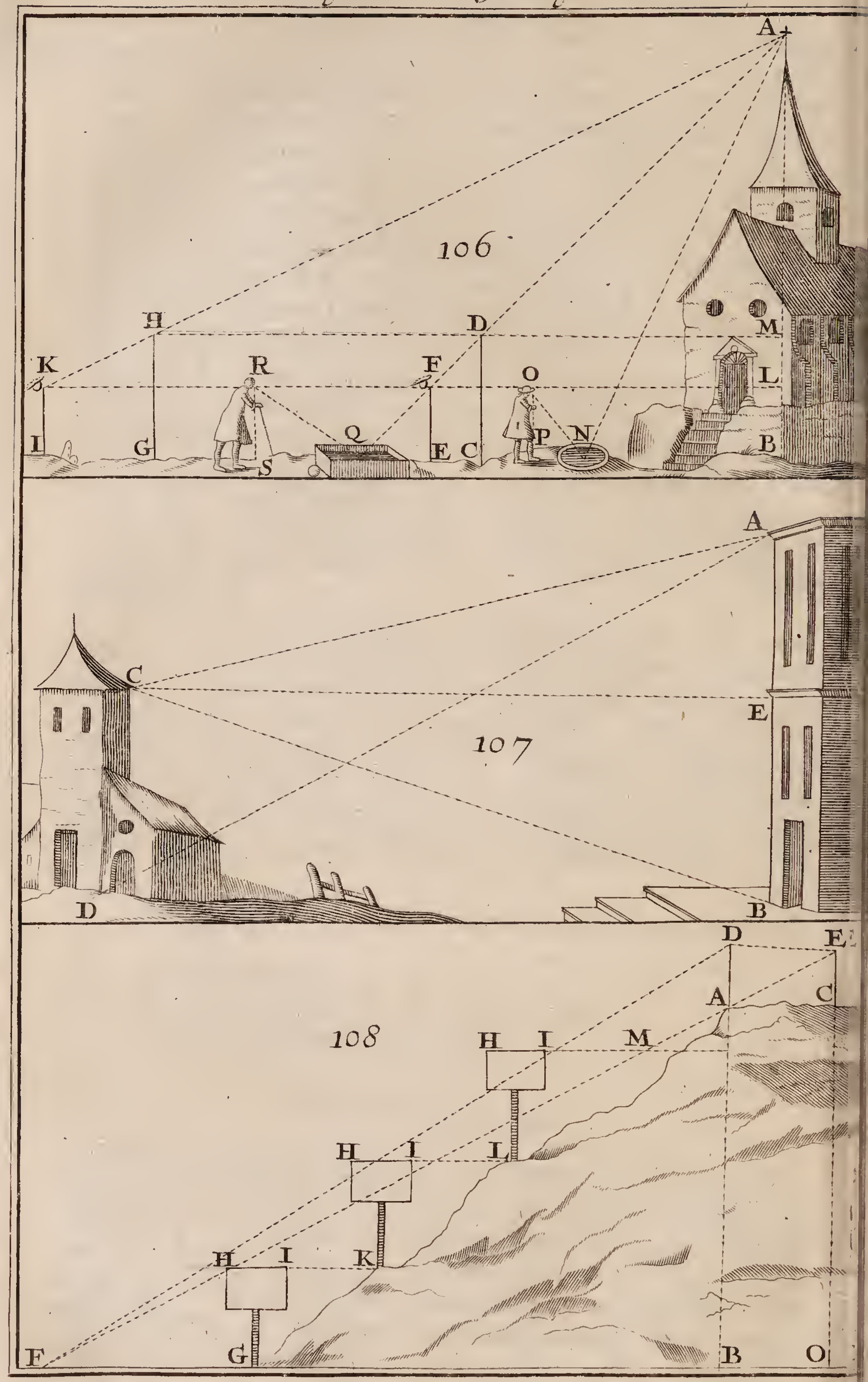
To measure this Height Trigonometrically, first measure with a Chain the Distance of the Stations CD, and with a Semicircle the Visual Angles AEG, AFG whose complements, namely, the Angles EAG, FAG, will be consequently known, and their Tangents, namely, the Lines EG, FG, in respect of the Radius AG will be also known, as well as thro' difference EF or CD, From whence you may easily draw this Proportion,

As the difference of the Tangents of the Angles EAG FAG,
To the Radius;
So is the Distance of the Stations CD,
To the Height sought AG.

SCHOLIUM.

If after having measured the Visual Angle AEG in the Point C of the first Station, you make a second at the Point D, so that the Visual Angle AFG be double the first AEG, and you measure the Line CD, in this Case equal to the Visual Ray AF; you may find the Height AG, by making in the Right-angled Triangle AFG, this Proportion,

As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As the whole Sine or Radius
to the Line AF or CD,
So is the Sine of the Angle AFG,
to the Height AG.

If you would uſe no Calculation, make the firſt Station at the Point C, ſo that the Viſual Angles AEG, be 26 Degrees and about 34 Minutes; and the ſecond Station at the Point D, ſo that the Viſual Angle AFG be preciſely 45 Degrees, and then the Height AG will be equal to the Diſtance CD, of the two Stations, as alſo to the Line BD.

If you would find the Height AG by Protraction, let KL be equal, in number of Parts, to CD, make at the Points K and L, the Angles NKM, NLM, equal to the viſual Angles AEG, AFG, draw KLN, then will the Perpendicular MN ſhew the Height AG required. *Plate XIII Fig.* 106.

When the Height AB is not very great, you may meaſure it very eaſily, by two unequal Sticks after this manner: Erect two unequal Sticks parallel to one another, and to the Height to be meaſured AB, ſo that there the Points B, C, E, be in a right Line, and that the Top A may be ſeen by the two Extremities F, D. Then make a ſecond Station in a right Line, at the Points G, I, that you may erect as before the two Scicks GH, IK, equal to the two former, ſo that the Top A may be ſeen after the ſame manner by the two Extremities H, K, imagine the two right Lines HDM, KFL, drawn through the Points, H, K, parallel to one another and to the Horizon IB; then to find the Height AM of the Top A above the longer Stick CD or GH; meaſure exactly the length of the Sticks, and the Diſtances IG, EG, GC, to make this Proportion,

As the difference of the Diſtances, IG, EC,
to the difference of the Sticks, CD, EE;
So is the Diſtance GC,
to the Height AM.

In the ſimilar Triangles AKF, AHD, you know *by* 46. that the four Lines AK, AH, FK, DH, are proportional, likewiſe in the ſimilar Triangles AKL, AHM, that the four Lines AK, AH, AL, AM are proportional, conſequently, that the four Lines FK, DH, AL, AM, or EI, CG, AL, AM, are alſo proportional; wherefore *by Diviſion* you may find that the Difference of the two firſt EI, CG, or IG, EC, is to CG, as the difference

LM of the two last AL, AM, or of the two Stick CD, EF, is to AM. *Which was to be Demonstrated.*

The Height proposed AB may be found directly by Reflection after this manner. Having placed a piece of Glass Horizontally in an advantageous Place as the Point N, on a Level with the Base B of the Height AB to be measured, retire to P, so that having your Eye at the Point O of the Line of Reflection NO you may discern the Top A, by the Angle of Reflection PNO always equal to the Angle of Incidence ANB. Then place another or the same piece of Glass at some other advantagious Point as Q, but on a level and in a right Line with N and B, that it may be Horizontal, then retire as before, till, being for Instance in E, you may discern the same Top A, by the Angle of Reflection SQR equal to the Angle of Incidence AQB. *Lastly*, measure the Height of the Eye PO or RS exactly, and the Distances NP, NQ, QS, that you may make this Proportion, which will give you at once the Height AB.

Plate XIII. Fig. 106.

As the Difference of the Distances SQ, PN,
To the Height of the Eye, RS, or OP,
So is the Distance QN,
to the Height propos'd, AB.

DEMONSTRATION.

In the similar Triangles ANB, OPN, the four Lines BN, PN, AB, OP are proportional *by* 4. 6. and by substituting BQ, QR, instead of the two last AB, OP, or AB, RS, that are in the same Ratio *by* 4. 6. by reason of the similar Triangles ABQ, RSQ, you will find that BN, PN, BQ, QS are also proportional; wherefore *by Compounding* you will find BP, PN, BS, QS, proportional, and by *Permutation,* that BP, BS::PN, QS, and *by Division* that PS, BP : : QS—PN, PN, and by *Permutation* PS, QS—PN :: BP, PN, and *by Division*, that QN, QS—PN : : BN, PN, and if in the room of the two last BN, PN you substitute AB, OP that are in the same Ratio, *by* 4. 6. the Triangles ABN, OPN being similar, you will find that QN, QS—PN, AB, OP are proportional, and lastly, *by Conversion* that QS—PN, QN, OP, AB are proportional. *Which was to be Demonstrated.*

PROB-

PROBLEM IX.

To measure from a small Height a greater where the Base is visible.

TO measure the Height AB, whose Base B is visible from the Point C of the small Height CD, whose Base D is supposed upon a level with the Base B, that is to say, the Line BD is supposed parallel to the Horizon, imagin the Line CE drawn parallel to the Line DB, and measure with a Semicircle the Quantity of the visual Angles ACE, BCD, and with a Plummet the Height CD, then you may find in the Triangle CDB, Rectangled in D, the Line DB, or CE its equal, and in the Triangle AEC rectangled in E, the Line AE, to which adding the Height BE, or CD its equal, you will have AB the Height sought. Or which is as well in the Rightangled Triangle BDC, find the Hypotenuse BC, and in the Oblique-angled Triangle ACB, find the Side or Height AB sought. The two Proportions, that are to be made in the two Rightangled Triangles ACE, CDB, to find the Height AE, may be reduced very easily to this, *Plate XIII. Fig.* 107.

As the Square of the Radius,
To the Rectangle of the Tangents of the visual Angles ACE, BCD;
So is the Height CD,
To the Height AE sought.

PROBLEM. X.

To measure from a greater Height a less, when the Base is visible.

WHen you are to measure a Height as CD, whose Base D is visible, from the Top A of a greater Height AB, whose Base B is supposed on a level with the Base D, that is to say, the Line BD is supposed parallel to the Horizon, imagine a Line CE drawn parallel to the Line BD, and measure with a Semicircle the quantity of the visual Angles BAC, BAD, and with a Plummet the Height AB, and then you may find in the Triangle ABD, Rightangled in B, the Line BD, or EC its equal, and in the Triangle AEC, the Line AE, which being substracted from the Height AB, leaves the Height *Fig.* 107.

Plate XIII. Fig. 107. BE or CD ſought. Or in the Rightangled Triangle ABD, find the Hypotenuſe AD, and in the Oblique-angled Triangle ACD, you may find the Side or Height ſought CD.

The two Proportions to be made, in the two Triangles ABD, AEC, to find the Line AE, may be eaſily reduced to this,

As the Square of the Radius,
To the Product of the Tangents of the viſual Angles BAD, BAC;
So is the Height AB,
To the Line AE.

PROBLEM XI.

To meaſure the Height of a Tower ſituated on a Mountain.

Fig 108. 'TIS evident, that if *by Probl.* 10. you meaſure the height of the top of the Tower above the Plane of the Horizon, and the Height of the Mountain, and then ſubſtract that Height, there will remain the Height of the Tower above the Mountain. As if AD repreſent a Tower upon the Mountain GAC, ſubſtracting from the whole Height DB of the top D above the Plane of the Horizon BF, the Height AB of the Mountain, you will have the Height ſought AD.

SCHOLIUM.

When the Mountain GAC is not very high, and it may be gone over from top to bottom, that is to ſay from A to G, you may eaſily meaſure the Height AB and its *Talû* BG by a very ſimple Method call'd *Cultellation*, and may be very eaſily put in practiſe by the help of the Univerſal Inſtrument, as you ſhall ſee.

Having placed the Univerſal Inſtrument perpendicularly on its Staff faſtened to the Point G. So that the Line of Direction HI, be Horizontal, direct by its two Sights the viſual Ray HK, and the Inſtrument being placed after the ſame manner at the Point K, draw the Ray HL in like manner, to get the Point L on the Declivity of the Mountain, whither the Inſtrument being tranſported after the ſame manner, direct the Ray HM, and ſo continue till you come to the top A; and then 'tis evident that all the Lenghts of the Lines HK, HL, HM, &c. which are eaſily meaſured, being added together will give the *Talû* BG, and the Height of the firſt Point

Point H, being multiplied by the number of Stations, will give the Height ſought AB. *Plate XIII Fig. 108.*

The ſame Height AB may alſo be eaſily meaſured by Trigonometry, if you meaſure with a Chain the Length AG, and with a Semicircle the Angle BAG, whoſe Complement will give the Angle AGB, becauſe the Angle B is right: For in the right-angled Triangle AGB, the Height AB may be found by this Proportion,

As the whole Sine or Radius,
To the Length AG;
So is the Sine of the Angle AGB,
To the Height AB.

If the Declivity of the Mountain be too unequal to be commodiouſly meaſured, but there is a pretty large Plane above, make two Stations at two commodious Places, as A, C, from whence you may ſee the ſame Point of the Field as F, and meaſure the Height AB thus,

Having made your firſt Station in A, meaſure with a Semicircle ſuſtained by a perpendicular Staff AD, the viſual Angle ADF, and having in like manner made your ſecond Station in C, meaſure with a Semicircle ſuſtained by the perpendicular Staff CE equal to the firſt AD, the viſual Angle CEF, which ſubſtracted from the right Angle CED gives the Angle DEF as the firſt Angle ADF being added to the right Angle ADE, gives the Angle EDF, and conſequently the Angle EFD. Thus in the Triangle FDE, by knowing the Angles, and the Side DE, equal to the Diſtance AC, that may be meaſured with a Chain, wherefore you may find the Line DF, and in the Triangle DBF rightangled in B, you may find the Height DB, from whence take the Height of the Stick AD, and you have the Height ſought AB.

The Height DB may be found much eaſier, reaſoning thus. Since in the rightangled Triangle DBF, the Angle BDF is known, its Tangent BF in regard of the Radius BD is alſo found in the Table of Tangents, and in like manner ſince in the rightangled Triangles EOF, the Angle OEF is known, its Tangent OF, OE equal to BD being Radius, is alſo known; For the Lines DB, EO being perpendicular to the Horizon may paſs for Parallels. Wherefore ſubſtracting the Tangent BF from the Tangent OF, the difference of the two Tangents will be BO or AC, or the Radius DB, or EO, equal to 10000000 parts: And ſince the Line AC is already known,

you may find its value in regard of the Radius BD, by this Proportion,

Plate XIII. 108.

As to the Difference of the Tangents,
To the Radius :
So is the Distance AC of the Stations,
To the Height sought BD.

After the same Method you may measure any Depth, as you shall see in

PROBLEM XII.

To measure a Depth.

Plate XIV. Fig. 109.

TO measure the Depth AB, first by the help of the Universal Instrument, chuse such commodious Points, as may be in a right Line with the Point B, on the same Level, and as far distant from one another and the Point B as possible, as C, D, for the Points of Station, where place the Instrument after this manner.

Having fix'd the Center of the Alidad of the Universal Instrument in a fit Point of the Line of Direction GH, as in E, and the Instrument being erected perpendicularly, so as that the Point E, may answer perpendicularly to the Point C, and the Line of Direction GH to the Line CD, turn the Alidad towards the Point A, so that you may see the Point A thro' the Sights, and draw on the Surface of the Instrument along the Line of Sight, the right Line EF.

After that advance the Center of the Alidad from E to G, as many equal Parts of the Line of Direction GH, as the Line CD contains Feet, and the Universal Instrument being erected as before Perpendicular to the Point D, so as that the Point G answer perpendicularly to the Point D, and the Line of Direction GH to the Line CD, turn after the same manner the Alidad toward the Point A, and draw on the Surface of the Instrument along the Line of Sight, the right Line GI, meeting the former EF at the Point I.

Lastly, imagine a Perpendicular from the Point I to the Line of Direction GH, and this Line GH will represent the Depth sought AK, and to gain the Length of that Perpendicular, without actual drawing it, place one Point of the Compasses at the Point I, and describe upon that Point an Arc of a Circle touching the Line of Direction GH, and the Aperture of the Compasses will give the Length

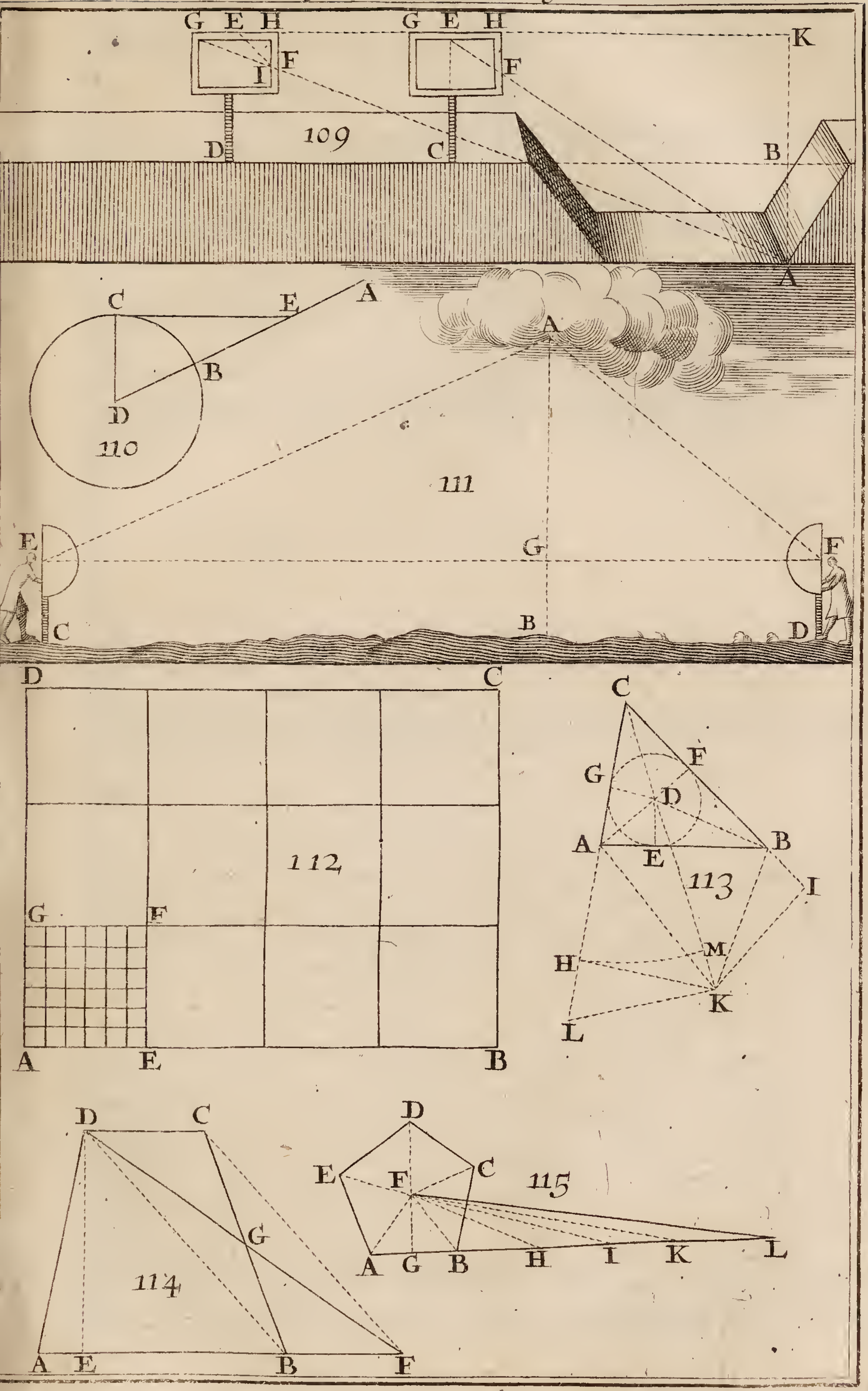
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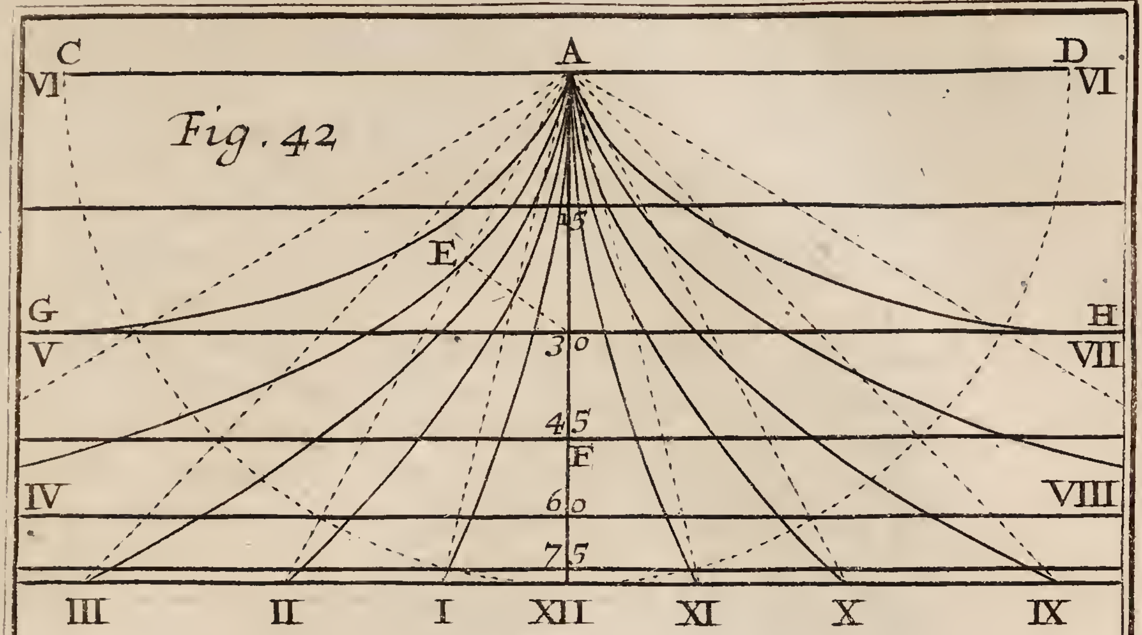
Fig. 42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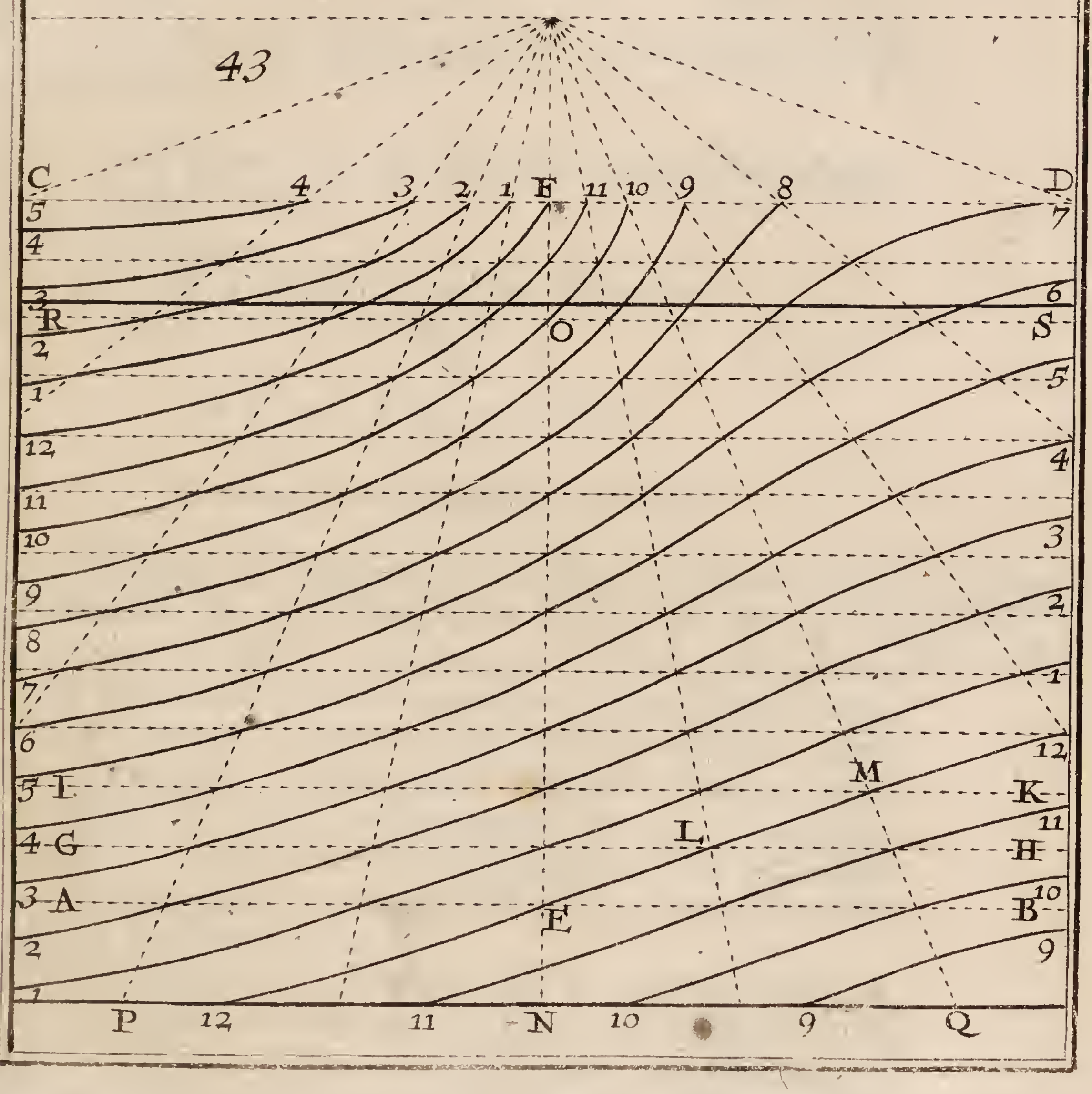
C 4 3 2 1 F 11 10 9 8 D
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Length of the Perpendicular, which applied to the Line *Plate XIV. Fig.* 109. of Direction GH, will give a number of equal Parts equal to the number of Feet contained in the Height of the Eye above the Point A, that is to say, tho Line AK, which being lessen'd by the Line BK, or CE, that is to say, the Height of the Eye above the Horizon CD, will give the Depth sought AB.

To measure the Line AK by Trigonometry, first measure with a Chain the Distance CD of the two Stations, equal to the Line GE, and then measure with a Semicircle the quantity of the visual Angle CEA, DGA, and then in the oblique-angled Triangle GAE, find the Line AE, and in the rightangled Triangle AKE, the Line AK: Or in the oblique-angled Triangle GAE, find the Side AG, and in the rightangled Triangle AKG, the Line AK.

But these two Proportions may be reduced to this one very easily,

As the Difference of the Tangents of the visual Angles
CEA, DGA,
To the Radius:
So is the Distance CD of the Stations,
To the Line AK sought.

PROBLEM XIII.

To measure the Height of a Cloud.

IN measuring the Height of a Cloud, choose a time *Plate XIV. Fig.* 111. when no Wind is stirring, that the Cloud may be as little moved as possible, let the two Observers, at a considerable Distance from one another in the same Plane, observe at the same time, a certain Point agreed upon, with a Semicircle, which each ought to have, under what Angle the Point shall appear, for then measuring the Distance of the two Observers exactly, you will have Triangle, in which you may find by Trigonometry the Height of the Point proposed.

Thus to measure the Height AB, or AG, of a Cloud, choose in a Plain two Points considerably Distant, as C, D, where the two Observers must take their Stations, to measure at the same time with their Semicircles, the quantity of their visual Angles AEF, AFE, looking at the Point A, agreed upon, when they separated. But that the time of taking the Observations may be the same, some sensible Sign, as the report of a Musket, or

Plate XIV. Fig. 111. the like, must be pitch'd upon, to inform the other when he must apply himself to observe the Point A, otherwise the motion of the Cloud may cause some Error. Knowing therefore the Angles AEF, AFE, the Angle EAF, will be also known by 32. 1. Wherefore measuring the Distance CD or EF its equal, you may find in the Oblique-angled Triangle AEF, the Side AE, and in the rightangled Triangle AGE, the Side AG, or the Height sought AB, the difference BG being of little Consequence.

SCHOLIUM.

Because in this Example the Distance of the two Stations CD ought to be very great, just as when a very high Mountain is to be measured, and the Line CD being on the Surface of the Earth, which is Spherical, not right; we shall here determine the Error arising from taking CD for a right Line, as we have always done.

Fig. 110. Suppose the Line CB, that is on the Circumference of the Earth, whose Center is D, and Semidiameter DC or DB, to be 7608 Yards, or 22824 Feet, in which case the Arc CB or the Angle D will be about 4 Minutes, because by Observation a Minute of the Circumference of a great Circle of the Earth is about 1902 Yards, or 5706 Feet, as we shall shew more particularly in Geography.

But since in Practise the Arc CB is taken for a right Line, it may consequently be taken for the Line CE, that touches the Earth in the Point C: Wherefore if AB represent the Height of a Cloud, the Line AE may be taken for that Height, so the Line BE will be the Difference or Error, and may be found by Trigonometry, measuring the Hypotenuse DE of the Rightangle Triangle CDE, where, besides the Angles, is known the Side CD or Semidiameter of the Earth, which is about 6538594 Yards, or 19615182 Feet, as we shall shew more particularly in Planimetry, namely by making this Proportion,

As the Radius	10000000
To the Secant of the Angle D,	10000007
So is the Semidiameter CD	19615782
To the Line DE,	19615795.9

which you will find about 19615795 Feet and 9 Inches, from whence substracting the Semi-diameter DB, supposed to be 19615782 Feet, and the remainder 13 Feet 9 Inches, gives the Error or Difference sought BE.

THE

The THIRD PART.

OF

PLANIMETRY.

PLanimetry, call'd alſo *Surveying*, as has been ſaid already, ſhews how to meaſure a Plane, and any other kind of Surface. And as in *Longimetry* we have meaſured Lines by leſs ones, ſo in *Planimetry*, Surfaces are meaſured by leſs ones, that are generally Squares, and ſometimes Oblongs. Geometers having pitch'd upon a right Angle rather than an acute or obtuſe one, becauſe theſe two may be infinitely varied, but a right one is invariable, and the only one of its kind.

The number of Square Meaſures contain'd in a Superficies, is always found by Multiplication, becauſe 'tis conceiv'd equal to a Rectangle, whoſe Area is found by multiplying the Length into the Breadth. Thus if you multiply the Length AB, of the Rectangle ABCD, four Fathoms, by the Breadth AD three Fathoms, you will have twelve ſquare Fathoms for the Surface of the Rectangle, form'd by the Interſections of the right Lines drawn lengthwiſe, and croſs thro' the Diviſions of the oppoſite Sides.

Whence it follows, that a Fathom long as AE, contains ſix Feet long, a Fathom ſquare contains 36 Feet Square, for multiply 6 by 6, and the Product is 36: And a Foot long containing 12 Inches long, a Foot Square will contain 144 ſquare Inches, becauſe the Product of 12 by 12 is 144. Hence we ſay an Acre contains 160 Perches ſquare, becauſe we allow 40 Perches to the length of one its Sides, and 4 to the other, ſo multiplying 40 by 40, the Product is 160. And ſo of the reſt. *Plate XIV. Fig.* 112.

Wherefore when Square Yards are given to be reduced into ſquare Feet, inſtead of multiplying them by 3, you muſt multiply by 9. Thus knowing the Area of the Rectangle ABCD 12 Yards ſquare, to know how many ſquare Feet it contains, you muſt multiply them by 9, and you will have 108 ſquare Feet for the Content of the Rectangle ABCD. But on the contrary, when Feet are to be turn'd into ſquare Yards, inſtead of dividing by 3, divide by 9. And ſo with the reſt.

Plate XIV. Fig. 112. I said Surfaces are sometimes measured by Oblongs, which is done chiefly for ease of Calculation, when different Species are to be multiplied together. And then a Rectangle containing three Square Feet is call'd a *Foot of a square Yard*, as a Yard long contains three long Feet, this Rectangle being used in Practise instead of a Yard square, containing 9 square Feet, and is call'd a *Foot of a square Yard*, because it has a Foot of Breadth to a Yard of length.

In like manner a Rectangle, whose Superficies contains twelve Inches square, is call'd *an Inch of a Foot square*, as a Foot long contains twelve Inches long, this Rectangle being in Practise put instead of a Foot square, that contains 144 Inches square, and is call'd an *Inch of a Foot square*, because it has an Inch of Breadth to a Foot of Length.

After the same manner, we ought to call an *Inch of a Square Yard*, a Rectangle whose Breadth is an Inch, and Length a Yard, or 36 Inches, and whose Area consequently is 36 Square Inches, just as a Yard long is three Feet long. Hence it is that Surveyors say, Yards long multiplyed by Yards long produce *Yards Square*: Feet long multiplyed by Feet long produce *Feet Square*, and so on. But that Yards long multiplyed by Feet long, produce *Feet of a Square Yard*, and thus Feet long multiplyed by Inches long produce *Inches of a Square Foot*.

CHAPTER I.

THEOREMS.

THO' the Problems in Planimetry are easie in Performance, yet the Theory is somewhat hard. Wherefore to render the Practise as easie as I can, I thought it proper to separate the Theory from the Practise, that such as do not care to work without a Reason, may find the Demonstrations of all the Rules of Practise, that shall be laid down, and such, as shall be contented with the Practise only, may find it disengaged from the Theory, and so may take it up more easily.

THEOREM I.

The Area of a rectilinear Triangle is a fourth proportional to the Radius, the Tangent of half one of the three Angles, and to the Rectangle under half the Perimeter of the Triangle, and the excess of that half above the opposite Side to the same Angle.

Plate XIV. Fig. 113. I Say the Ratio of the Radius to the Tangent of half the Angle C, of the Triangle ABC, is equal to that

of

of the Rectangle under half the Perimeter, that is to say, half the Sum of the three Sides of the Triangle ABC, and the Excess of that half above the Side AB opposite to the same Angle C, to the Area of the Triangle ABC. *Plate XIV. Fig.* 113.

PREPARATION.

Inscribe *by* 4. 4. in the Triangle ABC, the Circle EFG, and its three Radius's DE, DF, DG, that terminate in the Point of Contact, E,F,G, will be perpendicular to the Sides they fall on *by* 18. 3. so that all the Angles made by the Radius's at the Points of Contact, E,F,G, will be right, and the two Lines AE, AG, will be equal, because Tangents to the Circle drawn from the same point A, since *by* 36. 3. the Square of each is equal to the same Rectangle, besides the two right Angled Triangles AGD, AED, are equal, &c. By the same way of reasoning, you may find that the two Lines BE, BF, are equal, as well as CF, CG. Whence 'tis easie to conclude that the three Lines AE, BE, CF, or the two AB, CF, alone are equal to half the Perimeter of the Triangle ABC, consequently CF, or CG, is the excess of half the Perimeter above the Side BA. Thus we have to demonstrate, that the Radius is to the Tangent of the Angle ACD, or BCD, half the Angle ACB *by Constr.* As the Rectangle under CF, or CG and half the Perimeter of the Triangle ABC, to the Area of the same Triangle ABC.

DEMONSTRATION.

In the Triangle CGD right Angled in G, the Radius is to the Tangent of the Angle GCD, as CG to DG, by *Theor.* 1. *Chap.* 1. *L.* 2. *Trigon.* Wherefore allowing the two latter Terms CG, DG, half the Perimeter of the Triangle ABC, for the common Height, you will find *by* 1. 6. that the Radius is to the Tangent of the Angle GCD, as the Rectangle under CG, and half the Perimeter is to the Rectangle under DG, and half the Perimeter, that is to say, to the Area of the Triangle ABC, because *by* 41. 1. The Triangle ADC is equal to the Rectangle under DG and half AC, in like manner the Triangle CDB is equal to the Rectangle under DF equal to DG, and half BC, also the Triangle ADB is equal to the Rectangle under DE equal to DG, and half AB, *Which was to be Demonstrated.*

THEO-

THEOREM. II.

The Area of a Rectilinear Triangle is a mean Proportional between the Rectangle under half the Perimeter and the Exceſs of that half above one Side, and the Rectangle under the two Exceſſes of the ſame half above each of its two other Sides.

Plate XIV. Fig. 113. I Say the Area of the Triangle ABC is a mean Proportional between the Rectangle under half its Perimeter or half the Sum of the three Sides, and the Exceſs of that half above the Side AB, and the Rectangle under the two Exceſſes of the ſame half above each of the two other Sides AC, BC.

PREPARATION.

Having inſcrib'd as above in the Triangle ABC, the Circle EFG, whoſe Center is D, add to the Side AC, the Line AH equal to the Line BE, or BF, and the Line HL equal to the Line AE, or AG, and then the Line AL will be equal to the Side AB. Add alſo to the Side BC, the Line BI equal to the Line AE, or AG, and then the Line CI will be equal to the Line CH, each being equal to half the Perimeter of the Triangle ABC; For as in *Theor.* 1. the two Lines AB, CF, are together equal to half the Perimeter, after the ſame manner you may find that the Sum of AC, BE, or AC, AH, or the ſingle Line CI is half the Perimeter, becauſe we have already found that CF or CG, is the Exceſs of half the Perimeter above the Side AB, and BE or BF, the exceſs of half the Perimeter above the Side AC, and AE or AG is the exceſs of half the Perimeter above the Side BC. Thus we have to demonſtrate, that the Area of the Triangle ABC is a mean Proportional between the Rectangle under the Lines AB, CF, and the Rectangles under the Lines AE, BE, which we ſhall do after we have drawn a Perpendicular HK to the Line CH, from the point H, cutting the Line CD produced in the point K, thro' which drawing the right Lines KA, KB, and further the right Line KI perpendicular to the right Line CI and equal to KH, becauſe the two right Angled Triangles CHK, CIK, are equal *by* 4. 1. whence KHL, KIB, are alſo equal *by* 4. 1. as well as ABK, ALK *by* 8. 1. and conſequently the Angle KAL is equal to the Angle KAB, that is to ſay, the Line AK divides the Angle LAB, juſt as the right Line AD does the Angle EDG, into two equal

equal Parts, because the two right angled Triangles AED, AGD, are equal. Whence it follows, that the Angle ADE is equal to the Angle HAK, for the Angles E, G, being right, the opposite Angles GAB, GDE, of the Quadrilateral Figure AEDG, will be equal to two right *by* 32. 1. and consequently equal to GAB, HAB, which taken together are equal to two right *by* 13. 1. *Plate XIV. Fig.* 113.

Wherefore taking from each Side the common Angle GAB, there will remain the Angle GDE, equal to the Angle HAB, and the half ADE equal to the half HAK: Which makes the two right angled Triangles AED, AHK, similar: Wherefore *by* 4. 6. the four Lines DE, AE, AH, KH, are proportional, and *by* 16. 6. the Rectangle of the Extreams DE, KH, is equal to the Rectangle of the means AE, AH, or AE, BE.

DEMONSTRATION.

If in the Triangle CHK right angled at H, you make the Side CH, or half the Perimeter Radius, the other Side HK, will become the Tangent of the Angle HCK half the Angle ACB, as you will find by describing from the point C, thro' the point H, the Arc of the Circle HM: And because by *Theor.* 1. the Radius CH, or half the Perimeter, is to the Tangent KH, as the Rectangle under CF and half the Perimeter, so the Triangle ABC. If to the two first Terms, namely, half the Perimeter and Line KH, you give the common Height DE, then *by* 1. 6. the Rectangle under half the Perimeter and the Line DE, that is to say, the Triangle ABC, as was seen in *Theor.* 1. is to the Rectangle under the Lines KH, DE, or the Rectangle of the Lines AE, BE, as the Rectangle under half the Perimeter and the Line CF, to the Triangle ABC, and consequently the Triangle ABC is a mean Proportional between the Rectangle under half the Perimeter and the Line CF, and the Rectangles under the Lines AE, BE. *Which was to be Demonstrated.*

THEOREM. III.

The Area of a rectilineal Triangle having an Angle right is equal to a Rectangle under half its Perimeter, and the Excess of that half above the Hypotenuse: Or to a Rectangle under the two Excesses of the same half above each of the two Sides.

THE Demonstration of this Theorem will be evident, when we have once demonstrated *that half the Perimeter*

meter of a right angled Triangle, and the three Exceſſes of that half above the two Sides, and Hypotenuſe, are four proportional Quantities, which we have diſcover'd by Syntheſis, and ſhall demonſtrate by Analyſis, as you ſhall ſee.

To find therefore, whether taking ſeparately from the half of the Perimeter $\frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c$, of the right-angled Triangle a, b, c, the two Sides a, b, and Hypotenuſe c, this half $\frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c$, and the three Remainders or Exceſſes. $-\frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c$, $\frac{1}{2}a - \frac{1}{2}b + \frac{1}{2}c$, $\frac{1}{2}a + \frac{1}{2}b - \frac{1}{2}c$, make four proportional Quantities, reaſon thus.

SYNTHESIS.

If $\frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c, -\frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c :: \frac{1}{2}a - \frac{1}{2}b + \frac{1}{2}c, \frac{1}{2}a + \frac{1}{2}b - \frac{1}{2}c$, doubling the Terms $a + b + c, -a + b + c :: a - b + c, a + b - c$, and taking the Squares of all the Terms, $aa + 2ab + bb + 2ac + 2bc + cc, aa - 2ab + bb - 2ac + 2bc + cc :: aa - 2ab + bb + 2ac - 2bc + cc, aa + 2ab + bb - 2ac - 2bc + cc$, and putting $aa + bb$ in the room of cc, you will have this Proportion, $2aa + 2bb + 2bc + 2ab + 2ac, 2aa + 2bb + 2bc - 2ab - 2ac :: 2aa + 2bb - 2bc - 2ab + 2ac, 2aa + 2bb - 2bc + 2ab - 2ac$, and taking half all the terms $aa + bb + bc + ab + ac, aa + bb + bc - ab - ac :: aa + bb - bc - ab + ac, aa + bb - bc + ab - ac$, and compounding $2aa + 2bb + 2bc, aa + bb + bc + ab + ac :: 2aa + 2bb - 2bc, aa + bb - bc - ab + ac$, and taking half the Antecedents, $aa + bb + bc, aa + bb + bc + ab + ac :: aa + bb - bc, aa + bb - bc - ab + ac$, and dividing $ab + ac, aa + bb + bc :: -ab + ac, aa + bb - bc$, and by Permutation $ab + ac, -ab + ac :: aa + bb + bc, aa + bb - bc$, and by dividing $2ab, ab + ac :: 2bc, aa + bb + bc$, and taking half the Antecedents $ab, ab + ac :: bc, aa + bb + bc$, and putting cc in the room of $aa + bb$, you will have this Proportion leſs compounded, $ab, ab + ac :: bc, cc + bc$, and laſtly dividing the two firſt Terms by a, and the two laſt by c, you will have this laſt Proportion, $b, b + c :: b, c + b$, which being known, will ſerve to make a Demonſtration by Analyſis thus.

ANALYSIS.

If you make this Proportion $b, b + c :: b, c + b$, and multiply the two firſt Terms by a, and the two laſt by c, you will have this Proportion $ab, ab + ac :: bc, cc + bc$, and if in the room of cc you put $aa + bb$, you will have this,

this, ab, $ab + ac :: bc$, $aa + bb + bc$, and doubling the Antecedents $2ab$, $ab + ac :: 2bc$, $aa + bb + bc$, and by Division, $ab + ac, - ab + ac :: aa + bb + bc, aa + bb - bc$, and Permutation $ab + ac$, $aa + bb + bc :: + ab + ac$, by $aa + bb - bc$, and by compounding $aa + bb + bc$, $aa + bb + bc + ab + ac :: aa + bb - bc$, $aa + bb - bc - ab + ac$, and by doubling the Antecedents $2aa + 2bb + 2bc$, $aa + bb + bc + ab + ac :: 2aa + 2bb - 2bc$, $aa + bb - bc - ab + ac$, and dividing $aa + bb + bc + ab + ac$, $aa + bb + bc - ab - ac :: aa + bb - bc - ab + ac$, $aa + bb - bc + ab - ac$, and doubling all the Terms $2aa + 2bb + 2bc + 2ab + 2ac$, $2aa + 2bb + 2bc - 2ab - 2ac :: 2aa + 2bb - 2bc - 2ab + 2ac$, $2aa + 2bb - 2bc + 2ab - 2ac$, and putting $1cc$ in the room of $1aa + 1bb$, you will have this other Proportion $aa + bb + cc + 2bc + 2ab + 2ac$, $aa + bb + cc - 2ab - 2ac + 2bc :: aa + bb + cc - 2bc - 2ab + 2ac$, $aa + bb + cc - 2bc + 2ab - 2ac$, and taking the Square Root of each Term, $a + b + c$, $a + b - c :: a - b + c$, $a + b - c$, and lastly taking half all the Terms $\frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c$, $- \frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c :: \frac{1}{2}a - \frac{1}{2}b + \frac{1}{2}c$, $\frac{1}{2}a + \frac{1}{2}b - \frac{1}{2}c$. *Which was to be demonstrated.*

DEMONSTRATION.

Because half the Perimeter of the Rightangled Triangle is to the excess of that half above one of the two Sides, as the excess of the same half above the other Side, to the excess of the same half above the Hypotenuse, as has been demonstrated, whence it follows by 16. 6. that the Rectangle under half the Perimeter, and the excess of that half above the Hypotenuse is equal to the Rectangle under the two excesses of that half above each of the two Sides : And because the Area of the same Triangle is a mean Proportional between the two equal Rectangles, by *Theor.* 2. it must necessarily be equal to each of the two Rectangles. *Which was to be demonstrated.*

THEOREM IV.

The Area of a Trapezoid is half the Rectangle under the Sum of the two Parallel Sides, and the Perpendicular drawn between the two Sides.

I Say the Area of the Trapezoid ABCD, whose two Sides AB, CD, are parallel, is equal to half a Rectangle, that has for its Length the Sum of the two parallel Sides AB, CD, and for the Breadth the Perpendicular *Plate XIV. Fig.* 114.

Plate XIV. dicular DE, drawn between the two parallel Sides AB, CD.
Fig. 114.

DEMONSTRATION.

If you draw CF, parallel to the Diagonal DB, thro' the Point C, meeting the Side AB produced in the Point F, and join the right DF, you will find by 37. 1. the Triangle CDF equal to the Triangle CBF, wherefore if from each you take away the Triangle CGF, you will have the Triangle CGD equal to the Triangle BGF, and if you add to each of the two equal Triangles CGD, BGF, the Trapezium ABGD, you will find the Trapezoid ABCD equal to the Triangle ADF, that is to say, by 41. 1. to half the Rectangle under the Perpendicular DE, and Sum AF of the two parallel Sides AB, CD, because the Figure BFCD being a Parallelogram *by Constr.* the two opposite Sides BF, CD are equal by 34. 1. *Which was to be demonstrated.*

THEOREM V.

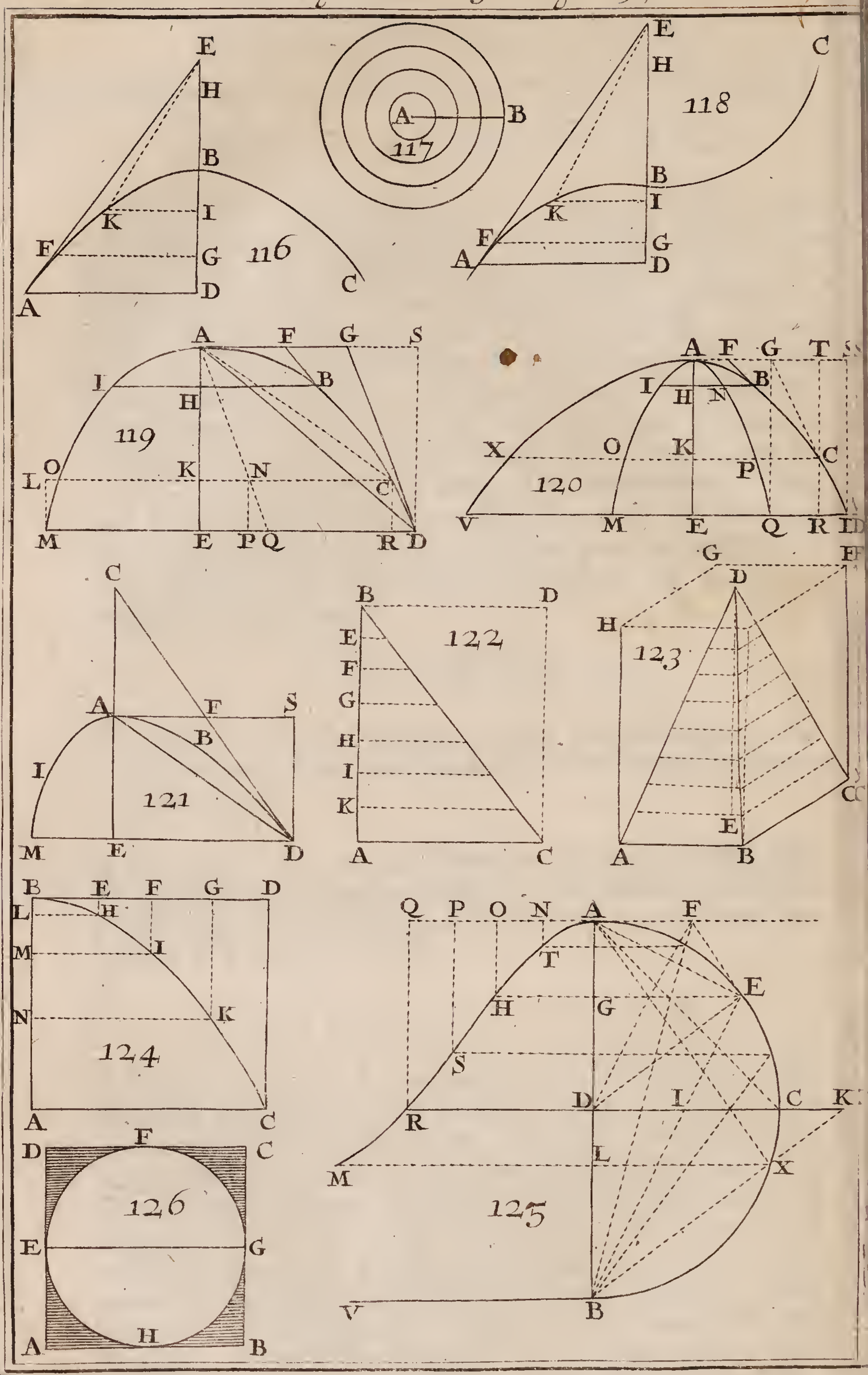
The Area of a regular Polygon is half the Rectangle under its Perimeter, and a Perpendicular drawn from the Center to one Side.

Fig. 115. I Say the Area of the Regular Pentagon ABCDE, is half the Rectangle under its Perimeter or Circumference AL, and the Perpendicular FG drawn from the Center F to the Side AB.

DEMONSTRATION.

Because the Line AL represents the Perimeter of the Polygon ABCDE, if you divide it into as many equal Parts as the Polygon has Sides, that is, in this case, into five, at the Points H, I, K, each Part will be equal to a Side of the Pentagon, and if you draw to the Center F, thro' the Points H, I, K, L as many right Lines, you will have the great Triangle AFL made up of as many equal Triangles as those of the Pentagon ABCDE, that are made at the Center. Whence it follows, that the Pentagon ABCDE is equal to the Triangle AFL, that is to say, by 41. 1. to half the Rectangle under the Perimeter AL, and Perpendicular FG. *Which was to be demonstrated.*

THEO-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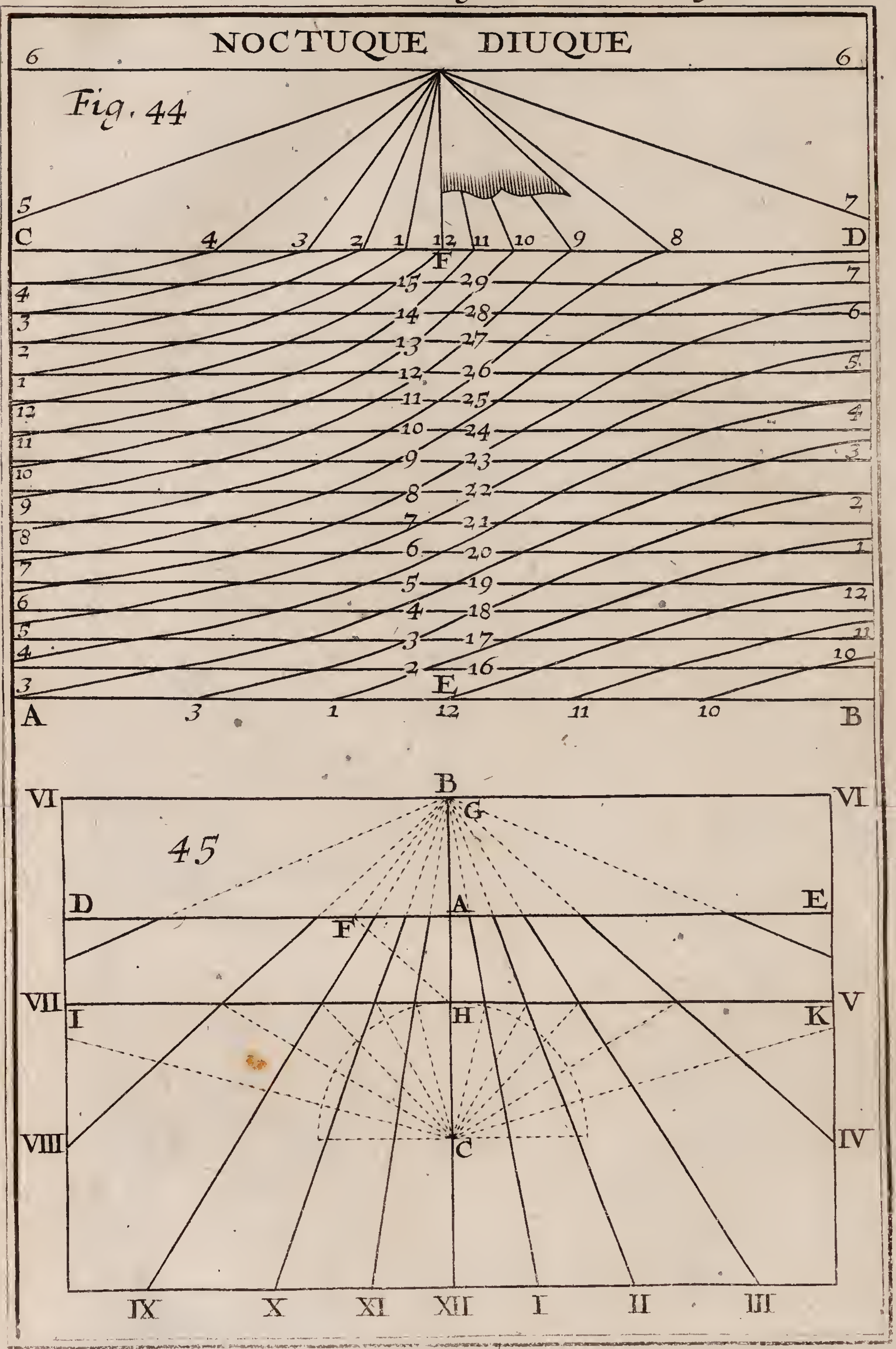
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Fig. 44
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THEOREM VI.

If thro' a Point of the Perimeter of the Quadratic Parabola, an Ordinate be drawn within the Parabola, and a Line without cutting the Diameter in a Point, as far distant from the Vertex as the Ordinate; that Line will be a Tangent to the Parabola in that Print.

I Say, if an Ordinate AD to the Diameter BD be drawn from the Point A in the Perimeter of the Quadratic Parabola ABC, and a right Line AE cutting the Diameter produced in E as far distant from the Vertex B as the Point D, that right Line AE will be a Tangent to the Curve ABC at the Point A, so as not to meet it in any other Point, as F. *Plate XV. Fig.* 116.

PREPARATION.

For if you would have it that the right Line AE should touch the Curve ABC in the Point F also, imagine an Ordinate FG drawn to the Diameter BD from that Point F, and make BH equal to BG, that EH may be equal to DG, BD, BE being equal by *Supposition.*

DEMONSTRATION.

Because by *Def.* 36. you have this Proportion, BG, BD :: FG*q* AD*q*, doubling the first terms, BG, BD, you will have this GH, DE :: FG*q* AD*q*, and giving the common Height DE, to the two first terms BG, BD, you will have DEGH, DE*q* :: FG*q*, AD*q*, and if instead of the two last terms FG*q*, AD*q*, you put GE*q*, DE*q*, that are in the same Ratio, the Triangles FGE, ADE being similar, you will have DEGH, DE*q* :: GE*q*, DE*q*, where the two Consequents being equal, the two Antecedents, namely, the Rectangle DEGH, and Square GE will be also equal, and by 17. 6. you will have this Proportion DE, GE :: GE : GH, and *by dividing*, you will have this, DG, GE :: EH : GH, whose two Antecedents being equal DG, EH, their Consequents GE, GH must be equal, which being impossible, 'tis impossible also, that the Line AE should touch the Parabola ABC in any other Point but A. Wherefore 'tis a Tangent to it in the Point A. *Which was to be demonstrated.*

SCHOLIUM.

The Converse of this Theorem is also true, namely, that if the right Line AE be a Tangent to the Parabola ABC at the Point A, the two Lines BD, BE will be equal; for if one of the Lines, as BD, be greater than BE, cut off BI equal to BE, and draw from the Point I, an Ordinate IK to the Diameter, and then the right Line EK will be Tangent to the Parabola ABC at the Point K, according to the preceding Demonstration: And because we suppose the right Line AE a Tangent at the Point A, these two Tangents AE, KE must necessarily meet one another near the Points of Contact A, K, without the Parabola; and since they meet also at the Point E, they will contain a Space, which is impossible, consequently that BD, BE, should be equal, is so too.

THEOREM VII.

If from a Point of the Circumference of a Cubic Parabola, you draw within it an Ordinate to a Diameter, and without it a right Line cutting the Diameter in a Point, whose Distance from the Vertex is double that of the Ordinate, that right Line will be a Tangent to the Parabola in that Point.

Plate XV. Fig. 118. I Say, that if from a Point A of the Circumference of a Cubic Parabola ABC, an Ordinate AD be drawn within it to the Diameter BD, and a right Line AE, without cutting the Diameter BD produced at the Point E, so that the Part BE be double the Part DB, that right Line AE will be a Tangent to the Point A of the Parabola, that is to say, cutts it in no other Point, as F.

PREPARATION.

For if you say the right Line AE meets the Circumference of the Cubic Parabola ABC, in the Point F also; imagine an Ordinate FG drawn to the Diameter BD from the Point F, and make BH double BG, that GH may be triple BG, as DE is triple BD.

DEMONSTRATION.

Because by the Property of this solid Parabola, you have this Proportion BG, BD :: FG*c*, AD*c*, tripling the

the two first Terms, you will have GH, DE : : FGc, ADc, and placing GEc, DEc instead of the two last FGc, ADc, that are in the same Ratio, because the Triangles FGE, ADE, are similar, you will have this other Proportion GH, DE : : GEc, DEc, and giving the two first Terms GH, DE, the Square DE for a Base, you will have this GHDEq : DEc : : GEc : DEc, where since the Consequents are equal, the Antecedents must also be equal, namely, the Solid under GH, and the Square DE, and the Cube GE, which is impossible, for then the Line BO would be equal to its Part BG, as you shall see. Plate XV. Fig. 118.

Put a for AD, b for FG, c for BD, d for BG, and you will have $3d$ for GH, $3c$ for DE, $2c + d$ for GE, the Cube of which is $8c^3 + 12ccd + 6cdd + d^3$, and because the Solid GHDEq, or $27ccd$ should be equal to DEc, you will have this Equation, $27ccd = 8c^3 + 12ccd + 6cdd + d^3$, or $d^3 + 6cdd - 15ccd + 8c^3 = 0$, which being divided by $dd + 7cd - 8cc$ gives $d - c = 0$, or $d = c$, or BG = BD, which being impossible, 'tis also impossible that the Solid GHDEq should be equal to the Cube DE, and consequently that AE should meet the Curve of the Parabola in any other Point, but whence it follows, that 'tis a Tangent to that Point. *Which was to be demonstrated.*

SCHOLIUM.

The Converse of this Theorem is also true, namely that if a right Line AE be a Tangent to the Parabola in the Point A, the Part BE will be double the Part BD, for if it were not so, but BD were greater than half BE, make BI equal to half BE, and draw the Ordinate IK to the Diameter BD, from the Point I, and then the right Line EK will be a Tangent to the Parabola ABC in the Point K; according to the former Demonstration, which is impossible, because the Line AE, that goes from the same Point E, is a Tangent *by Supp.* as has been shewn in *Theor.* 6.

In imitation of the two foregoing Demonstrations, you will find the Part BE is triple the corresponding Part BD, in the Biquadratic Parabola, and quadruple in a Parabola a Degree higher, and so on. Thus you see that in all these infinite Parabola's, the Property of the Tangent is this, that the Part BD is to the corresponding Part BE, as Unity to the natural Numbers, 1, 2, 3, 4, &c. Which is worth remarking, for I shall make use of it afterwards.

THEOREM VIII.

Plate XV. Fig. 119. *If you form upon the Diameter AE of the Curve ABCD, whose Tangent at the Vertex A is AS parallel to the Ordinate DE, the Curve AIOM, so as that one of its Ordinates, as HI be equal to the Part AF of the Tangent at the Vertex AS, terminated in F by the correspondent Tangent BF, and in like manner the Ordinate EM equal to the Part AG of the same Tangent, bounded in G by the correspondent Tangent DG, and so of the rest, draw the right Line AD, and it shall cut off the Segment ADCBA equal to half the corresponding Space AEMOIA.*

PREPARATION.

MAke DQ equal to the Tangent AG, and join the right Line AQ, equal and parallel to the Tangent DG, by 33. 1. so that the Figure AGDQ will be a Parallelogram. Imagine in the Curve ABCD, the Point C infinitely near the Point D, in which case the Part CD may pass for a right Line, and consequently for a part of the Tangent DG, and draw thro' the Point C, the Line CL parallel to DM, and CR parallel to the Diameter AE, and thro' the Points M, N, the right Lines LM, NP, parallel to the same Diameter AE. Lastly, join the right Line AC.

DEMONSTRATION.

Because the two parallel Lines DM, CL, are supposed infinitely near, the two Points L, O, are also infinitely near, so that the mixtilineal Figure MOKE, is the same with the Parallelogram EMLK: And because the Line EM is by the Generation of the Curve AIOM equal to the Line AG, or CN, the Parallelogram EMLK, or mixtilineal Figure EMOK, by 36. 1. is equal to the Parallelogram PNCR, or by 35. 1. to the Parallelogram QNCD, that is to say by 41. 1. to double the Triangle ACD. Whence 'tis easy to conclude, drawing other Parallels, and other corresponding Lines from the Point A, that the mixtilineal Figures, of which AEMI is composed, are double the correspondent Triangles, of which the Segment ADCB is composed, and consequently the Figure AEMI is double the Figure ADCB. *Which was to be demonstrated.*

SCHO-

SCHOLIUM.

If you draw the Line DS parallel to the Diameter AE, thro the Point D, and make the Curve ANPQ similar to the Curve AIOM, the Curve propos'd ABCD will divide the Space AQDS into two equal Parts, that is the two Spaces AQDB, ABDS will be equal. Plate XV. Fig. 120.

PREPARATION.

Take as before, in the Curve ABCD, the Point C, infinitely near the Point D, and draw thro' the Point C, the right Line TR parallel to the Diameter AE, or the Line DS. Join the right Line GQ, which will be parallel to the Diameter AE, or Line TR, because AG, EQ, are equal by the Generation of the Curve ANPQ.

DEMONSTRATION.

Because the Triangle GQD, CRD are similar, you will find by 4. 6. that the four Lines DR, CR, DQ, GQ or DS, are proportional, and by 16. 6. the Rectangle of the two Extremes DR, DS, that is to say, the Parallelogram RDST, is equal to the Rectangle of the means CR, DQ, that is to say, to the mixtilineal Figure DCPQ, which may pass for a Parallelogram: Wherefore if from each of the two equal Planes you take away the common Triangle CRD, there will remain the Trapezium CDST equal to the Trapezium QRCP. Whence 'tis easy to conclude, drawing other Parallels, that all the Trapeziums that make up the Figure ABDS, are equal to all the Trapeziums corresponding, that make up the Figure AQDB, and consequently the two Figures AQDB, ABDS are equal. *Which was to be demonstrated.*

COROLLARY.

It follows, that if you describe a new Curve AXV, so as that the Part MV be double the corresponding Part QD, and in like manner the Part OX double the corresponding Part PC; and so of the rest; the Space AEVX is equal to the Parallelogram AEDS: because AEMO, AEQP, are equal, *by Constr.* and AMVX, AQDS are equal, each being double the space AQDC; because the first AMVX has its Lines double those of the Space AQDC, *by Constr.* and the second AQDS has been de-

monstrated double the same Space AQDC. Whence it follows, that the three Spaces AMV, MAQ, AQDS are equal.

THEOREM IX.

The Quadratic Parabola is to a Parallelogram of the same Base and Height, as 2 to 3.

Plate XV. Fig. 121.

I Say, that if ABD be the Curve of a Qradratic Parabola, whose Diameter is AE, and Ordinate ED, *the Parabolic Space* AEB is to the Parallelogram AEDS form'd on the same Base DE, and of the same Height AE, as 2 to 3.

PREPARATION.

Form the Curve AIM by *Theorem* 8. from the Curve ABD, where we have demonstrated, that the Space AEMI is double the *Parabolic Segment* ADB, draw the Tangent DF, to the Point D, and produce it, till it meet the Diameter in C, and then the Line AF will be equal to the Line EM by the Generation of the Curve AIM, and the Line AC equal to the Line AE, by *Theorem* 7.

DEMONSTRATION.

Because the Triangles CAF, CED, are equiangular, and the Side CE double CA, by the Property of the Tangent of this sort of Parabola, the Side ED will also be double the Side AF or EM its equal, by the Generation of the Curve AIM; after the same manner it may be demonstrated by drawing other Tangents, that all the Ordinates of the Space AEDB are double all the corresponding Ordinates of the Space AEMI, and consequently the Space AEDB is double the Space AEMI, which being double the Space ADB, by *Theor.* 8. the Space AEDB will be quadruple the Space ADB, and the Triangle ADE by consequence triple the same Segment ADB; wherefore the Parallelogram AEDS, that is double this Triangle, *by* 34. 1. will be Sextuple of the Segment ADB; and since the Parabolic Space AEDB is quadruple that Segment, it follows that the Parabola AEDB is to the Parallelogram AEDS, as 4 to 6, or 2 to 3. *Which was to be demonstrated.*

SCHO-

SCHOLIUM.

Because the Triangle ADE, or ADS, is triple the Parabolic Segment ADB, it follows, that the Space ABDS, which we will call the *Parabolic Complement*, is double the same Segment ADB: And because the Parallelogram AEDS is Sextuple of the Segment ADB, it follows that the Parabolic Complement ABDS is to the Parallelogram AEDS, as 2 to 6, or 1 to 3. *Plate XV. Fig. 121.*

When the Parabola ABD is a Cubic one, by the same Method of Reasoning, you may find that the Parabolic Space AEDB is to the Parallelogram AEDS as 3 to 4, as you shall see. Because the Triangles CAF, CED, are equiangular, and the Side CE to the Side CA, as 3 to 2, AC being double AE, by the Property of the Tangent of this kind of Parabola; the Side ED will also be to the Side AF or EM as 3 to 2, and after the same manner you may find, drawing other Tangents, that all the Ordinates of the Space AEDB are to all the corresponding Ordinates of the Space AEMI, as 3 to 2, consequently the Space AEDB is to the Space AEMI, as 3 to 2, and its half ADB, as 3 to 1, that is to say the Parabolic Space AEDB is triple the Parabolic Segment ADB, and the Triangle ADE double the Segment ADB; wherefore the Parallelogram AEDS will be quadruple the Segment ADB; since therefore the Parallelogram AEDS is quadruple the Segment ADB, and the Parabolic Space AEDB quadruple the same Segment ADB, it follows, that the Cubic Parabola AEDB is to the Parallelogram AEDS as 3 to 4.

Since the Triangle ADE or ADS is double the Segment ADB, it follows that the Parabolic Complement ABDS is equal to the same Segment ADB; and because the Parallelogram AEDS is Quadruple the Segment ADB, 'twill also be quadruple the Parabolic Segment ABDS, that is to say, the Parabolic Segment ABDS in the Cubic Parabola is to the Parallelogram AEDS, as 1 to 4.

When the Parabola ABD is a Biquadratic one, you may find by the like method of reasoning, that the Parabola AEDB, is to the Parallelogram AEDS as 4 to 5, and the Parabolic Complement ABDS is to the same Parallelogram AEDS, as 1 to 5: And when the Parabola is one degree higher, 'twill be to its Parallelogram, as 5 to 6, and the Parabolic Segment to the same Parallelogram, as 1 to 6. Thus you see in an infinity of Para-

bola's of different Degrees, the Parabola's are to their Parallelograms of the ſame Baſe and Height as 2 to 3, as 3 to 4, as 4 to 5, as 5 to 6, and ſo on; and their Parabolic Complements are to the ſame Parallelograms as 1 to 3, to 4, to 5, to 6, &c. by reaſon of the Propriety of the Tangent of all theſe Parabola's, which is that AF or EM is to ED, as 1 to 2, as 2 to 3, as 3 to 4, as 4 to 5, &c. Which makes the Parabolic Spaces in the ſame Ratio alſo.

Plate 15. *Fig.* 121.

THEOREM X.

The Sum of an Infinity of Quantities in an Arithmetic Progreſſion beginning at o, *is equal to half the Product under the greateſt, and the Number that expreſſes the Multitude of all the Quantities.*

Fig. 122.

TO demonſtrate this Theorem, conſider the greateſt of all theſe Quantities in Arithmetic Proportion, as the Baſe AC of a Rectangled Triangle ABC, and the number of their Multitude as the Height AB, ſuppoſing that the Height AB divided into an Infinite Number of equal Parts, and making right Lines paſs Parallel to the Baſe AC, thro' the Points of Diviſion, which may be taken for Quantities in Arithmetic Progreſſion, becauſe in effect they are Arithmetically Proportional, being the Homologous Sides of an Infinity of Similar Triangles that are to one another as their Heights, BE, BF, BG, BH, &c. that are in Arithmetic Progreſſion; and ſince all theſe parallel Lines, the leaſt of which is o, or the Point B, and the greateſt AC, compleat the Triangle ABC, which *by* 41. 1. is half the Rectangle ABDC, whoſe Area is equal to the Product under the Baſe AC, and Height AB, it follows that half the Product of the Greateſt AC and the number AB of the Multitude of Quantities Arithmetically Proportional, is equal to their Sum, which the Triangle ABC repreſents. *Which was to be demonſtrated.*

THEO-

THEOREM XI.

The Sum of the Squares of an Infinity of Quantities in an Arithmetic Progression beginning at 0, is equal to a third of the Product, under the greatest Square, and the number that expresses the Multitude of all the Quantities.

I Say that the ſum of the Squares 0, 1, 4, 9, 16, 25, 36, 49, 64, *&c.* of the natural Numbers, 0, 1, 2, 3, 4, 5, 6, 7, 8, *&c.* that repreſent Quantities in Arithmetic Progreſſion, is equal to a third of the Product under the greateſt and the Number that expreſſes the Multitude.

DEMONSTRATION.

If we conſider for inſtance the three firſt Squares whoſe ſum is 5, which, if the Number of Squares were Infinite, would be equal to a third of the Product 12, of the greateſt Square 4, and the Number 3 of their Multitude, and as the third of the Product 12 is but 4, whereas it ſhould be 5, the difference 1, which is $\frac{1}{5}$ of the true Sum 5, and $\frac{1}{4}$ of the falſe Sum 4, is ſo conſiderable only in the Number of three Squares.

If you conſider a greater Number of Squares, for inſtance, the ſix firſt, 0, 1, 4, 9, 16, 25, whoſe Sum is 55, this Sum ought to be equal to a third of the Product 150, of the greateſt Square 25, multiplied by the Number of their Multitude 6.

0	0		
1	1		
2	4		
Sum	5	$\frac{1}{5}$	$\frac{1}{4}$
3	9		
4	16		
5	25		
Sum	55	$\frac{1}{11}$	$\frac{1}{10}$
6	36		
7	49		
8	64		
Sum	204	$\frac{1}{17}$	$\frac{1}{16}$

And

And since the third of the Product 150 is but 50, whereas it ought to be 55, the difference 5 which is $\frac{1}{11}$ of the true Sum 55, or $\frac{1}{10}$ of the false Sum 50, is still pretty considerable; but not so much as the first, because the Number of Squares is greater.

Let us take then still a greater Number of Squares, as the nine first, 0, 1, 4, 9, 16, 25, 36, 49, 64, whose Sum is 204, which if the Number of Terms had been infinite, ought to be equal to a third of the Product 576, of the greatest Square 64, multiplied by 9 their Multitude, and since the third of the Product 576, is but 192, instead of being 204, the difference 12, that is $\frac{1}{17}$ of the true Sum 204, or $\frac{1}{16}$ of the false one 192, is still less considerable than the former.

Since therefore by equally increasing the Number of the Squares, the difference always lessens after the same manner, as may be seen by the three Fractions $\frac{1}{5}$, $\frac{1}{11}$, $\frac{4}{17}$, or by the three $\frac{1}{4}$, $\frac{1}{10}$, $\frac{1}{16}$, whose Denominators exceed one another by 6, 'tis easie to conclude the more Squares you take, the less considerable the difference will be, so that it will become nothing, when the Number of Squares shall be infinite, and thus the Sum of an Infinity of Squares is precisely equal to a third of the Product of the greatest and the Number of their Multitude. *Which was to be demonstrated.*

Another Demonstration.

Since the Sides of the Squares are in continual Arithmetic Proportion, they may be all consider'd in a Pyramide, for instance, ABCD, whose Base ABC is the greatest Square, and the Vertex D the least, or 0, so that all the Squares that compose the Pyramid, divide the Height DE, into an infinite Number of equal Parts, and DE will represent the Multitude of all the Squares, whose Sides will be in continual Arithmetic Proportion, namely, in the same proportion as the Parts of the Height DE, reckoning them from the Vertex D. Conceiving therefore all the Squares clapp'd one upon another to fill the Pyramid ABCD, which *by* 7. 12. is a third of the Prism ABCFGH, of the same Base and Height, and whose Solidity is equal to the Product of the Base ABC, and Height DE, or AH, it follows that the third of the Product of the greatest Square ABC, and Number DE

Plate XV. Fig. 123.

of

of the Multitude of all the Squares is equal to their Sum, that the Pyramid ABCD represents. *Which was to be demonstrated.*

A Third Demonstration.

Describe on the Ordinate AC, to the Axe AB, of the Parabola ABC, the Rectangle ACDB, divide its Side BD into an infinite number of equal Parts, that you may draw thro' the Points of Division E, F, G, the Parallels EH, FI, GK, to the Side CD, bounded by the Circumference of the Parabola at the Points H, I, K, thro' which draw HL, M, KN parallel to the Ordinate AC. 'Tis evident, that all the Parallels EH, FI, GK, make up the Parabolic Complement BCD, and the Line BD represents the number of the Parallels whose values are in the Ratio of the Squares of the natural numbers 0, 1, 2, 3, 4, &c. because they are equal to the Parts of the Axe BL, BM, BN, that are in the same Proportion by the nature of the Quadratic Parabola, and thus all the Parallels EH, FI, GK, DC, of the Parabolic Segment BCD, may be consider'd as the Squares of Quantities in Arithmetic Progression, whose greatest is CD, and least 0, or the Point B: wherefore the Sum of all these Parallels, or the Parabolic Complement BCD, being *by Theorem* 9. equal to a third of the Rectangle ACDB, whose Area is equal to the Product of the Lines BD, CD, it follows that the Sum of the Squares of the Infinity of Quantities Arithmetically Proportional, beginning at 0, is equal to a third of the Product of the greatest CD, and the number BD of their multitude. *Which was to be Demonstrated.* *Plate* XV. *Fig.* 124.

THEOREM XII.

The Sum of the Cubes of an Infinity of Quantities in Arithmetic Progression, beginning at 0, *is equal to a fourth of the Product of the greatest Cube by the number of Terms.*

I Say the Sum of the Cubes, 0, 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, &c. of the natural numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, that represent Quantities in an Arithmetic Progression is equal to a

a Quarter of the Product of the greateſt of theſe Cubes by the number that expreſſes the Multitude.

DEMONSTRATION.

Let us conſider for inſtance the firſt four Cubes, 0, 1, 8, 27, whoſe Sum is 36, which, if the number of Cubes were infinite, ought to be equal to a quarter of the Product 108, of the greateſt Cube 27, by 4 the number of their multitude: And ſince a quarter of the Product 108, is but 27, inſtead of being 36, the difference 9, that is $\frac{1}{4}$ of the true Sum 36, or $\frac{1}{3}$ of the falſe one 27, is pretty conſiderable in the number of only four Cubes.

0	0
1	1
2	8
3	27
Sum	36 — $\frac{1}{4}$ — $\frac{1}{3}$
4	64
5	125
6	216
7	343
Sum	784 — $\frac{1}{8}$ — $\frac{1}{7}$
8	512
9	729
10	1000
11	1331
Sum	4356 — $\frac{1}{12}$ — $\frac{1}{11}$

If you conſider a greater number of Cubes, for inſtance the eight firſt Cubes 0, 1, 8, 27, 64, 125, 216, 343, whoſe Sum is 784, this Sum 784 ought to be equal to a quarter of the Product 2744 of the greateſt Cube 343 by the number of Terms 8; and ſince a quarter of the Product 2744, is but 686, whereas it ought to be 784, the difference 98, that is $\frac{1}{8}$ of the true Sum 784, or $\frac{1}{7}$ of the falſe one

one 686, is ſtill pretty conſiderable, but not ſo much as the firſt, becauſe the number of Cubes is greater.

Let us take then ſtill a greater number of Cubes, as the twelve firſt, 0, 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, whoſe Sum is 4356, which if the number of Cubes were infinite, ſhould be equal to a quarter of the Product 15972 of the greateſt Cube 1331 by the number of Terms 12, and ſince a quarter of the Product 15972 is but 3993, when it ſhould be 4356, the difference 363, which is $\frac{1}{12}$ of the true Sum 4356, and $\frac{1}{11}$ of the falſe 3993, is ſtill leſs conſiderable than the former.

Since therefore in equally increaſing the number of the Cubes, the difference continually decreaſes after the ſame manner, as may be ſeen here by the three Fractions $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{12}$, or by three $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{11}$, whoſe Denominators exceed one another, by 4, 'tis eaſie to conclude, that the more Cubes you take the leſs conſiderable the Difference, ſo that it will become nothing, when the number of Cubes ſhall be Infinite, and thus the Sum of the infinite Cubes is preciſely equal to a quarter of the Product of the greateſt Cube by the number of the Terms. *Which was to be Demonſtrated.*

Another Demonſtration.

Deſcribe on the Ordinate AC to the Axe AB of the Cubic Parabola ABC, the Rectangle ACDB, and having compleated the reſt as in the preceding Theorem; *Plate XV. Fig.* 124. you will find all the Parallels EH, FI, GK compoſing the Parabolic Complement BCD, and the Line BD repreſenting the number of the Parallels, whoſe values are in the Ratio of the Cubes of the numbers 0, 1, 2, 3, 4, *&c.* becauſe they are equal to the Parts of the Axe BL, BM, BN, that are in the ſame Proportion by the nature of the Cubic Parabola; and thus all the Parallels EH, FI, GK, DC of the Parabolic Segment BCD, may be conſider'd as the Cubes of Quantities in Arithmetic Progreſſion, whoſe greateſt is CD, and leaſt 0, or the Point B; wherefore the Sum of all the Parallels or the Parabolic Complement BCD, being *by Theorem* 9. equal to a quarter of the Rectangle ACDB, whoſe Area is equal to the Product of the Lines BD, CD, it follows that the Sum of the Cubes of the Infinity of Quantities Arithmetically proportional, beginning at 0, is equal to a quarter of the Product of the greateſt CD, by the number BD of their Multitude. *Which was to be Demonſtrated.*

SCHO-

SCHOLIUM.

In imitation of this and the foregoing Theorem, 'tis easie to demonstrate, taking ABC for a Biquadratic Parabola, that the *Sum of the Biquadrates of Quantities in continual Arithmetic Proportion, beginning at* 0, *is equal to a fifth part of the Product of the greatest by their number*: And supposing ABC to be a Sursolid Parabola, that the *Sum of the Sursolids of an Infinity of Quantities in an Arithmetic Progression beginning at* 0, *is equal to a sixth part of the Product of the greatest by their number. And so on.*

Plate XV. Fig. 124.

Thus you see, by this and the two foregoing Theorems, that the *Sums of the Squares, Cubes, Biquadrates and Sursolids, and other Powers raised higher and higher, of an Infinity of Quantities in Arithmetic Proportion, are to the Products of the greatest by the number of their Terms, as Unity is to the following natural numbers*, 2, 3, 4, 5, 6, &c. Which deserves to be taken notice of, because it serves to Square the Circle or Hyperbola.

THEOREM XIII.

A Circle is equal to half the Rectangle under the Perimeter and its Radius.

THIS Theorem is evident by what has been demonstrated of a Regular Polygon in *Theor.* 5. because the Circle is properly a regular Polygon of an infinite number of Sides, whose Perpendicular is equal to the Radius.

Fig. 117. Or imagine the Radius AB divided into an infinite number of equal Parts, and describe on the Center A, thro' the Points of Division, as many Circumferences of Circles, which will be in Arithmetic Progression, because their Radij increases equally, and the Circumferences are in the same Ratio with their Radij, and because the Sum of all the Circumferences or Quantities in Arithmetick Progression, composing the Circle, is equal to half the Product of the greatest, which is the Circumference of the Circle, by their number, which is the Radius AB, *by Theor.* 10. it follows that the Circle is equal to half the Rectangle under its Circumference and Radius. *Which was to be Demonstrated.*

SCHO-

SCHOLIUM.

After the ſame manner, you may demonſtrate that the Area of the Sector of a Circle, is equal to half the Rectangle under the Arc that ſerves for a Baſe, and Radius, which is the Side of the Sector.

THEOREM XIV.

The Diameter of a Circle is to its Circumference, as 100 *is to* 314, *nearly.*

I Say the Diameter AB of a Circle, whoſe Center is D, is to its Circumference, as 100 is to 314 nearly. *Plate XV. Fig.* 125.

PREPARATION.

Deſcribe as in *Theorem* 8. from the Curve or Semicircle ACB, the Curve ASM, which I ſhall call the *Geometric Quadratrix*, becauſe 'tis of uſe in the *Quadrature of the Circle*, that is to ſay, in the knowledge of the Area of the Circle: Namely by making the Ordinate GH equal to the correſponding part of the Tangent at the Vertex AF, or, which is eaſier, to the Line DI terminated by BE in the indefinite Line DK perpendicular to the Diameter AB, becauſe that Line DI is equal to the correſponding Tangent AF or EF, as may be ſeen by joyning the right Line DF, that biſects the Angle at the Center ADE, half of which ADF is equal to the Angle at the Circumference ABE, which makes the Rectangled Triangles DAF, BDI ſimilar and equal, &c.

'Tis evident that the Curve ASM being continued, will approach nearer and nearer to the right Line BV perpendicular to the Diameter AB, without ever meeting it, and ſo the right Line BV will become its Aſymptote: And the indefinite ſpace terminated by the indeterminate Curve ASM, its Axe AB, and Aſymptote BV is equal to a Circle whoſe Diameter is AB, becauſe the Space AGHT is double the correſpondent Segment AE, *by Theorem* 8. and in like manner the ſpace ADRH is double the correſpondent Segment ACE, and ſo the ſpace ALMS, double the correſpondent Segment AXE, and ſo on to the indefinite Space ABVM, which will be double

double the Semicircle ABC, or equal to the Circle that has for its Diameter AB.

'Tis evident alſo that the Ordinate DR, to the Center D, is equal to the Radius DA, and conſequently the Parallelogram RDAQ is a ſquare, whoſe Side AQ muſt be divided into an infinite number of equal Parts, to draw the Lines NT, OH, PS, &c. thro' the Points of Diviſion, that may fill up the complement ARQ, whoſe Area being determined in Analytic Terms will ſerve for the

DEMONSTRATION.

Plate XV. Fig. 125. Put a for the Radius AD or BD, conſequently $2a$ for the Diameter AB, x for the Ordinate GH, or AF or DI, and y for the Part AG, and you will have $2a-y$ for BG, and conſequently $2ay-yy$ for the ſquare GE, equal to the Rectangle of the Lines AG, BG, *by* 35. 3. becauſe the four Lines, BD, DI, BG, GE are proportional *by* 4. 6. the Triangles BDI and BGE, being ſimilar, their Squares alſo will be proportional *by* 22. 6. and you will have in Analytic Terms this Proportion $aa, xx :: 4aa-4ay+yy, 2ay-yy$, dividing their laſt Terms by $2a-y$, you you will have this, $aa, xx :: 2a-y, y$, conſequently this Equation $aay=2axx-xxy$, in which you will find y, or $AG=\frac{2axx}{aa+xx}$, and dividing the Numerator $2axx$ by the Denominator $aa+xx$, you will have $AG=\frac{2xx}{a}-\frac{2x^4}{a^3}+\frac{2x^6}{a^5}-\frac{2x^8}{a^7}$ &c. if inſtead of x the Ordinate GH, you put $2x$, you will have $AG=\frac{8xx}{a}-\frac{32x^4}{a^5}+\frac{128x^6}{1a^5}-\frac{512x}{a^7}$ and putting $3x$ for the ſame Ordinate GH, you will find $AG=\frac{18xx}{a}-\frac{162x^4}{a^3}+\frac{1458x^6}{a^5}-\frac{13122x^8}{a^7}$, &c. and ſo on.

Wherefore if you put x for the firſt part AN, you will have $2x$ for the ſecond AO, $3x$ for the third AP, and ſo for the reſt, to the Line $AQ=a$, that expreſſes the number of the Ordinates NT, OH, PS, &c. and fill up the Complement ARQ: And then you may find

NT=

$$NT = \frac{2xx}{a} - \frac{2x^4}{a^3} + \frac{2x^6}{a^5} - \frac{2x^8}{a^7}, \text{ \&c.}$$ *Plate* 15. *Fig.* 125.

$$OH = \frac{8xx}{a} - \frac{32x^4}{a^3} + \frac{128x^6}{a^5} - \frac{512x^8}{a^7} \text{ \&c.}$$

$$PS = \frac{18xx}{a} - \frac{162x^4}{a^5} + \frac{1458x^6}{a^6} - \frac{13122x^8}{a^7}, \text{ \&c.}$$

And ſo for the reſt, to the greateſt $QR = a$.

You may find by the help of *Theor.* 11. and 12. the Sum of all the Ordinates infinite in number, that is to ſay the Complement ARQ, becauſe all the ſimilar Terms, whoſe Numerators are the values of all the Lines, are double the Powers, whoſe Sides are in the Progreſſion of the natural numbers, 1, 2, 3, 4, *&c.* and conſequently in an Arithmetic Progreſſion. So you may find *by Theorem* 11. that the Sum of the Numerators of all the firſt terms $2xx$, $8xx$, $18xx$, and is equal to $\frac{2}{3}a^3$, and *by Theor.* 12. that the Sum of the Numerators of the ſecond Terms $2x^4$, $32x^4$, $162x^4$ and is equal to $\frac{2}{5}a^5$, and the Sums of the Numerators of all the third Terms $2x^6$, $128x^6$, $1458x^6$, *&c.* is equal to $\frac{2}{7}a^7$, and ſo on. Whence it follows, that the Sum of all the Infinite Lines NT, OH, PS, *&c.* or the Complement ARQ is equal to $\frac{2}{3}aa - \frac{2}{5}aa + \frac{2}{7}aa - \frac{2}{9}aa$, *&c.* from whence taking the Square RDAQ, equal to aa, there will remain $aa - \frac{2}{3}aa + \frac{2}{5}aa - \frac{2}{7}aa - \frac{2}{9}aa$, *&c.* for the ſpace ARD, whoſe half $\frac{1}{2}aa - \frac{1}{3}aa - \frac{1}{5}aa - \frac{1}{7}aa + \frac{1}{9}aa$, *&c.* will be *by Theor.* 8. the Area of the Correſpondent Segment ACE, to which adding the Area of the rect-angled Iſoſceles Triangle ADC, equal to $\frac{1}{2}aa$, you will have $aa - \frac{1}{3}aa + \frac{1}{5}aa - \frac{1}{7}aa + \frac{1}{9}aa$, *&c.* for a quarter of the Circle ADCE, whoſe Quadruple $4aa - \frac{4}{3}aa + \frac{4}{5}aa - \frac{4}{7}aa + \frac{4}{9}aa$, *&c.* will be conſequently the Area of the whole Circle, which being equal to half the Rectangle under the Circumference and Radius *by Theorem* 13. divide it by half the Radius, that is by $\frac{1}{2}a$, you will have $8a - a\frac{8}{3} + \frac{8}{5}a - \frac{8}{7}a + \frac{8}{9}a$, *&c.* for rhe Circumference of the Circle. Whence it follows, that the Diameter of a Circle is to its Circumference, as $2a$ is to $8a - \frac{8}{3}a + \frac{8}{5}a - \frac{8}{7}a + \frac{8}{9}a$, *&c.* or as 1 to $4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9}$, *&c.* or as 1 to $\frac{8}{7} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} + \frac{8}{483} + \frac{8}{675}$, *&c.*

'Tis eaſie to continue theſe Fractions *in Infinitum*, becauſe they have the ſame Numerator 8, and their Denominators 3, 35, 99, 195, *&c.* that are ſquare Numbers leſſen'd by Unity, are in a Progreſſion of the ſecond Degree, becauſe the Differences of their Differences are equal, namely 32. 'Tis evident the more you continue theſe Fractions, the more you approach the Ratio of the Diameter to the Circumference, ſo that if you continue the whole, and find their Sum, you would have their Ratio preciſely. We have continued it in the following Table to the Number of 315 Denominators, which is the Ratio of the Diameter of a Circle to the Circumference to a Hundredth Part.

A Table

A Table of the Denominators of 315 *Fractions, whose common Numerator is* 8, *and that make up the Circumference of a Circle, whose Diameter is* 1.

3	14883	58563	131043	232323	362403
35	15875	60515	133955	236195	367235
99	16899	62499	136899	240099	372099
195	17955	64515	139875	244035	376995
323	19043	66563	142883	248003	381923
483	20163	60643	145923	252003	386883
675	21315	70755	148995	256035	391875
899	22499	72899	152099	260099	396899
1155	23715	75075	155235	264195	401955
1443	24963	77283	158403	268323	407043
1763	26243	79523	161603	272483	412163
2115	27555	81795	164835	276675	417315
2499	28899	84099	168099	280899	422499
2915	30275	86435	171395	285155	427715
3363	31683	88803	174723	289443	432963
3843	33123	91203	178083	293763	438243
4355	34595	93635	181475	298115	443555
4899	36099	96099	184899	302499	448899
5475	37635	98595	188355	306915	454275
6083	39204	101123	191843	311363	459683
6723	40803	103683	195363	315843	465123
7395	42435	106275	198915	320355	470595
8099	44099	108899	202499	324899	476099
8835	45795	111555	206115	329475	481635
9603	47525	114243	209763	334083	487203
10403	49283	116963	213443	338723	492803
11235	51075	119715	217155	343395	498435
12099	52899	122499	220899	348099	504099
12995	54755	125315	224675	352835	509795
13923	56643	128163	228483	357603	515523

521283	708963	925443	1170723	1444803
527055	715715	933155	1179395	1454435
522899	722499	940899	1188099	1464099
538755	729315	948675	1196835	1473795
544643	736163	956487	1205603	1483523
550563	743043	964323	1214403	1493283
556585	749955	972195	1223235	1503075
562499	756899	980099	1232099	1512899
568515	763875	988035	1240995	1522755
574563	770883	996003	1249923	1532643
580643	777923	1004003	1258883	1542563
586755	784995	1012035	1267875	1552515
592879	992099	1020099	1276899	1562499
599075	799235	1028195	1285955	1572515
605283	806403	1036323	1295043	1582563
611523	813603	1044483	1304163	
617795	820835	1052675	1313315	
624099	820099	1060899	1322499	
630 35	835395	1069155	1331715	
636803	842723	1077443	1340963	
643203	850083	1085763	1350243	
649635	857475	1094115	1359555	
656099	864899	1102499	1368899	
662595	872355	1110915	1378275	
669123	879843	1119363	1387683	
675683	887363	1127843	1397123	
682275	894915	1136355	1406595	
688899	902499	1144899	1416099	
695555	910115	1153475	1425635	
702243	917763	1162083	1435203	

These Fractions may be changed into whole Numbers, and so fitted for Addition, if instead of supposing the Diameter to be 1; you suppose it to be 100000000, and so the Numerator instead of being 8, will become 80000000, which divided by the

the Denominators of the foregoing Table, namely, first by 3; and then by 35, by 99, by 195, by 723, &c. will give the following Numbers instead of the foregoing.

A Table of 315 *Parts of the Circumference of a Circle, whose Diameter is* 100000000.

266666667	312236366	313178896	313501379
22857143	71206	18852	8544
8080808	66121	18141	8325
4102564	61562	17469	8114
2476780	57459	16834	7911
1656315	53753	16233	7716
305840277	312546467	313266425	313541989
1185185	50394	15663	7528
889878	47340	15123	7346
692641	44556	14611	7171
554400	42010	14124	7003
453772	39677	13661	6840
309616153	312770444	313339607	313577877
378250	37532	13220	6682
320128	35557	12800	6531
274442	33734	12400	6384
237883	32047	12019	6242
208171	30484	11655	6105
311035027	312939798	313401701	313609821
183697	29033	11307	5972
163299	27683	10974	5844
146119	26424	10656	5719
131514	25250	10352	5599
118994	24152	10060	5482
311778650	313072340	313455450	313638437
108181	23125	9780	5369
98778	22161	9513	5260
90549	21257	9255	5153
83307	20407	9009	5050
76901	19606	8772	4950
312236366	313178896	313501379	313664219

313664219	313777588	313848706	313897483
4853	2891	1917	1363
4759	2848	1893	1349
4668	2805	1870	1335
4579	2764	1848	1322
4492	2723	1825	1308
313687570	313791619	313858059	313904160
4408	2684	1804	1295
4327	2645	1782	1282
4247	2607	1761	1269
4170	2569	1740	1256
4095	9533	1720	1244
313708817	313804657	313866866	313910506
4022	2497	1700	1231
3951	2462	1680	1219
3881	2428	1661	1207
3814	2395	1642	1196
3748	2362	1623	1184
313728233	313816801	313875172	313916543
3684	2330	1605	1173
3622	2298	1587	1161
3561	2267	1569	1150
3501	2237	1552	1139
3443	2207	1535	1128
313746044	313828140	313883020	313922294
3387	2178	1518	1118
3332	2150	1501	1107
3278	2122	1485	1097
3226	2095	1469	1087
3175	2068	1453	1077
313762442	313838753	313890446	313927780
3125	2041	1438	1067
3076	2016	1422	1057
3028	1990	1407	1047
2981	1965	1392	1038
2936	1941	1378	1028
313777588	313848706	313897483	313933017

313933017	313956008	313974757	313990339
1019	823	678	569
1010	816	673	565
1002	810	668	561
991	803	664	557
983	797	659	554
313938022	313960057	313978099	313993145
975	790	654	550
966	784	649	546
958	778	645	543
949	772	640	539
941	766	635	536
313942811	313963947	313981322	313995859
933	760	631	532
925	754	627	529
917	748	622	525
909	742	618	522
902	737	613	519
313947397	313967688	313984433	313998486
894	731	609	515
886	726	605	512
879	720	601	509
872	715	597	506
864	709	592	
313951792	313971289	313987437	314000528
857	704	588	
850	699	584	
843	694	580	
836	688	577	
830	683	573	
313956008	313974757	313990339	

If you add these Numbers, as we have done here, by fives, their Sum 314000528 will be the Circumference of a Circle, whose Diameter is 100000000. Thus you see the Diameter of a Circle is to its Circumference, as about 100000000 to 314000528, or 100 to 314. *Which was to be demonstrated.*

SCHOLIUM.

Ludolp. Van Ceulen has found by Polygons inscrib'd and circumscrib'd, after *Archimede's* Method, that the Diameter of a Circle being 1000, 000, 000, 000, 000, 000, 000, its Circumference is between these two Numbers,

314, 159, 265, 358, 979, 323, 846, 264, 338, 327, 950,
314, 159, 265, 358, 979, 323, 846, 264, 338, 327, 951,

In small Calculations, *Archimede*'s Ratio will serve, which is that of 7 to 22, and differs from that of 100 to 314, in this, that 22 is a Number too great, 314 too little; but the defect is not so considerable as the Excess, so that in Practice 'tis better to use the Ratio of 100 to 314, in in small Calculations, but in large ones the Ratio of 1000000, &c. to 314159, &c.

THEOREM XV.

The Area of a Circle is to the Square of its Diameter as 785 to 1000 nearly.

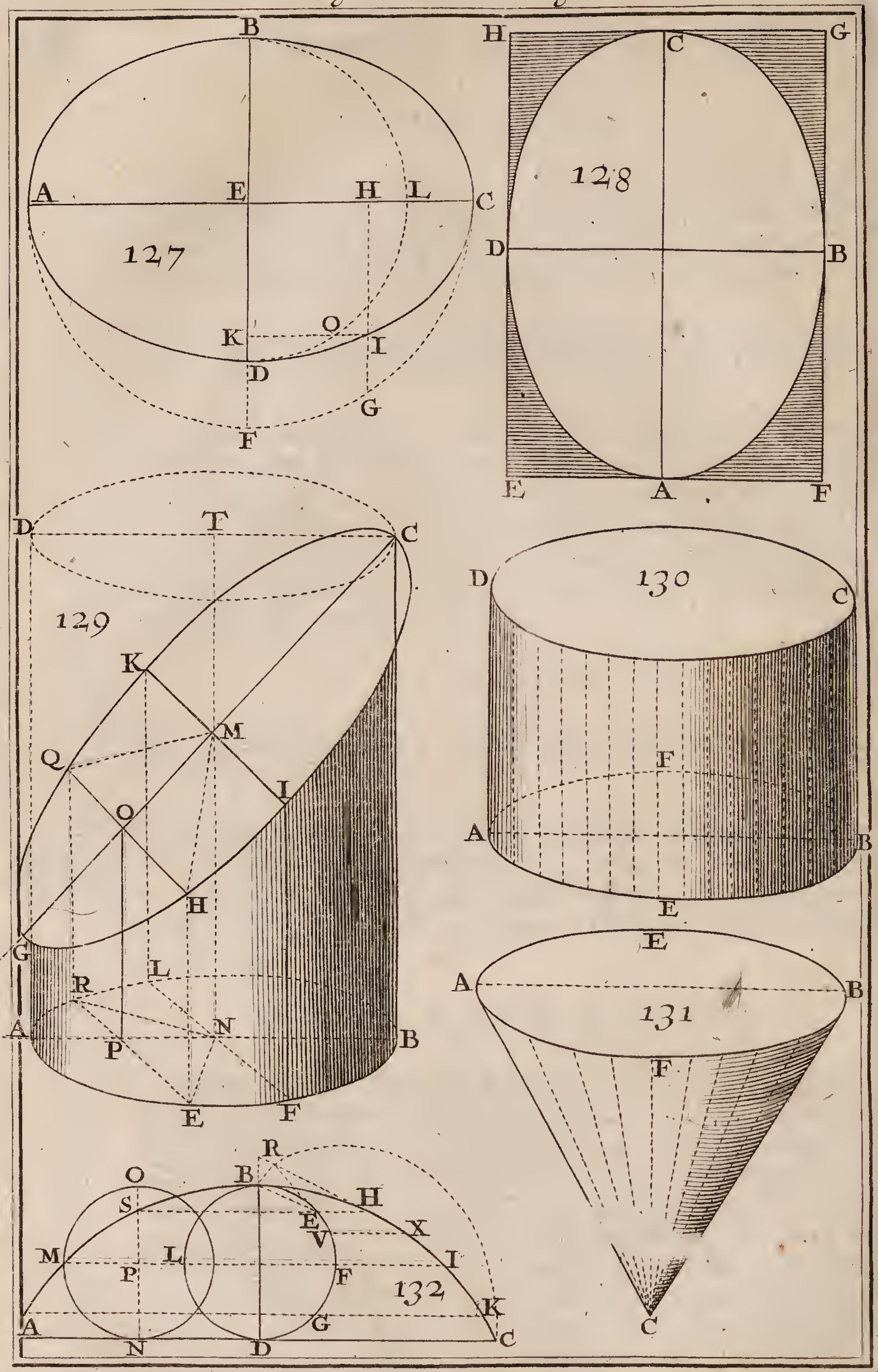
Plate XV. Fig. 126.

I Say the Area of the Circle EFGH, is to the Square ABCD of its Diameter EG, as 785, to 1000, that is to say, if the Area of the Circle EFGH, is 785 Square Feet, the Circumscrib'd Square ABCD, will contain about 1000.

DEMONSTRATION.

If you suppose the Diameter EG 100 Feet long, the Area of the Square ABCD will be 10000 Feet Square, and the Circumference EFGH will be 314 Feet long, by *Theor.* 14. and the Rectangle under the Diameter and Circumference will be 31400 Square Feet, its half will give 15700 Square Feet, for the double of the Area of the Circle by *Theor.* 13. wherefore taking half this half, you will have 7850 Feet Square, for the Area of the Circle EFGH. Thus you see the Area of the Circle EFGH, is to the Square ABCD of its Diameter EG, as 7850 to 10000, or in less Numbers as 785 to 1000. *Which was to be Demonstrated.*

SCHO-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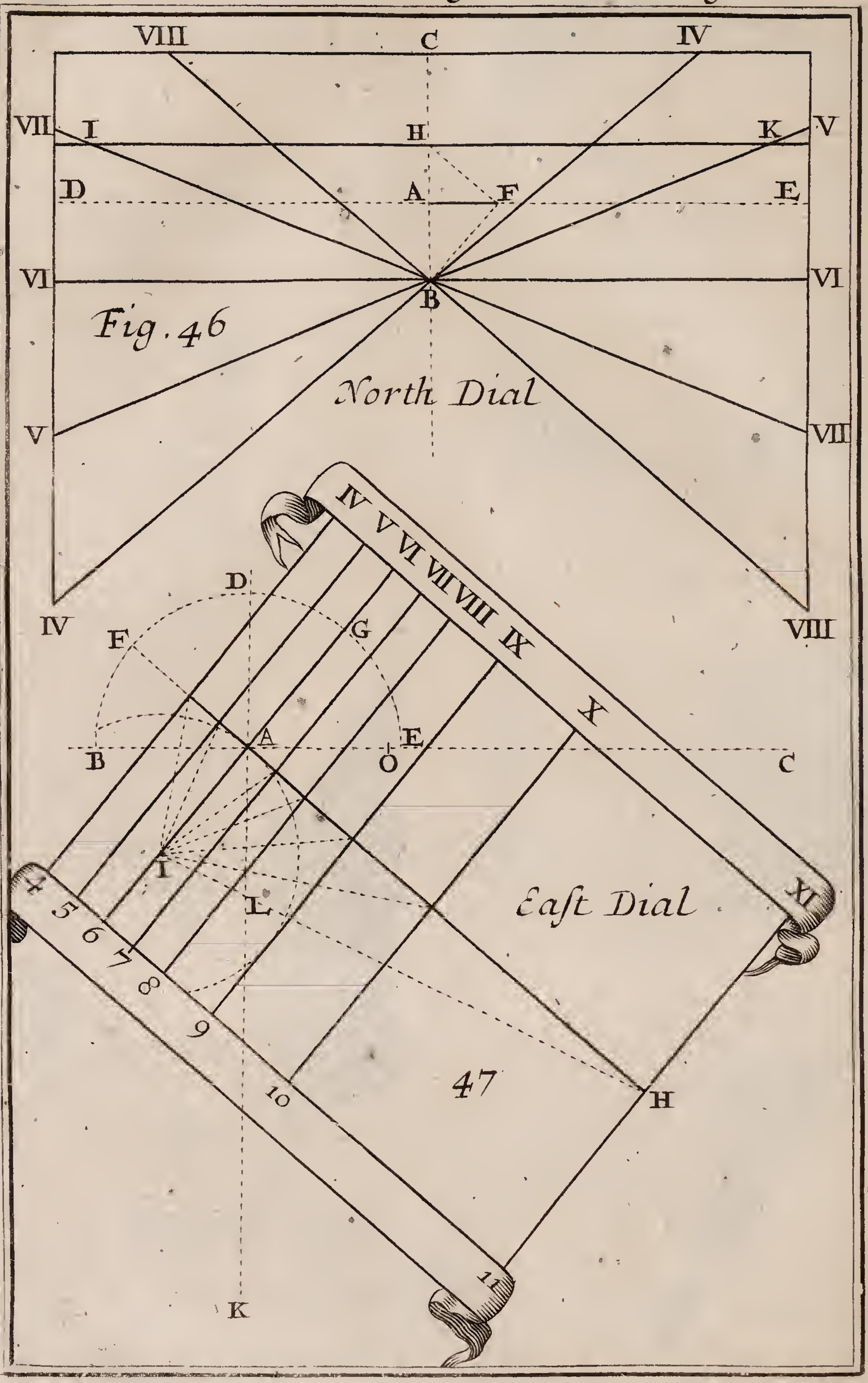
Fig. 46
North Dial
VIII
C
IV
VII
I
H
K
V
D
A
F
E
VI
B
VI
V
VII
IV
VIII
IV V VI VII VIII IX
X
XI
D
F
G
B
A
E
O
C
I
L
East Dial
4 5 6 7 8 9 10 11
47
H
K

SCHOLIUM.

One might leſſen the Terms of this Ratio, by dividing each by their common meaſure 5, but 'tis better to leave it as it is, becauſe the number 1000 is more commodious than any other, ſince by this means you avoid Multiplication or Diviſion, which in Practice you would be forced to do by any other; we ſhall therefore neglect the Ratio of 11 to 14, commonly uſed, becauſe not ſo commodious, nor exact as that of 785 to 1000, and if we would be more exact, will uſe that of 785398 to 1000000.

THEOREM XVI.

An Ellipſe is equal to a Circle, whoſe Diameter is a mean Proportional between the two Axes of the Ellipſe.

I Say the Ellipſe ABCD, whoſe two Axes are AC, BD, is equal to a Circle whoſe Diameter is a mean Proportional between the two Axes AC, BD. *Plate XVI. Fig.* 127.

PREPARATION.

Make upon the Centre E of the Ellipſe, and the two Axes AC, BD, two Semi-circles AFC, BLD, and draw from the Point I, taken at diſcretion in the Circumference of the Ellipſe, the right Lines IK, GH, parallel to the two Axes AC, BD.

DEMONSTRATION.

Becauſe the right Lines ED, HI, are two Ordinates to the Axe AC, by *Def.* 10. the Rectangle of the Parts AE, EC, or the Square EF, the two Parts being equal, is to the Rectangle of theſe two AH, HC, or the Square GH, *by* 35. 3. as the Square DE to the Square IH, by *Def.* 35. Whereſore *by* 22. 6. the four Lines EF, GH, DE, IH, are proportional, and doubling the firſt EF, and third DE, which are the Antecedents of this Proportion, you will find the Ratio of the Line GH, that is an Ordinate in the Circle AC is to the Ordinate IH of the Ellipſe, as the great Axe AC, to the little one BD. Whence 'tis eaſie to conclude *by* 12. 5. drawing other Ordinates common to the Circle and Ellipſe, that all the Ordinates of the Circle AC, that is to ſay, the Circle AC,

Plate XVI. Fig. 127. AC, is to all the Ordinates of the Ellipse, or to the Ellipse as the great Axe AC, is to the little one BD.

In like manner because the right Lines CE, IK, are two Ordinates to the Axe BD, by *Def.* 10. the Rectangle of the Parts BE, ED, or the Square BE, because the two Parts are equal, is to the Rectangle of the two BK, DK, or the Square KO, *by* 35. 3. as the Square CE, to the Square KI, by *Def.* 35. Wherefore by 22. 6. the four Lines BE, KO, CE, KI, are proportional, and doubling the first BE, and third CE, that are the Antecedents of the Proportion, you will find the Ratio of the Line KO, that is an Ordinate in the Circle BD, is to the Ordinate KI in the Ellipse, as the little Axe BD, to the great one AC. Whence 'tis easie to conclude *by* 12. 5. drawing other Ordinates common to the Circle BD, and Ellipse, that all the Ordinates of the Circle BD, that is to say the Circle BD, is to all the Ordinates of the Ellipse, or the Ellipse, as the less Axe BD, to the greater AC.

Since therefore an Ellipse is to a Circle AC, as the great Axe AC is to the less BD, and the Circle BC is to the Ellipse, as the great Axe AC, to the less BD, it follows that the Ellipse is a mean proportional between the two Circles AC, BD, and consequently equal to a Circle, whose Diameter is a mean proportional between the two Axes AC, BD. *Which was to be demonstrated.*

PROBLEM XVII.

The Area of an Ellipse is to the Rectangle of its two Axes, as 785 *to* 1000, *nearly.*

Fig. 128. I Say the Ellipse ABCD is to the Rectangle EFGH, of the two Axes AC, BD, as 785 to 1000, so that if the Area of the Ellipse ABCD be 785 square Feet, that of the circumscrib'd Rectangle EFGH will be about 1000 square Feet.

DEMONSTRATION.

Because a Circle is to the Square of its Diameter, as 785 to 1000, *by Theor.* 15. I say, instead of the two first Terms, namely the Circle, and the Square of its Diameter, you put an Ellipse, and the Rectangle under its two Axes, that are in the same Ratio, because *by Theor.* 16. an Ellipse is equal to a Circle, whose Diameter is a mean proportional between its two Axes, and consequently such an

an one as that the Square of its Diameter is equal to the Rectangle under its two Axes, by 17. 6. you will find an Ellipse is to the Rectangle under its two Axes, as 785 to 1000. *Which was to be demonstrated.* *Plate XVI. Fig. 128.*

SCHOLIUM.

The Ratio of 11 to 14 may also serve, but not so exact, if you would have one very exact, you may use that of 785398 to 1000000, as in the Circle. One easily sees by this Theorem, that Ellipses are to one another, as the Rectangles under their two Axes, and that such, as have two equal Axes, are to one another as the two other Axes. Whence 'tis easy to conclude, that two Ellipses are equal, when their Axes are reciprocally proportional.

THEOREM XVIII.

If you cut a right Cylinder by a Plane inclin'd to its Base, the Section will be an Ellipse, whose lesser Arc will be equal to the Diameter of the Base of the Cylinder.

I Say, if you cut the right Cylinder ABCD, whose Base is the Circle AEBL, and Axe the Line TN perpendicular to the same Base, by a Plane GHCK inclined to that Base, the Section GICK will be an Ellipse, whose less Axe will be equal to the Diameter AB of the Base. *Fig. 129.*

PREPARATION.

Make a Plane pass thro' the Axe TN, and Diameter AB, which being perpendicular to the Plane of the Base AEBL, by 18. 11. will cut the Cylinder by the Rectangular Plane ABCD, and the Section GICK, by the right Line CG, that will represent the length of the Curve GICK, and which being produced will meet the Diameter AB, also produced in a Point as S, where the Angle of the Inclination of the two Planes GICK, AEBL, and the Triangular Plane SBC, that is perpendicular to the Plane of the Base AEBL, is terminated, so that all the Points of the Line CG, correspond perpendicularly to all the Points of the Diameter AB, as all the Points of the Plane GICK correspond perpendicularly to all the Points of the Plane AEBL. Draw at Pleasure in the Plane GICK, the two Lines IK, HQ, perpendicular to the Line CG, and on the Surface of the Cylinder thro' the

Plate XVI. *Fig.* 129. the two Points H, I, the two Lines HE, IF, perpendicularly to the Base AEBL, lastly, in the Plane of the Circle AEBL, thro' the Points E, F, the two Lines ER, FL, perpendicular to the Diameter AB, which will divide them into two equal Parts at the Points P, N, by 3. 3. This Construction is equivalent to that made by cutting the Cylinder by the two Planes IKLF, HQRE, perpendicular to the Base AEBL, and to the Triangular Plane SBC, which makes it evident that all the Points of the common Sections IK, FL, correspond perpendicularly, as well as those of the two common Sections HQ, ER, and consequently all the Points of the mixtilineal Figure HIKQ, correspond perpendicularly with all the Points of the mixtilineal Figure EFLR.

DEMONSTRATION.

'Tis evident by 6. 11. that the two Lines IK, FL, being perpendicular to the same Plane SBC, are parallel, and equal also by 34. 1. because the Figure IKLF is a Parallelogram, one may find after the same manner, that the two Lines HQ, ER, are equal and parallel, because the Figure HQRE, is also a Parallelogram.

'Tis evident also, that the two Points I, F, being equally distant from the Plane SBC, the Line IF being parallel to the Plane, the two Lines IM, FN, made perpendicular to the same Plane, are also equal: And as the Line FN is half the Line FL, or IK its equal, the Line IM will also be half the Line IK. After the same manner it may be demonstrated, that the Line HO is half the Line HQ. Whence it follows by *Def.* 10. that the Line CG, is a Diameter in respect of the Ordinates IK, HQ, that are parallel, by 29. 1. because each was made perpendicular to the Diameter CG, which consequently will be an Axe. Thus you may know the Line CG is the great Axe of the Curve GICK, and consequently IK, the less. Supposing the Point M in the middle of the Axe CG, in which Case the Point N will be the Center of the Circle AEBL, whose Diameter AB, or FL is consequently equal to the less Axe IK. It remains therefore to be demonstrated, that the Curve GICK is the Circumference of an Ellipse.

Because the two Planes HQRE, IKLF are parallel, being perpendicular to the same Plane AEBL the two Lines AB, CG, are cut proportionally by the two Planes,

Planes, by 17. 11. and becauſe the Rectangle of the Lines GO, OC, is by 23. 6. to the Rectangle of the Lines GM, MC, in a Ratio compounded of that of the Sides GO, GM, or of the Lines AP, AN, and of that of the Sides OC, MC, or of the Lines PB, NB: and in like manner the Rectangle of the Lines AP, BP, or by 35. 3. the Square EP or HO, is to the Rectangle of the Lines AN, NB, or to the Square FN or IM, in a Ratio compounded of the ſame Ratio's; it follows, that the Rectangle of the Lines GO, OC, is to the Rectangle of the Lines GM, MC, as the Square HO, is to the Square IM, and *by Def.* 35. that the Curve GICK is the Circumference of an Ellipſe. *Which was to be demonſtrated.* *Plate XVI. Fig.* 129.

SCHOLIUM.

As all the Points of the Ellipſe GICK correſpond with all the Points of the Circle AEBL, and the Diameters AB, CG are cut proportionally by all the Planes perpendicular to the Plane SBC, 'tis eaſy to conclude, by inſcribing after *Archimedes*'s Method a regular Polygon of an infinite number of Sides in the Circle AEBL, that the Circle is to the Ellipſe, as the Diameter AB equal to the leſs Axe TK, is to the great Axe CG, as we have demonſtrated, in *Theor.* 16. and that the Segment of the Ellipſe GHQ, is to the whole Ellipſe GICK, as the Segment of the correſpondent Circle AER is to the whole Circle AEBL; and in like manner that the Sector of the Ellipſe GQMH is to the whole Ellipſe GICK, as the Sector of the correſponding Circle ARNE, is to the whole Circle AEBL, &c.

THEOREM XIX.

The Space bounded by the Cycloid and the Circumference of the generating Circle, that ſerves for the Baſe, is triple the ſame generating Circle.

I Say the Space ACB bounded by the Cycloid ABC, whoſe Axe is BD, and by the Baſe AC, equal to the Circumference of the generating Circle BFDL, whoſe Diameter is the ſame Axe BD, is triple the ſame generating Circle BFDL: Which we ſhall demonſtrate, by making it appear that the Space BFDCH is equal to the generating Circle. To that end we muſt firſt demonſtrate, that as the Baſe AC is equal to the Circumference BFDL, or half the Baſe AD is equal to the corresponding Semi- *Fig.* 132.

circum-

Plate XVI. Fig. 132. circumference BLD, so also any right Line ML parallel to the Semi-base AD is equal to the corresponding Arc BL.]

DEMONSTRATION.

Give the disposition MNO to the generating Circle, by placing it in N, that the Diameter NO be parallel to the Diameter BD, or perpendicular to the Base AC, you know by the Generation of the Cycloid; that the Part AN is equal to the corresponding Arc MN or DL, and consequently the Part ND, or PQ, or ML, is equal to the Arc OM, or BL. *Which was first to be demonstrated.*

Imagine the Semi-circle BFD divided into an infinite number of equal Parts at the Points E, F, G, so that the Arcs BE, BF, BG, be in an Arithmetic Progression, and thro' the Points E, F, G, *&c.* draw to the Base AC, as many Parallels, as EH, FI, GK, *&c.* that being equal to their corresponding Arcs, will also be in an Arithmetic Progression. And by *Theor.* 10. you know their Sum or the Space BFDCH is equal to half the Product of the greatest CD, equal to the Semi-circle BFD, and the Diameter BD, that may pass for their Number, for tho' they don't divide it into equal Parts, yet that inequality is made up by the uniformity found on both Sides of the Center Q of the generating Circle. But as the Product of the Radius DQ by the Semi-circle CD is the Area of the Circle, by *Theor.* 13. and the same Product is half the Product of the Diameter BD, by the Semi-Circle CD, it follows, that the Space BFDCH is equal to the Area of the generating Circle BFDL. *Which was to be demonstrated.*

SCHOLIUM.

I shall demonstrate by the way, what I have advanced without proof in my *Mathematical Dictionary*, namely, that two corresponding Tangents of the Cycloid and generating Circle, placed in the middle of the Space bounded by the Cycloid, as HR, ER, intersect in the Point R, that belongs to the *Line of evolution* of the Semi-circle BFD, that is to say to the Curve BRC, described by the continual Motion of the extremity of a Thred stretched and fasten'd to the Point D by the other extremity, after it has been apply'd to the Semi-Circle BFD, and then the Part of the String, when stretched, as ER, will touch the Curve BFD at the Point andE,and will be equal to the corresponding Arc EB,consequently to the correspon-

ding

ding Line EH. We muſt demonſtrate therefore that the Tangent ER is equal to the correſpondent Line EH. *Plate XVI. Fig.* 132.

To that end, imagine the right Line VX parallel, and infinitely near the Line EH, in which caſe the Arcs EV, HX, may paſs for right Lines, and for Parts of the Tangents ER, HR, and the difference of the Parallels EH, VX, or of the Arcs BE, BV will be equal to the Line EV, and laſtly in the ſimilar Triangles REH, RVX, you will have by 4. 6. this Proportion, RV, is to RE : : VX : EH, and *dividing*, you will have EV, RE : : EV, EH, which ſhews you that the Tangent RE is equal to the Line EH, and conſequently the Point R, belongs to the Line of Evolution BRC. *Which was to be demonſtrated.*

Whence 'tis eaſy to conclude, that the Tangent HR of the Cycloid is parallel to the correſponding BE that divides the Angle SER into two equal Parts, &c.

THEOREM XX.

The Convex Surface of a right Cylinder, is equal to a Rectangle under its height, and the Circumference of its Baſe.

I Say the Convex Surface of the right Cylinder ABCD is equal to a Rectangle, that has for its Baſe the Circumference of the Baſe AEBF, and for its Height the Height AD, or BC of the Cylinder. *Fig.* 130.

DEMONSTRATION.

If thro' all the Points of the Circumference AEDF, you imagine as many Lines parallel and equal to the Height AD or BC, you will ſee the Surface of the Cylinder ABCD is compoſed of an infinite Number of little equal Rectangles, whoſe common Height is the ſame with that of the Cylinder: And as the Sum of all the infinite Rectangles, or the Surface of the right Cylinder ABCD, is by 1. 2. equal to the Rectangle under the common Height AD, or BC of the Cylinder, and the Sum of the Baſes of the ſame Rectangles, it follows, that the ſame Surface is equal to the Rectangle, under the Height AD, or BC of the Cylinder and the Circumference AEBF of the Circle that ſerves for the Baſe of the Circle. *Which was to be demonſtrated.*

SCHO-

SCHOLIUM.

The Truth of this Theorem will be further evident, by imagining the Surface of the Cylinder ABCD to roll upon a Plane; for you may easily see that by an intire Revolution, the Surface of the Cylinder so extended, as it were, will take up on the Plane, a Rectangle that has for its Length the Circumference AEBF, now extended in the form of a right Line, and for its Breadth the Height AD, or BC of the Cylinder.

THEOREM XXI.

The Convex Surface of a right Cylinder is to a Rectangle under its Height and the Diameter of the Base, as 314 to 100, quam proxime.

Plate XVI. Fig. 130. I Say the Surface of the right Cylinder ABCD, is to the Rectangle under its Height AD or BC, and the Diameter AB of the Base AEBF, as 314 to 100, nearly.

DEMONSTRATION.

Because the Convex Surface of the right Cylinder ABCD is equal to the Rectangle under the Height AD or BE and the Circumference of the Base AEBF, *by Theor.* 20. this Rectangle, or the Surface of the Cylinder ABCD will be to the Rectangle under the same Height AD, or BC, and Diameter AB of its Base, as the Circumference AEBF, is to the Diameter AB, by 1. 6. Wherefore if instead of the Circumference AEBF, and its Diameter AB, you put 314, and 100, that are in the same Ratio, *by Theor.* 14. you will find the Surface of the Cylinder ABCD is to the Rectangle under the Height AD, or BC, and the Diameter AB of its Base, as 314 is to 100. *Which was to be demonstrated.*

THEOREM XXII.

The Convex Surface of a right Cone is equal to half the Rectangle under the Side of the Cone, and the Circumference of its Base.

Fig. 131. I Say the Convex Surface of the right Cone ABC is equal to half the Rectangle under the Side AC or BC, and the Circumference of its Base AEBF.

D E-

DEMONSTRATION.

If thro' all the Points of the Circumference AEBF, you draw Lines to the Point C, these right Lines will form an infinite number of little equal Isosceles Triangles, and of the same Height with the side of the Cone, and because each of these equal Triangles is equal to half the Rectangle of the same Base and Height, *by* 41. 1. it follows, that their Sum, or the Surface of the Cone is equal to half the Sum of all the Rectangles, that is to say, to the single Rectangle under the side of the Cone AC, or BC, and the Circumference of the Base AEBF. *Which was to be Demonstrated.* *Plate XVI. Fig.* 131.

Or, imagine the side AC or BC, of the Cone to be divided into an infinite nnmber of equal Parts, and describe on the Point C, as a Pole, as many Circles through the points of Division, and they will be in an Arithmetic Progression: And because the Sum of all the Circles or Quantities in Arithmetic Progression, that fill the Surface of the Cone, is equal to half the Product of the greatest, which is the Circumference of the Base AEBF, and their Number, which is the side AC, or BC, by *Theor.* 10. It follows, that the Surface of the Cone ABC, is equal to half the Rectangle under the side AC, or BC, and the Circumference of the Base AEBF. *Which was to be Demonstrated.*

THEOREM XXIII.

The Convex Surface of a right Cone is to a Rectangle under its Side and Diameter of its Base, as 157 *to* 100 *nearly.*

I Say the Convex Surface of the right Cone ABC is to the Rectangle under its Side AC or BC, and the Diameter AB of its Base AEBF, as 157 to 100.

DEMONSTRATION.

Because the Surface of the Cone ABC, is by *Theor.* 22. equal to half the Rectangle under the Side AC or BC, and Circumference AEBF of the Base, or equal to the Rectangle under the Side AC or BC, and Semi-circle AEB; this Rectangle, or the Surface of the Cone ABC, will be to the Rectangle under the same Side AC, or BC, and the Diameter AB of the Base, as the Semi-circle AEB,

to the Diameter AB, that is to ſay, by *Theor.* 14. as 157 to 100. *Which was to be Demonſtrated.*

THEOREM XXIV.

The Convex Surface of the Fruſtum of a right Cone is equal to half the Rectangle under its Side, and the Sum of the Circumferences of the two oppoſite and parallel Baſes.

Plate XVII. Fig. 133. I Say the Convex Surface of the Fruſtum of a Cone ABCD, is equal to half the Rectangle under its Side AD or DC, and the Sum of the Circumferences of the two oppoſite and parallel Baſes AEBF, CODG.

DEMONSTRATION.

Imagine each of the two Circumferences AEBF, CODG divided into the ſame Number of infinitely ſmall Parts, and thro' the oppoſite Points of the Diviſions as many right Lines drawn, that will be equal to one another, and to the Side AD or BC, and you will find the Surface of the Fruſtum of the Cone ABCD, is compos'd of an infinite Number of little equal Trapezoids, of the ſame Height with the Side AD or BC of the Fruſtum: And becauſe each of theſe equal Trapezoids, is equal to half the Rectangle under the ſame Height, or under the ſame ſide AD or BC, and the Sum of the two oppoſite and parallel Sides, by *Theor.* 4. it follows that their Sum or the Surface of the Fruſtum of the Cone is equal to half the Sum of all the Rectangles, that is to ſay, to the ſingle Rectangle under the Side AD or BC of the Fruſtum of the Cone, and the Sum of the two oppoſite Circumferences AEBF, CODG. *Which was to be Demonſtrated.*

THEOREM XXV.

The Convex Surface of the Fruſtrum of a right Cone is to the Rectangle under its Side, and the Sum of the Diameters of its two oppoſite Baſes, as 157 *to* 100 *nearly.*

I Say the Convex Surface of the Fruſtrum of a right Cone ABCD, is to the Rectangle under its Side AD or BC, and the Sum of the Diameters AB, CD, of the the two oppoſite Baſes AEBF, CODG, as 157 to 100.

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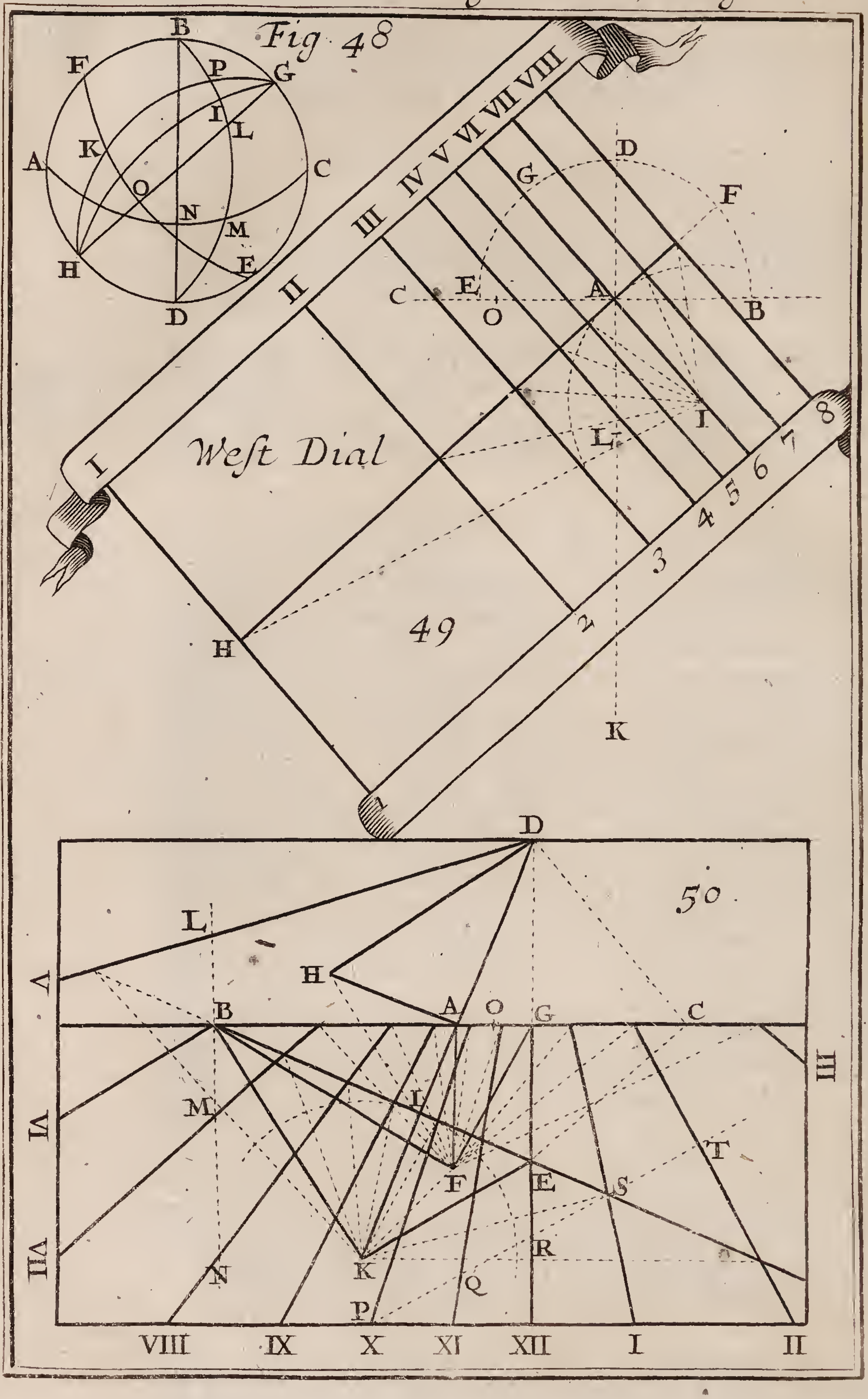
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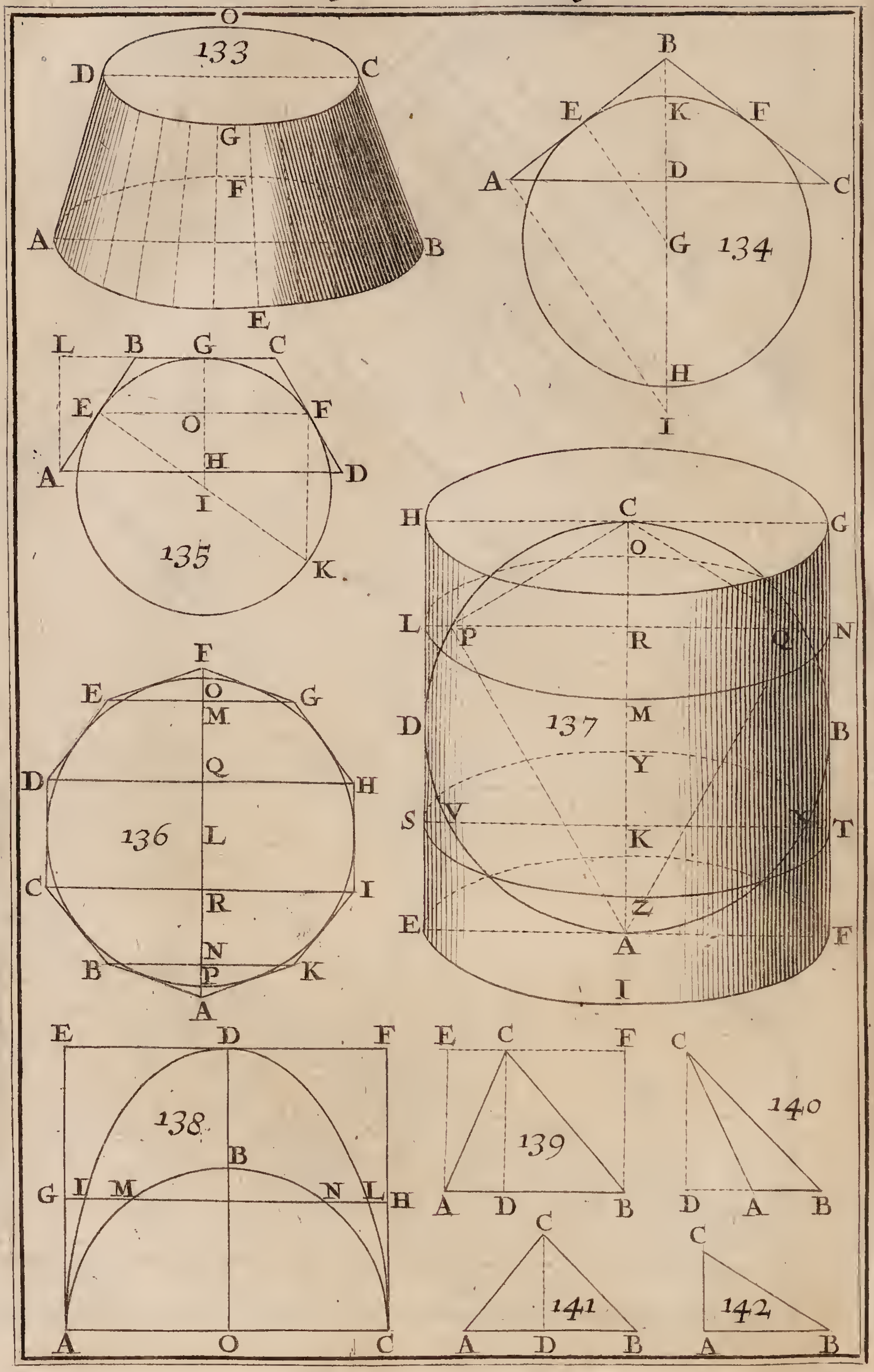
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DEMONSTRATION.

Becauſe the Surface of the Fruſtum of a right Cone ABCD, is equal to half the Rectangle under the Side AD, or BC, and the Sum of the two Circumferences AEBF, CODG, or equal to the Rectangle under the ſide AD or or BC, and half the Sum of the two Circumferences AEBF, CODG, by *Theor.* 24. This Rectangle, or the Surface of the Fruſtum of a right Cone ABCD will be to the Rectangle under the ſame Side AD or BC, and the Sum of the two Diameters AB, CD, as the Sum of the two Semi-circles AEB, COD, to the Sum of the two Diameters AB, CD, that is to ſay, by *Theor.* 14. as 157 to 100. *Which was to be Demonſtrated.* *Plate XVII. Fig.* 133.

THEOREM XXVI.

If a Circle be deſcribed touching the two equal Sides of an Iſoſceles Triangle, and dividing them into two equal Parts at the Points of Contact, the Surface of a right Cone, that has for its Side one of the two equal Sides of the Triangle, and for its Baſe a Circle whoſe Diameter is equal to the Baſe of the ſame Triangle, is equal to the Rectangle under the Height of the Triangle, and the Circumference of the Circle that touches the two Sides.

I Say that, if you biſect at the Points E, F, each of the equal Sides AB, BC, of the Iſoceles Triangle ABC, whoſe Height BD biſects the Baſe AC in the Point D, and draw EG, thro' the Point E, perpendicular to the Side AB, and interſecting the Height BD produced, in the Point G, on which as a Center deſcribe the Circle EFH, touching the two Sides AB, BC, at the two Points EF, the Surface of a right Cone having the Line AB or BC for its Side, and AC for the Diameter of its Baſe, is equal to the Rectangle under the Height BD, and Circumference EFH. *Fig.* 134.

PREPARATION.

Draw thro' the Point A, to the Line EG, the parallel AI, interſecting the Height BD produced in the point I, which will be double the Radius GE of the Circle EFH, and conſequently equal to its Diameter HK, becauſe AB is double BE, & the two Triangles ABI, EBG, are Similar, &c.

DEMONSTRATION.

Plate XVII. Fig. 134. Because *by* 86. the two Rectangled Triangles ABD, ADI, that compose the Rectangled Triangle ABI, are Similar, and *by* 4. 6. the Ratio of the two Lines AB, BD, is equal to that of the Line AI or KH, to the Line AD, if instead of the two last KH, AD, consider'd as Diameters, you substitute their Circumferences that are in the same Ratio, you will find the Ratio of the two Lines AB, BD, is equal to the Ratio of the Circumference EFH, to the Circumference to the Diameter AD, half the Circumference to double the Diameter AC, and *by* 16. 6. the Rectangle under AB, and the Circumference to the Diameter AD, that is to say by *Theor.* 22. the Circumference of the right Cone ABC is equal to the Rectangle under BD, and the Circumference EFH. *Which was to be Demonstrated.*

THEOREM XXVII.

If a Circle be described touching the two equal Sides, and the less of the two other parallel Sides of an Isosceles Trapezoid, and bisecting each of the three Sides in the Points of Contact, the Surface of the Frustum of a right Cone having for its Side one of the two equal Sides of the Trapezoid, and for its Base a Circle whose Diameter is equal to the greater of the two parallel Sides, is equal to the Rectangle under the Perpendicular drawn between the two parallel Sides, and the Circumference of the Circle touching the three Sides.

I Say if you bisect at the Points E, F, G, each of the two parallel Sides AB, CD, and the lesser BC of the two parallel Sides AD, BC, of the Isosceles Trapezoid *Fig.* 135. ABCD, and describe thro' the Points E, F, G, the Circle EGFK, touching the same Points E, F, G, the three Sides AB, BC, CD, and having its Center in the Line GH, and dividing at right Angles, and into two equal Parts the two parallel Sides AD, BC, the Surface of the Frustum of a right Cone, having the Line AB, or CD for a Side, and AD for the Diameter of its Base, is equal to the Rectangle under the Height GH, and Circumference EGFK.

PREPARATION.

Draw thro' the Point E, the Diameter EK, which will be

be Perpendicular to the Side AB, *by* 18. 3. and thro' the two Points E, F, the right Line EF, that being parallel *Plate XVII. Fig.* 135. to the two Sides AD, BC, will cut at right Angles and into two equal Parts at the point O, the right Line GH, draw alſo from the point A, to the Side BC produced, the Perpendicular AL, that will be equal to the Line GH, and will make the Rectangled Triangle ALB, Similar to the right-angled Triangle EOI, as will be evident to one that conſiders, that the Angle L being right, the two other LAB, LBA, taken together are equal to a Right one, *by* 32. 1. and conſequently to the two LAB, BAH, or EIO, or LBA, &c. Whence it follows that the right-angled Triangle ALB is alſo ſimilar to the Triangle EFK, right-angled in F, *by* 31. 3.

DEMONSTRATION.

Becauſe the Line EF, biſects the two Sides ABCD and ſo is an Arithmetic mean Proportional between the two Sides AD, BC, to which it is pa allel, it will be equal to half their Sum, by *Prop.* 3. *Chap.* 4. *L.* 1. *Trigon.* Wherefore conſidering thoſe three Lines Arithmetically proportional BC, EF, AD, as Circumferences of Circles, the Circumference of the mean EF will be equal to half the Sum of the Circumferences of the two extreams AD, BC.

In the ſimilar Triangles ALB, EFK, *by* 4. 6. the Line AB is to the Line AL or GH its equal, as the Diameter EK, to the Diameter EF : Wherefore inſtead of the two Diameters EF, EK, ſubſtituting their Circumferences that are in the ſame Ratio, you will find the Line AB is to the Line GH, as the Circumference EGKF is to the Circumference EF, or to half the two AD, BC, and *by* 16. 6. the Rectangle under the Side AB, and half the Sum of the two Circumferences AD, BC, is equal to the Rectangle under the Height GH, and the Circumference EGFK, that is to ſay by *Theor.* 24. to the Surface of the Fruſtrum of the right Cone ABCD, *Which was to be Demonſtrated.*

THEOREM XXVIII.

If a regular Polygon of an even Number of Sides be circumscrib'd about a Circle, and the Circle with its Polygon be made to revolve about a Diameter passing thro' the two opposite Angles, the Circle will form by that intire Circumvolution a Sphere, and the Polygon a Body bounded by several convex Surfaces, whose Sum is equal to the Rectangle under the Circumference of the Circle, and the right Line or Axe drawn thro' the two opposite Angles of the Polygon.

Plate XVII. Fig. 136. I Say if you circumscribe about a Circle, whose Center is L, and Diameter OP, a regular Polygon of an even number of Sides, for instance, the Decagon ABCD EFGHIK, and make this Polygon revolve upon the Line AF, as an Axe, it will form by this entire Circumvolution a Body, whose Convex Surfaces, that bound it, namely those of the two right Cones EFG, BAK, of the two Frustrums of right Cones DEGH, BCIK, and of the Cylinder CDHI, are together equal to the Rectangle under the Axe AF, and the Circumference of the generating Circle, whose Diameter is OP.

DEMONSTRATION.

Because by *Theor.* 26. the Surface of the right Cone EFG, is equal to the Rectangle under the Circumference to the Diameter OP, and the Part of the Axe FM, and in like manner the Surface of the right Cone BAK equal to the Rectangle under the Circumference to the Diameter OP, and corresponding Part of the Axe AN : and by *Theor.* 27. the Surface of the Frustum of the right Cone DEGH, is equal to the Rectangle under the Circumference to the same Diameter OP, and corresponding part of the Axe MQ, and in like manner the Surface of the Frustum of a right Cone BCIK, equal to the Rectangle under the Circumference to the Diameter OP, and part of the Axe RN : and lastly by *Theor.* 20. the Surface of the Cylinder CDHI, is equal to the Rectangle under the Circumference to the Diameter of its Base CI, equal to the Diameter OP, and its Height CD, or corresponding part of the Axe QR ; 'tis easie to conclude *by* 1. 2. that all the Surfaces together are equal to the Rectangle under the Circumference to the Diameter OP, and the whole Axe AF. *Which was to be Demonstrated.*

THEO-

THEOREM XXIX.

The Surface of a Sphere is equal to the Rectangle under its Diameter, and the Circumference of the Circle to that Diameter.

I Say the Surface of the Sphere ABCD, whose Diameter AC, is equal to the Rectangle under the Diameter AC, and Circumference ABCD of the Circle to that Diameter, which is the same with that of a great Circle of the Sphere. *Plate XVII. Fig. 137.*

DEMONSTRATION.

Because a Circle may be consider'd as a Regular Polygon of an Infinite Number of sides, and a Sphere is generated by the entire Revolution of the Circle or Polygon ABCD, on its Diameter AC, as an Axe, all the Convex Surfaces produc'd by that Revolution being the same with that of the Sphere, will be equal by *Theor.* 28. to the Rectangle under the Axe AC, and Circumference of the generating Circle ABCD. *Which was to be Demonstrated.*

COROLLARY I.

From this Theorem it follows, that *the Surface of a Sphere is quadruple to that of its great Circle*, because the Rectangle under its Diameter and Circumference, to which the Surface of the Sphere is equal, is quadruple the Circle to that Diameter, since by *Theor* 18. this Circle is equal to half the Rectangle under its Circumference and Diameter.

COROLLARY II.

It follows also, that *the Surface of the Sphere ABCD, is equal to that of a Circumscribed Cylinder EFGH,* whose Height EH, or FG, is equal to the Diameter AC of the Sphere, and the Base EIFK, has for a Diameter the Line EF equal to the same Diameter AC: Because by *Theor.* 20. the Surface of this Cylinder is as the Surface of the Sphere, equal to the Rectangle under its Height EH, or AC, and the Circumference of the Circle EIFK, whose Diameter EF is equal to the same Diameter AC of the Sphere.

COROLLARY III.

Plate XVII. Fig. 137. Hence it follows alſo further, that the Surface of the Sphere ABCD is to the Square EFGH, of its Diameter AC, as 314 to 100 nearly, becauſe by *Theor.* 21. the Surface of the Circumſcribed Cylinder EIFGH, is to the ſame Square EFGH, which is the Rectangle under the Height EH, and Diameter EF of its Baſe EIFK, in the ſame Ratio of 314 to 100.

COROLLARY IV.

It follows ſtill, that if you cut the Sphere ABCD, and Circumſcrib'd Cylinder EIFGH by a Plane perpendicular to the Axe AC, as the Plane LMNO, the Surface of the part of the Cylinder HLMNG is equal to that of the correſponding Portion of the Sphere PQC, and in like manner the Part of the Cylinder LEIFNO is equal to the correſponding Portion of the Sphere PAQ.

COROLLARY V.

From hence it follows, that the Surface of a Segment of a Sphere, as PCQ, is equal the Rectangle under its Height CR, and the Circumference ABCD of a great Circle of the ſame Sphere, becauſe the Surface of the Cylinder HLMNG, to which 'tis equal, by *Corol.* 4. is equal to a ſimilar Rectangle by *Theor.* 20. and conſequently the Surface of the ſame Segment of the Sphere PQ, is to the Rectangle HLNG, under the Height CR, and Diameter LN of the Sphere, as 314 to 100 nearly, becauſe the Surface of the Cylinder HLMNG, to which 'tis equal by *Coroll.* 4. is to the ſame Rectangle HLNG, in the ſame Ratio of 314 to 100. by *Theor.* 21.

THEOREM XXX.

The Surface of the Segment of a Sphere is equal to a Circle, whoſe Radius is equal to the Chord of half the Arc of the Segment.

Fig. 137. I Say the Surface of the Segment of the Sphere PCQ, is equal to a Circle, whoſe Radius is equal to the Chord CP or CQ of half the Arc PQC: And in like manner the Surface of the Segment of a Sphere ADPQB, is equal to a Circle, whoſe Radius is equal to the Chord AP or AQ of half the Arc PAQ.

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DEMONSTRATION.

Because the Angle APC is right by 31. 3. and the Line PR, perpendicular to the Hypotenuse AC of the right-angled Triangle APC, you will find *by* 8. 6. each of the two right-angled Triangles ARP, CRP, is equiangular to the great one APC, and *by* 4. 6. that the Line CP is a mean Proportional between the two AC, CR, and in like manner the Line AP a mean Proportional between AC, AR; wherefore *by* 17. 6. the Square CP will be equal to the Rectangle of the Lines AC, CR, and in like manner the Square AP will be equal to the Rectangle of the Lines AC, AR: And since these two Rectangles have the same Height AC, they will be to one another as their Bases CR, AR, *by* 1. 6. Thus the Square CP will be to the Square AP, as CR to AR, or as LH to LE, and if to these two last Lines LH, LE, consider'd as Heights, you allow the Circumference ABCD of a great Circle of a Sphere for a common Base; you will find *by* 1. 6. the Square CP is to the Square AP as the Rectangle under LH, and Circumference ABCD is to the Rectangle under LE, and the same Circumference ABCD, that is to say by *Corol.* 5. *Theor.* 29. as the Surface of the Segment of the Sphere PCQ, is to the Surface of the Segment of the Sphere ADPQB. *Plate XVII. Fig.* 137.

Since therefore the Square CP is to the Square AP as the Surface of the Segment PCQ, is to the Surface of the Segment ADPQB, you will find by *Compounding*, the Square AC is to the Square AP, as the Surface of the Sphere ABCD, to the Surface of the Segment ADPQB: And instead of the Squares of the Lines AC, AP, consider'd as Radij of a Circle, you substitute the Areas of the Circles that are in the same Ratio, *by* 2. 12. you will find the Circle to the Radius AC is to the Circle to the Radius AP, as the Surface of the Sphere ABCD, to the Surface of the Segment ADPQB; and because the two Antecedents are equal, namely, the Circle of the Radius AC, and the Surface of the Sphere ABCD, because *by* 2. 12. the Circle to the Radius AC, is quadruple a great Circle of a Sphere, whose Radius is but half the Radius AC, and the Surface of the Sphere ABCD is also quadruple the Area of its great Circle, by *Corol.* 1. *Theor.* 29. It follows, that the two Consequents are also equal, namely, the Circle to the Radius AP, and Surface of the Segment ADPQB, and consequently the Circle to the Radius CP is also equal to the Surface of the Segments PQC. *Which was to be Demonstrated.*

THEO-

THEOREM. XXXI.

The Surface of a Zone is equal to that of a Cylinder of the same Height, and having for its Base a great Circle of a Sphere.

Plat XVII. Fig. 137. I Say if you cut the Sphere ABCD, and circumscribed Cylinder EIFGH, by the two Planes LMNO, SZTY, perpendicular to the Axe AC, the Surface of the Zone VPQX, is equal to that of the Cylinder LSZTNO, whose Height LS is equal to the Height RK of the Zone, and Base SZTY is equal to a great Circle of a Sphere.

DEMONSTRATION.

Because the Surface of the Segment of a Sphere CPQ, is equal to that of the Cylinder HLMNG, by *Cor.* 4. *The.*29. and in like manner the Surface of the Segment VPCQX, is equal to that of the corresponding Cylinder HSZTG, the difference of the Surfaces of the two Segments, that is the Surface of the Zone VPQX, will be equal to the difference of the Surfaces of the two Cylinders, that is, the Surface of the Cylinder LSZTNO. *Which was to be Demonstrated.*

COROLLARY.

It follows from this Theorem, that the Surface of a Zone, for instance VPQX, is equal to the Rectangle under its Height RQ, and the Circumference ABCD of a great Circle of a Sphere, because the Surface of the Cylinder LSZTNO, to which 'tis equal, is equal to the same Rectangle, by *Theor.* 20.

SCHOLIUM.

Fig. 138. I shall conclude this Theory by rectifying, by the way, a Mistake that several have run into, believing that as the Surface of the Zone AMNC of the Hemisphere ABC, whose Centre is O, and Diameter AC, is equal to the Surface of the corresponding Cylinder AGHC; so the Surface of the Zone AILC, of the Semi-Spheroid ADC, whose lesser Axe is the same Diameter AC of the Hemisphere ABC, and half the great Axe is OD, is equal to the Surface of the same Cylinder AGHC, of the same Base and Height with the Hemisphere, and that the Semi-Spheroid

mi-Spheroid, and consequently the Surfaces of the two Zones AIL, AMNC, are equal; which is not true, it being certain, that of the Zone AILC of the Semi-Spheroid is less than that of the Zone AMNC of the Hemisphere, or than that of the Zone AGHC of the Cylinder, as has been very well demonstrated by *R. F. Nicholas* of the Society of *Jesus*, the best Geometer of the Age, that I know. Whence 'tis easie to conclude, that the Surface of the Cylinder AEFC, is greater than the Surface of the Inscrib'd Semi-Spheroid ADC, consequently what a Modern Author advances in a Book publish'd at *Paris*, 1691, is not true, as also what *Errard* says, in his *Geomet. lib.* 3. *Chap.* 10. namely, that the Surface of an Oblong Spheroid is to the Surface of a Sphere inscrib'd in the same Spheroid, as the great Axe to the little one. See the end of *Sect.* 2. *Chap.* 3. *Lib.* 2. *Mechan.* *Plate XVII. Fig.* 138.

CHAPTER. II.

PROBLEMS.

I Have now explain'd the Theory separately, and am come to the Practical Part, which I shall dispatch with annexing no other Demonstration, but a Quotation of the Theorem it depends upon, that it may be more agreeable and proper for such as content themselves with the Practice thus disengaged from the Theory.

PROBLEM I.

To measure a Triangle.

THE Area of a Triangle may be found two ways, namely, by the help of one of its Sides known, and its Perpendicular, that may be measured Mechanically on the Ground: Or by its three Sides being known, without a Perpendicular, as you shall see. *Plate XVII. Fig.* 139.

To measure therefore the Area of the Triangle ABC, by the help of the known Side AB, of 28 Yards for instance, and its Perpendicular CD, which we will suppose 24 Yards: Multiply 28 and 24 together, the two Numbers of the Side AB, and its Perpendicular CD, and half the Product 672, will give 336 Yards Square for the Area of the Triangle proposed ABC, because multiplying the Side AB by its Perpendicular CD, the Product 672 is the Area of the Rectangle AEFB, of the same Base and Height, but that is double the Triangle by 41. 1.

To

Plate XVII. Fig. 139. To meaſure the ſame Triangle ABC, by the help of its three Sides being known, AB ſuppoſed to be 28 Yards AC 26 Yards, BC 30 Yards.

Add the three Numbers together, 28, 26, 30; and the Sum 84 will be the Perimeter of the Triangle ABC, whoſe half is 42. Subtract ſeparately from that half 42, the Sides 28, 26, 30, to obtain the three exceſſes 14, 16, 12. Multiply the ſame half 42, by one of the three foregoing exceſſes, for inſtance the firſt 14, and the Product 588, by one of the other two exceſſes 16, 12, for inſtance 16, and the ſecond Product 9408, by the laſt exceſs 12, to obtain a third Product 112896, whoſe Square Root will give 336 ſquare Yards for the content of the Triangle propoſed ABC, as is evident by *Theor.* 2.

		42	42	42
The Sides	28	28	26	30
	26	—	—	—
	30	14	16	12

84 *the Perimeter of the Triangle.*

42 *half the Perimeter.*
14 *the firſt Exceſs*

588 *the firſt Product.*
16 *the ſecond Exceſs.*

9408 *the ſecond Product.*
12 *the third Exceſs.*

11 | 28 | 96 *the third Product.*
3 | (336 *the Area of the Triangle.*

228
63

3996
666

000

Remarks on the firſt Method.

'Tis evident that inſtead of multiplying the Baſe AB by the Height CD, and taking half the Product to get the Area of the Triangle ABC, you may multiply the Baſe

Baſe by half the Perpendicular: or half the Baſe AB *Plate XVII.* by the Perpendicular CD; but in Practiſe 'tis better to *Fig.* 139. multiply the two Lines together, and take half their Product, becauſe they can't always be exactly halved; and multiplication of Fractions is always more troubleſome than that of Integers.

When the Perpendicular CD is not very long, it may be eaſily meaſur'd with a Cord faſtned to the Point C, extending it till it touches the Side AB: Otherwiſe, that is, when the Perpendicular CD is pretty long, in which caſe a Cord can't be ſo well uſed, you may find by the Staff of the Surveying Inſtrument, or ſome other way the Point D, of the Perpendicular, as has been ſhown in my *Introduction into Mathematicks.*

And then it will not be difficult to meaſure with a Chain and Sticks the length of the Perpendicular CD, provided you can but run all along it, whether it fall within or without the Triangle, but in Practiſe 'tis better to make it fall within, becauſe then you are freed from the trouble of producing the Side that the Perpendicular muſt be let fall on.

If you can't actually meaſure the Length of the Perpendicular CD, but can meaſure the three Sides AB, AC BC, you may find, by the help of them, the Perpendicular CD *by* 13. 2. when it falls within the Triangle *Fig.* 140. ABC, or *by* 12. 2. when it falls without.

But if you can meaſure but two Sides, for inſtance, AB, AC, inſtead of the third Side BC, meaſure the Angle A, and ſo find by *Probl.* 2. *Chap.* 3. *B.* 2. *Trig.* the Perpendicular CD, and then the Area of the Triangle ABC, which may be found by *Theor.* 1.

If the Triangle ABC be Iſoſceles, for inſtance, its two *Fig.* 141. Sides AC, BC, equal, there is no need of Calculation to find the Segments AD, BD, one of which muſt be known to find the Perpendicular CD, becauſe each in this caſe is equal to half the Baſe AB.

There will be leſs need ſtill of Calculation in finding the Perpendicular, if the Triangle be Rectangled, for if *Fig.* 142. for inſtance, the Angle A be right, the Side AC will ſerve for a Perpendicular in regard of the Baſe AB; and then you have no more to do but to multiply the two Sides AB, AC, together, and take half the Product, to gain the Area of the Triangle propoſed ABC. Thus if the Side AC be 15 Yards, and the Side AB 20, multiplying theſe two Numbers 15 and 20 together, and taking half their Product 300, you will have 150 Yards ſquare for the Area of the Right-angled Triangle ABC.

If

If the two Sides AB, AC, are expreſs'd by Yards *Plate XVII.* and Feet, reduce them into Feet, by multiplying the *Fig.* 142. Yards by 3, becauſe a Yard long contains three Feet long, and adding to the Product the odd Feet, and then find the Area in Square Feet, which may be reduced if you pleaſe, into ſquare Yards by dividing them by 9, becauſe a ſquare Yard contains 9 ſquare Feet. This will be better underſtood in the following Problem.

Remarks upon the ſecond Method.

If you can't exactly halve all the Sides, but one muſt remain, that muſt not be neglected, becauſe the Area of the Triangle would then be very imperfectly found: But to avoid Fractions, that is to ſay, to cauſe that there be no Remainder, double all the Sides, and ſeek the Area of the Triangle, which will be quadruple that of the Triangle propos'd *by* 19. 6. by theſe Sides doubled, then divide the Area thus found by 4, and you will have that you inquire after.

Fig. 140. Thus if the Side AB be 6 Perches, the Side AC, 11, and the Side BC 12, doubling theſe three Sides you will have 12, 22, 24, for the Sides of a Triangle that is Quadruple, whoſe Area will be found to be 135 Perches Square, and a Quarter of it namely 33 $\frac{3}{4}$ Square Perches will give the Area of the Triangle propoſed ABC.

If the Sides of the Triangle be expreſs'd by Fractions of a different Denomination, reduce them into the ſame; then neglecting their common Denominator, conſider their Numerators as the Sides of the Triangle, and the Area found being divided by the common Denominator will give the Area of the Triangle propos'd.

Thus if the Side AB be 3 $\frac{1}{2}$ or $\frac{7}{2}$ Feet, AC 4 $\frac{2}{3}$ or $\frac{14}{3}$ Feet, and BC 5 $\frac{3}{4}$ or $\frac{23}{4}$ Feet, reduce theſe three improper Fractions $\frac{7}{2}$, $\frac{14}{3}$, $\frac{23}{4}$, into three other of the ſame Denomination $\frac{42}{12}$, $\frac{56}{12}$, $\frac{69}{12}$, and neglecting the common Denominator 12, find the Area of a Triangle, whoſe Sides are 42, 56, 69, which you will find is 4702 Feet Square, which divided by 144 the Square of the common Denominator, will give 32 $\frac{47}{72}$ Feet Square for the Area of the Triangle *Fig.* 142. propos'd ABC.

If the Triangle propos'd be Right-angled, as ABC is in A, only ſubtract from half the Perimeter of the Triangle, each of the two Sides AB, AC, and multiplying together the two Exceſſes: Or which is much eaſier, ſubtract the Hypotenuſe BC, and multiply the Exceſs by half the Perimeter, for by any one of theſe two Methods, you

you will find the Area of the Rightangled Triangle ABC, as 'tis evident by *Theor.* 3. *Plate XVII Fig.* 142.

Thus if the Side AB be 20 Yards, the Side AC 15, and consequently the Hypotenuse BC 25 Yards; half the Perimeter is 30, from whence subtracting each of the two Sides AB, AC, you will have the two Excesses 10, 15, whose Product 150 is the Area of the Triangle ABC, or subtracting from half the Perimeter 30 the Hypotenuse BC, you will find the Excess 5, by which multiplying the Semi-Perimeter 30, the Product is 150 Yards square, as before, for the Area of the Triangle propos'd ABC.

If the Triangle be equilateral, its Area is found by multiplying the Biquadrate of one of its equal Sides by 3, and dividing the Square Root of the Product *by* 4. As for instance, if each of the Sides were 6 Perches, the Biquadrate of 6 is 1296, and the square Root of its Triple 3888 is about 62, a Quarter of which is 15 $\frac{1}{2}$ Perches Square for the Area of an Equilateral Triangle, each of whose Sides is 6 Perches.

PROBLEM II.

To measure a Parallelogram.

'TIS evident, if the Parallelogram propos'd be a Rectangle, as ABCD, that you have no more to do, but to multiply its Length AB, by its Breadth AC, to find its Area. Thus if the Length AB be 125 Yards, and Breadth AC, 64 Yards, multiply these two Numbers together, the Product 8000 Yards Square gives the Area of the Rectangle propos'd ABCD. *Pl. XVIII. Fig.* 143.

'Tis evident also that if the Parallelogram propos'd be *Obliqueangled*, as ABCD, whose Angles are oblique, you must multiply one of its Sides as AB by its Perpendicular DE, that is drawn from the opposite Angle D, to find the Area, because by this Multiplication you find the Area of the Rectangle DEFC, which is equal to that propos'd ABCD, *by* 36. 1. We shall give no Example because the thing is so easie. *Fig.* 144.

SCHOLIUM.

Since the whole Artifice of this Problem consists in multiplying two Lines together, and it often times happens, that those Lines are express'd by Fractions, I thought it proper to give some instances here of the Difficulties, that may happen for such as are not much vers'd in Arithmetic. To

Plat XVIII Fig. 144. To find the Area of the Parallelogram ABCD, whose Base AB is for instance 5 Yards and 2 Feet, and Height ED, 4 Yards, reduce these 4 Yards into Feet, by multiplying them by 3, and you will have 12 Feet in their stead. Reduce also the Base AB by multiplying by 3 the 5 Yards it contains, and you will have 15 instead of them, to which adding the odd 2 Feet, you will have 17 Feet for the Base AB, which multiplied by the Height DE 12 Feet, gives 204 Square Feet, or 22 Yards square, and 6 Square Feet, for the Area of the Parallelogram propos'd ABCD.

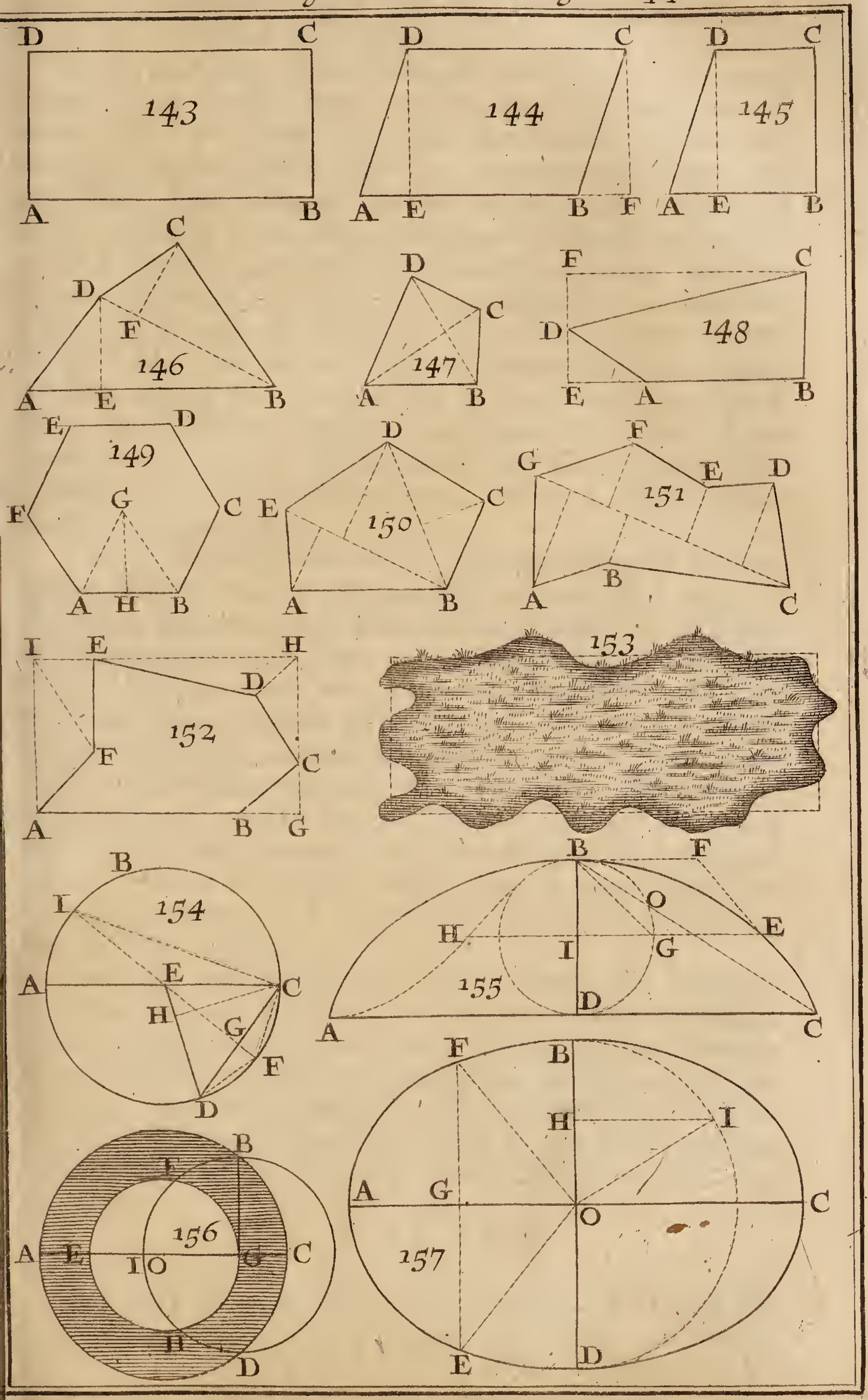
Or without reducing the Sides into Feet, multiply first the 5 Yards of the Base AB, by the 4 Yards of the Height ED, and you will have 20 Yards, then the 2 Feet of the Base AB, by the 4 Yards, or 12 Feet, the Height ED, gives 24 Feet Square, or 2 square Yards, and six Feet square, as before, for the Area of the Parallelogram sought.

This Multiplication also may be made by the Aliquot Parts, thus. Having multiply'd, as before, 5 Yards by 4 Yards, and to get the 20 square Yards, there remains 2 Feet to be multiply'd by 4 Yards, which may be done, by considering 2 Feet as $\frac{2}{3}$ of a Yard, but the $\frac{2}{3}$ part of 4 Yards, or $\frac{2}{3}$ multiplied by 4 Yards make 2 square Yards, and 6 square Feet, which added to 20 square Yards, makes 22 Yards, and 6 Feet as before; the Operation stands thus.

Yards.	*Feet.*
5	2
4	0
20	0
1	3
1	3
22	6

But since the Memory is too much charged by this Method, and 'tis easy to mistake in it, I think it better to reduce the Sides that are to be multiplied, into their lowest Terms, when they are of different kinds. As if the Base AB were 3 Yards and 2 Feet, and the Height ED two Yards and 3 Feet, each of these Lines must be reduc'd into Feet, and the 11 Feet of the Base must be multiply'd by the 9 Feet of the Height ED, and the Product will give 99 square Feet, or 11 Yards for the

Area



143
D C A B
144
D C A E B F
145
D C A E B
146
C D F A E B
147
D C A B
148
F C D E A B
149
E D F G C A H B
150
D E C A B
151
F G E D A B C
152
I E H D F C A B G
153
154
B I A E C H G F D
155
B F O H I G E A D C
156
B F A E I O G C H D
157
F B H I A G O C E D

Area of the Parallelogram ABCD, as is evident, since 9 square Feet make a yard square. *Pla.XVIII. Fig. 144.*

In like manner, if the Base AB, and Height ED were expreſs'd in Yards, Feet, and Inches, they muſt be reduced into the loweſt Denominator, namely Inches, by multiplying firſt the Yards by 3, becauſe each Yard long contains 3 Feet long, and you have the Feet, to which add the remaining Feet, then multiply the Sum by 12, becauſe a Foot long contains 12 Inches long, and you will have the Inches, to which add the odd Inches; then multiply the Inches of the Baſe AB, by the Inches of the Height ED, and you will have the Area ſought in ſquare Inches, which may be reduced, if you pleaſe, into ſquare Feet, by dividing them by 144, becauſe one ſquare Foot contains 144 ſquare Inches, and the Feet ſquare into Yards ſquare, by dividing by 9, becauſe a Yard ſquare contains 9 ſquare Feet.

Thus if the Baſe AB be for inſtance 5 Yards, 3 Feet, and 6 Inches, and the Height ED 2 Yards, 4 Feet, and 8 Inches, reduce each of theſe Lines into Inches, and you will have 222 for the Baſe, and 128 Inches for the Height ED: Multiply theſe two Numbers together, and you will have 28416 ſquare Inches for the Area of the Parallelogram ABCD, reduce theſe ſquare Inches into ſquare Feet, by dividing them by 144, and you will have 197 Square Feet, and 48 ſquare Inches: Reduce theſe 197 Square Feet into ſquare Yards, dividing them by 9, and you will have 21 ſquare Yards, and 8 ſquare Feet, in all 21 ſquare Yards, 8 ſquare Feet, and 48 ſquare Inches, for the Area ſought.

You may find the ſquare Yards at once, by dividing the 28416 ſquare Inches by 1296, the value of a ſquare Foot in Inches ſquare, becauſe a Yard long is 36 Inches long, for then you will have 21 ſquare Yards, and the remainder 1200 being taken for Square Inches and divided by 144 will give in the Quotient 8 Square Feet, and the remainder after the Diviſion will be 48 ſquare Inches as above.

SCHOLIUM.

You need not meaſure any Perpendicular, if the Parallelogram propos'd be a Rhombus, becauſe its Area is equal to half the Product under the two Diagonals, as 'tis eaſy to demonſtrate.

PROBLEM III.

To find the Area of a Trapezium.

Pl. XVIII. Fig. 145. FIRST, if the Trapezium propos'd be a Trapezoid, as ABCD, whose opposite Side AB, CD are parallel, measure exactly the length of each of these two parallel Sides AB, CD, as also that of their Perpendicular DE, and multiply the Sum of the same two Sides AB, CD by the Perpendicular DE, and half the Product shall be the Area of the Trapezoid propos'd ABCD, by *Theor.* 4. Thus if the Perpendicular DE be 24 Yards, and the Side AB 25, and the other Side parallel CD 16, the Sum of the two parallel Sides AB, CD is 41, which multiply'd by the Perpendicular DE, suppos'd to be 24 Yards, will give 984, whose half, 492 square Yards will give the Area sought.

Fig. 146. But if the Trapezium propos'd has no Sides parallel, as ABCD, whose Side AB suppose to be 21 Yards, BC 15, CD 8, and AD 10, then reduce it into two Triangles by the Diagonal BD, 17 Yards long. On this Supposition, the Area of the Triangle BCD, whose three Sides are known, will be 60 square Yards, and that of the Triangle ADB 84 Yards square, by *Prob.* 1. Wherefore add the two Areas found 60 and 84 together, and you will have 144 Yards square for the Surface of the Trapezium propos'd ABCD.

SCHOLIUM.

When you are to measure the Triangles ABD, BCD, by their Perpendiculars, you may let them fall from what Angle you please upon its opposite Side, as is most convenient. Thus here the Perpendicular DE is drawn from the Angle D to the opposite Side AB, and the Perpendicular CF from the Angle C, to its opposite Side or Diagonal BD. But it would be better, when it can be well done, to let fall Perpendiculars upon the same Diagonal BD, because the Diagonal BD being multiply'd by the Sum of its two Perpendiculars, and halved gives the Area of the Trapezium propos'd ABCD, as 'tis evident by 1, 2.

Fig. 147. The Diagonal BD must be measured upon the Ground, because it has no determin'd Ratio to the four Sides, except when the Quadrilateral ABCD is in a Circle, in which Case call the Side AB *a*, and the Side BC *b*, and the

the Side CD c, and the Side AD, d, and the Square of the Diagonal BD will be proportional to these three Planes $ad + bc$, $ab + cd$, $ac + bd$, and the Square of the other Diagonal AC will be proportional to the three Planes $ab + cd$, $ac + bd$, $ad + bc$. When therefore the four Sides shall be known, the Diagonals will also be known, and consequently the Area of the Trapezium ABCD. But this Area may be found independently upon the Diagonal AC or BD, by this Rule, which has its Demonstration. *Pl. XVIII. Fig.* 147.

Having added together the four Sides AB, BC, CD, AD, substract separately from half their Sum each Side, and multiply the four Excesses together, and the square Root of their Product will give the Area sought.

Thus if the Side AB be 100 Foot, the Side BC 63, Side CD 45, and the Side AD 80, half the Sum of the four Sides 100, 63, 45, 80, is 144, from which substracting separately the same Sides 100, 63, 45, 80, and there will remain these four Numbers or Excesses 44, 64, 81, 99, whose Product into one another is 22581504, and its square Root 4752 Feet Square, gives the Area of the Quadrilateral propos'd ABCD.

In like manner, if the Side AB be 18 Yards, BC 11, CD 2, and AD 23, its Perimeter will be 54 Yards, and its half consequently 27 Yards, from whence substracting separately these four Sides 18, 11, 2, 23, there will remain these four Numbers 9, 16, 25, 4, which being Squares, their Product into one another will be also a square Number, namely 14400, whose square Root 120 Yards is the Area sought.

If the Ground will not permit you to measure any of its Diagonals, describe about the Quadrilateral ABCD the Parallelogram EBCF, whose Area measure by *Prob.* 2. as also that of the two Triangles AED, CFD, then substract the Sum of these two Areas from that of the Parallelogram, and you will have remaining the Surface of the Trapezium propos'd ABCD. *Fig.* 148.

PROBLEM IV.

To find the Area of a regular Polygon.

To find the Superficies of a regular Polygon, for instance, of the Hexagon ABCDEF, whose Center G is the same with that of the circumscrib'd Circle, draw from the Center G, upon the middle of one of the Sides, as AB, the Perpendicular GH, which must be found first *Fig.* 149.

 of

of all, by the help of the known Side AB, which we will ſuppoſe to be 120 Fathoms, in this manner.

Pl. XVIII. Fig. 149. Divide the entire Circle, or 360 Degrees, by the number of the Sides of the Polygon, as here by 6, the Quotient will give 60 Degrees for the Angle at the Center AGB, wherefore its half, or the Angle AGH will be 30 Degrees, whoſe Complement GAH, which is half the Angle of the Polygon, will be conſequently 60 Degrees. Thus in the Right-angled Triangle AGH, there is known beſides the Angles, the Side AH 60 Fathoms, being half the Side AB, which we ſuppoſed 120 Fathoms; wherefore the other Side, or the Perpendicular GH may be found by this Proportion.

As the Radius	100000
To the Tangent of half the Angle of the Polygon	173205
So is half the Side of the Polygon	60
To the Perpendicular GH	103. 5. 6.

Which will be found to be 103 Fathoms, 5 Feet, and about 6 Inches, by which multiplying the Perimeter of the Hexagon, 720 Fathoms, half the Product will give 37410 Fathoms ſquare, for the Area of the Hexagon propos'd ABCDEF; the Demonſtration is evident by *Theor.* 5.

SCHOLIUM.

A diſtance never ſo little, from the true length of the Perpendicular, in neglecting the Fractions, will cauſe a ſenſible one from the true Area of the Polygon, eſpecially if its Side be of a conſiderable bigneſs: For by neglecting here the Fractions of the Inches in the length of the Perpendicular, we have come ſhort of the Area of the Hexagon, by two Fathoms ſquare, as may be found by following this particular Rule, for finding, as exactly as is poſſible, the Area of a regular Polygon, whoſe Side is known.

Multiply the Tangent of half the Angle of the Polygon by the Side of the ſame Polygon, and multiply the Product by the Perimeter of the Polygon, for a ſecond Product, divide a quarter of it by the Radius, and you will have the Area ſought.

As in this Example, half the Angle of the Polygon being 60 Degrees, multiply its Tangent 173205 by the Side of the Polygon, which we have ſuppoſed to be 120 Fathoms, and you will have the firſt Product 20784600, which multiply'd by the Perimeter of the Polygon, 720 Fathoms, will give

give a ſecond Product 14964912000, a quarter of which *Pl. XVIII.*
3741228000 being divided by the Radius 100000, gives *Fig.* 149.
37412 Fathoms ſquare for the Surface of the Polygon propos'd ABCDEF.

By this Rule we have computed the following Table, ſhewing the Areas of the regular Polygons, from a Triangle to a Dodecagon, the Side of the Polygon throughout being ſuppos'd to be 1000, and this Number pitch'd upon rather than any other, becauſe more commodious, not only for computing the Table, but for its Uſefulneſs, for by it, you may eaſily find the Area of a Polygon propos'd, whoſe Side is greater or leſs than 1000 Yards, or Feet, &c.

Triangle	433013
Square	1000000
Pentagon	1720477
Hexagon	2598075
Heptagon	3633526
Octagon	4828427
Enneagon	6181824
Decagon	7694209
Endecagon	9363808
Dodecagon	11196132

For as ſimilar Polygons are to one another as the Squares of their Homologous Sides, by 20. 6. 'tis eaſily known, you have nothing to do, but to multiply the Square of the Side of the Polygon propos'd, by the Area correſponding in the foregoing Table, and divide the Product by the Square of 1000, which is 1000000, which is done by cutting off ſix Figures to the right Hand. Thus here multiply the Area of the Hexagon found in the Table, namely 2590875 by the Square 14400 of the Side of the Hexagon, ſuppos'd to be 120 Fathoms, and divide the Product 37412280000 by 1000000, and the Quotient will give, 37412 ſquare Fathoms for the Area ſought, as before.

PROBLEM V.

To find the Area of an Irregular Polygon.

Plat XVIII. Fig. 150. AN Irregular Polygon may be meaſur'd juſt as the Trapezium, which we reduc'd into two Triangles by a Diagonal, namely, by reducing the Irregular Polygon into Triangles by Diagonals drawn, as you pleaſe, from one Angle to another, and meaſuring by *Probl.* 1. the Areas of all the Triangles, for their Sum gives the Area of the Polygon propos'd. Thus we have reduc'd the Pentagon ABCDE into three Triangles ABE, BDE, BCD, by the two Diagonals BE, BD: and after the *Fig.* 151. ſame manner you may reduce into Triangles the Irregular Heptagon ABCDEFG, or if it be more commodious, draw a right Line thro' the two moſt diſtant Angles, as CG, uſually call'd the *Fundamental Line*, to which let fall from all the other Angles as many Perpendiculars, and they will divide the Figure into Right-angled Triangles and Trapezoids, whoſe Areas may be found by *Probl.* 1. 2. and their Sum gives the Area ſought.

SCHOLIUM.

Fig. 152. When you can't draw Diagonals within the Polygon that is to be meaſured, for inſtance the Hexagon ABCDEF, to reduce it into Triangles, deſcribe about it the Parallelogram AGHI, whoſe Area may be found by *Probl*, 2. from whence ſubtracting the Areas of all the Triangles form'd without the Figure, and eaſily meaſur'd by *Probl.* 1. 2. what remains will be the Area of the Hexagon propos'd ABCDEF.

'Tis evident that by this Method one may eaſily plat a Field by Diameters taken without, as ABCDEF, becauſe having meaſured pretty exactly the Sides of all the Triangles form'd within the Parallelogram AGHI, 'twill be eaſie, by taking the meaſure of all the Lines upon a particular Scale, or upon the equal Parts of a Sector, to lay down all the Triangles, and ſo have the Figure reduc'd into a ſmall compaſs upon the Paper.

Since it often happens that the Grounds that are to be ſurvey'd, are bounded by Curve Lines, the Curve Lines may be taken for right ones, if the difference be but little, or you may divide them into leſſer Parts, that may paſs for right Lines, conſidering the Figure as a Polygon

lygon of a great number of little Sides, that may be measur'd as has been taught, without any considerable Error: Or by drawing right Lines cross the Curve ones, so that to the view they make up without, what they cut off from the Figure, without any sensible Error, especially if you were to measure the Content of a Lake, or County. *PlatXVIII. Fig.* 153.

PROBLEM. VI.

To find the Circumference of a Circle, its Diameter being given.

TO find the Circumference of the Circle ABCD, whose Diameter AC for instance is 125 Feet, multiply the Diameter 125 Feet by 314, and divide the Product 39250 by 100, and the Quotient will give 392 Feet, 6 Inches for the Circumference ABCD sought, as is evident by *Theor.* 14. which shews us also, that multiplying the Diameter 125 by 22, and dividing the Product 2750 by 7, we shall have 392 Feet, 10 Inches for the Circumference ABCD, which happens to be greater than the former, because as we took notice, the number 22 is greater in respect of the Divisor 7, than it should be, as the Number 314 is less than it should be in respect of 100, but the difference is not so great. Wherefore the Circumference ABCD will be nearer 392 Feet, 6 Inches, than 392 Feet, 10 Inches, but to have it nearer still, multiply the Diameter 125 by 314159, and divide the Product 39269875 by 100000, and then the Circumference will be found to be 392 Feet and about 8 Inches. *Fig.* 154.

PROBLEM VII.

To find the Diameter of a Circle, whose Circumference is given.

TO find the Diameter AC of a Circle whose Circumference ABCD is, for instance, 125 Yards, multiply this Circumference 125 by 100, and divide the Product 12500 by 314, and the Quotient will give 39 Yards, and about 3 Feet for the Diameter AC sought, as is evident by *Theor.* 14. Where we told you, that in great Numbers the Ratio of 100000 to 314159 must be used; thus to find the Diameter of the Earth, whose Circumference is 7200 Leagues, because a Degree of the Circumference of the Earth is 20 Leagues, multiply this Circumference 7200 by 100000, and divide the Product 720000000 by

314159, the Quotient will give about 2292 Leagues for the Diameter sought, or for the Distance from hence of our Antipodes, half whereof 1146 Leagues, gives the Semi-diameter of the Earth, or the Distance of the Center of the Earth from us.

Plat XLVII Fig. 154.

PROBLEM VIII.

To find the Area of a Circle, its Diameter being given.

TO find the Area of the Circle ABCD, whose Diameter AC is for instance 125 Feet, multiply the Diameter 125, by its Circumference, which we found to be 392 Feet 8 Inches, or 4712 Inches, by *Probl. 6.* and a quarter of its Product or 12270 Feet square, and about 120 Inches square, will be the Area of the Circle propos'd ABCD, as is evident by *Theor.* 13.

SCHOLIUM.

To avoid the Fractions, that happen in the Circumference; after you have multiplied the Diameter 125 by 314, don't divide the Product 39250 by 100, as ought to be done in finding the Circumference, but multiply it by the same Diameter 125, for a second Product 4906250, a quarter of which $1226562\frac{1}{2}$ divided by 100 gives 12265 Feet square, and 90 Inches square for the Area of the Circle propos'd ABCD.

Or because the Square of the Diameter of a Circle is to the Area of the same Circle as 1000 to 785 nearly, by *Theor.* 15. Multiply the Diameter 125 by it self, for its Square 15625, and this Square 15625 by 785, and divide the Product 12265625 by 1000, and you will have as before 12265 Feet square, and 90 Inches square for the Area sought.

This Area may be found more easily than by the first where we have omitted the Fractions of an Inch: But if you would be more exact, use the Ratio of 1000000 to 785398, instead of 100 to 785, that is to say, multiply the Square 15625 of the Diameter 125 by 785398, and divide the Product 12271843750 by 1000000, and the Quotient will give 12271 Feet square, and about 121 Inches square for the Surface of the Circle propos'd ABCD.

'Tis

'Tis by this Method I have made the following Table, that shews, in Integers, and Hundred-Thousandths Parts, separated from the Integers, by a Point to the right Hand, the Areas of Circles, whose Diameters are from 1 to 100.

A

A Table of Areas of Circles, whoſe Diameter is of a given Magnitude from 1 *to* 100.

Diam	*Areas.*	*Diam.*	*Areas.*
1	0.78540	31	754.76763
2	3.14159	32	804.24771
3	7.06858	33	885.29859
4	12.56637	34	907.92027
5	19 63495	35	962.11275
6	28.27433	36	1017.87601
7	30.48451	37	1075.21008
8	50.26548	38	1134.11494
9	63.61725	39	1194.59060
10	78.53981	40	1256 63706
11	95.03317	41	1320.25431
12	113.09733	42	1385.44230
13	132.73228	43	1452.20120
14	153.93804	44	1520.53084
15	176.73458	45	1590 43128
16	201.06192	46	1661.90251
17	226 98006	47	1734.94454
18	254.46900	48	1809 55736
19	283.52873	49	1885.74099
20	314.15926	50	1963.47540
21	346.36059	51	2042.82062
22	380.13271	52	2123.71663
23	475.47562	53	2206.18344
24	452.38934	54	2290.28104
25	490.87385	55	2375.82944
26	530.92915	56	2463.00864
27	572.55526	57	2551.75863
28	615.75216	58	2642.07942
29	660 51985	59	2733.97100
30	706.85834	60	2827.43338

Diam.

Diam.	*Areas.*	*Diam.*	*Areas.*
61	2922.46656	81	5152.99735
62	3019.07054	82	5281.01725
63	3117.24531	83	5410.60794
64	3216 99087	84	5541.76944
65	3318.30724	85	5674.50173
66	3421.19439	86	5808.80481
67	3535.65235	87	5944.67869
68	3631.68110	88	6082.12337
69	3739.28065	89	6221.13885
70	3848.45100	90	6361.72512
71	3959.19214	91	6503.88219
72	4071.50407	92	6647 01005
73	4185.38681	93	6792.90871
74	4300.84034	94	6039.77817
75	4417 86466	95	7088.21842
76	4536.45979	96	7238.22947
77	4656.62571	97	7389.81131
78	4778.36242	98	7542.96396
79	4901.06993	99	7697.68739
80	5026.54824	100	7853.98163

Thus you may find by this Table, the Area of a Circle whoſe Diameter is 56 Feet, to be 2463 Feet ſquare, and 864 Hundred-thouſandths which is about 1 Inch ſquare, as may be known by multiplying the Numerator 864 by 144, becauſe a Foot ſquare contains 144 Inches ſquare, and dividing the Product 1224416 by the Denominator 100000.

PRO-

PROBLEM IX.

To find the Area of a Circle, its Circumference being given.

Plat XVIII. Fig. 154. TO find the Surface of a Circle, whose Diameter is AC, its Circumference ABCD being given, for instance, 125 Fathoms, suppose it the Circumference of the Bason of a Fountain; multiply this Circumference 125 by the Diameter AC, which by *Probl.* 7. will be found to be 39 Fathoms, 5 Feet, or 239 Feet, a quarter of its Product will give 1244 Fathoms and 28 Feet square for the Area of the Circle propos'd.

SCHOLIUM.

To avoid the Fractions, that happen in the Diameter of the Circle, after you have multiply'd the Circumference 125 by 100, don't divide the Product 12500 by 314, as you ought to do in finding the Diameter, but multiply it by the same Circumference 125, for a second Product 1562500, a quarter of which 390625 divided by 314 will give 1244 Fathoms, and 1 Foot square for the Area sought.

Since the second Product 1562500 is nothing but the Product of the Square of the Circumference by 100, and the Ratio of 100000 to 314159 is more accurate than that of 100 to 314, as was seen in *Theorem* 14. The Area sought may be found more exactly by multiplying the Square 15625 of the Circumference 125 by 100000, and dividing 390625000, a quarter of the Product 1562500000, by 314159, the Quotient will give 1243 Fathoms, 14 Feet square for the Area of the Circle propos'd ABCD, which is very exact.

PRO-

PROBLEM X.

To find the Diameter of a Circle, its Area being given. Plat XVIII Fig. 154.

BEcause by *Th.* 13. the Area of a Circle is equal to half the Rectangle under the Circumference and Radius, consequently to a quarter of the Rectangle under the Circumference and Diameter, it follows, that if by the help of the following Problem, you find the Circumference, and divide the quadruple of the Area given by that Circumference, you will have the Diameter answering to that Area. But because we have not found the Circumference we must do thus.

To find the Diameter AC of the Circle ABCD, whose Area is 125 Feet square, we must recollect what was demonstrated in *Theor.* 15. namely, that the Area of a Circle is to the Square of its Diameter, as 785 to 1000 nearly: We must therefore multiply the Area 125 by 1000, and divide the Product 125000 by 785, and the square Root of the Quotient 159 gives 12 Feet 7 Inches for the Diameter sought.

SCHOLIUM.

The Ratio of 785398 to 1000000 being more exact than 785 to 1000, use the second Ratio, and you will find the Diameter with much greater exactness. Multiply therefore the Area given 125 by 1000000, and divide the Product 125000000 by 785398, and the Square Root of the Quotient 159, will give 12 Feet 7 Inches for the Diameter AC.

By this Method we have computed the following Table, showing in Integers and Hundred-thousandth Parts, that are separated from the Integers by a Point on the Right Hand, the Diameters of Circles, whose Areas are given, from 1 to 100.

A Table

A Table of the Diameters of Circles, whose Areas are given from 1 to 100.

Areas.	Diamet.	Areas.	Diamet.
1	1 12838	31	6.28254
2	1.59576	32	6.38304
3	1 95441	33	6.48204
4	2.25676	34	6.57952
5	2.52313	35	6.67558
6	2.76395	36	6.77028
7	2.98541	37	6.86366
8	3.19152	38	6.95579
9	3.38514	39	7.04673
10	3.56824	40	7.13648
11	3.74240	41	7.22515
12	3.90882	42	7.31272
13	4.06844	43	7.39928
14	4 22200	44	7.48480
15	4.37019	45	7.56939
16	4 51352	46	7.65304
17	4.65242	47	7.73578
18	4.78728	48	7.81764
19	4 91849	49	7.89865
20	5 04626	50	7.97885
21	5.17088	51	8.05824
22	5.29256	52	8.13686
23	5.41151	53	8.21481
24	5 52790	54	8.29185
25	5.64190	55	8.36829
26	5.75362	56	8.44400
27	5.86323	57	8.51907
28	5 97082	58	8.59347
29	6.07650	59	8.66724
30	6.18038	60	8.74038

Areas.

Areas.	*Diamet.*	*Areas.*	*Diamet.*
61	8.81292	81	10.15541
62	8.88487	82	10.21790
63	8.95623	83	10.28001
64	9.02703	84	10.34176
65	9.09727	85	10.40314
66	9.16700	86	10.46416
67	9.23618	87	10.52482
68	9.30484	88	10.58512
69	9.37302	89	10.64516
70	9.44069	90	10.70474
71	9.50789	91	10.76405
72	9.57456	92	10.82302
73	9.64072	93	10.88169
74	9.70668	94	10.94004
75	9.77205	95	10.99806
76	9.83698	96	11.05581
77	9.90148	97	11.11325
78	9.96557	98	11.17038
79	10.02924	99	11.22723
80	10.09253	100	11.28379

Thus you may find by this Table, that the Diameter of a Circle, whoſe Area is 30 Feet ſquare, is 6 Feet and 18038 Hundred-thouſandth parts of a Foot, which is about 2 Inches, as may be ſeen by multiplying the Numerator 18038 by 12, becauſe a Foot long contains 12 Inches long, and dividing the Product 216456, by the Denominator 100000.

PRO

PROBLEM XI.

To find the Circumference of a Circle, its Area being given.

Plat XVIII Fig. 154. BEcause by *Theor.* 13. the Area of a Circle is equal to half the Rectangle under the Circumference and Semi-diameter, and consequently four times the Area is equal to the Rectangle under the Circumference and Diameter, it follows, that if you divide four times the Area by the Diameter, which may be found by *Problem* 10, you will have the Circumference of the Circle.

Thus if the Area of the Circle ABCD were 125 Feet square, and its Diameter found by *Problem* 10. is 12 Feet 7 Inches, or 151 Inches, divide 500 Feet square or 72000 Inches square, which is the quadruple of the Area proposed 125, by the Diameter 151, you will have 477 Inches or 39 Feet, 9 Inches for the Circumference of the Circle ABCD, whose Area is 125 Feet.

SCHOLIUM.

THE Circumference may be found at once, *by multiplying* 500 *the quadruple of the Area given* 125, *by* 314, *and dividing* 396 *the Square Root of the Product* 15700 *by* 10, for the Quotient gives 39 Feet 7 Inches for the Circumference ABCD, exacter than the first: You may have a third more exact still, *by multiplying* 500, *the quadruple of the Area given* 125, *by* 3141592, *and dividing* 39633, *the Square Root of the Product* 1570796000, *by* 1000, the Quotient giving 39 Feet 7 Inches, as before, for the Circumference sought.

PROBLEM XII.

To find the Area of a Sector of a Circle, less than a Semi-circle.

TO find the Area of the Sector of a Circle CEDF, less than the Semi-circle ADC, its Diameter AC being given, for instance 125 Feet, and its Angle CED or Arc CFD being given, suppose 80 Degrees; the Area of the whole Circle ABCD will be found by *Problem* 8. to be 12271 Feet square, and about 121 Inches square, and because the Area of the Sector CEDF is such a Part of the Area of the whole Circle ABCD, as its Arc CFD is of

of the whole Circumference ABCD, that is to say, as 80 to 360, you plainly see the Area of the Sector is found by multiplying the Area of the Circle, which is 12271 Feet, and 121 Inches square, by 80 the Number of Degrees of the Arc CFD, or Angle CED, and dividing the Product 981747 Feet and 34 Inches Square by the Number of Degrees 360 contain'd in the Circumference, for the Quotient 2727 Feet and 10 Inches square gives the Area of the Sector propos'd CEDF. *Pla.XVIII. Fig.* 154.

SCHOLIUM.

To avoid the Fractions, that commonly happen in the Area of the Circle, *multiply the Square* 15625 *of the Diameter* 125 *always by* $21816\frac{1}{2}$, *and that Product* $340882812\frac{1}{2}$ *again by* 80 *the Number of Degrees of the Angle of the Sector, for a second Product* 27270625000, *which divide by* 10000000, and the Quotient 2727 Feet, 9 Inches square will give the Content of the Sector CEDF.

As by multiplying the Circumference ABCD by the Radius AE or EC, and taking half, you find the Area of the Circle, by *Theor.* 13. So after the same manner multiply the Arc CFD by the Radius CE or ED, and take half, and you will have the Area of the Sector CEDF, so that to find the Area independently on the Circle, you must know how to measure the Arc CFD, which may be done thus.

Because the Arc CFD is such a Part of the Circumference ABCD, as the number of Degrees it contains is of 360 the number of Degrees the Circumference contains; finding by *Problem* 6. the Circumference of the Circle ABCD, 392 Feet, 8 Inches, then multiply it by 80 the Number of Degrees of the Arc CFD, and divide the Product 31413 Feet, 4 Inches, by 360 the Number of Degrees in the Circumference of the Circle, the Quotient 87 Feet, and about 3 Inches will give the Quantity of the Arc CFD.

This Arc CFD may also be found more exactly, by adding to double the Chord FC or FD of half the Arc CFD, a third of the Excess of that double above the Chord of the intire Arc CD.

The double of the Chord CF will be found to be 85 Feet, 6 Inches, whence subtracting the Chord CD, 80 Feet, 4 Inches, the Remainder is 5 Feet, 2 Inches, whose third 1 Foot, 9 Inches, added to double the Chord CF, or 85 Feet, 6 Inches, gives 87 Feet, 3 Inches for the Arc CFD, as before.

The double of the Chord CF or FD was found by this Proportion,

As the Radius	100000
To double the Diameter AC	500
So is the Sine of a Quarter of the Angle CED,	34202
To double the Chord CF	85. 6

Pl. XVIII. Fig. 154.

And the Chord CD was found by this Proportion.

As the Radius	100000
To the Diameter AC	125
So is the Sine of half the Angle CED	64278
To the Chord CD	80. 4

From these two Proportions the following Rule may be drawn for finding, with ease and exactness, the Arc CFD. *Subtract the Sine* 64278 *of half the Angle of the Sector, from* 273616 *eight times the Sine* 34202 *of a Quarter of the same Angle, and multiply the remainder* 209338 *by the Diameter* 125, *for the Product* 26167250, *and the Quotient of* 8722417 *its third Part divided by the Radius* 100000, *will give* 87 *Feet and* 3 *Inches for the Quantity of the Arc CED.*

PROBLEM XIII.

To find the Area of a Sector of a Circle, greater than a Semi-circle.

Fig. 154. TO find the Area of the Sector of a Circle EDABCE greater than the Semi-circle ABC, whose Diameter AC; suppose to be 125 Feet, and the Arc DABC 280 Degrees, subtract these Degrees from 360, and the Remainder will be 80 Degrees for the Arc CFD. Find by *Probl.* 12. the Area of the Sector CEDF, which you will find 2727 Feet, and 9 Inches Square, and subtract the Area of the whole Circle, which you will find by *Probl.* 8. to be 12271 Feet and 121 Inches Square, the Remainder will give the Area of the Sector propos'd EDABCDE, 9544 Feet, and 112 Inches Square.

PRO-

PROBLEM XIV.

To find the Area of a Segment of a Circle leſs than a Semi-circle.

TO find the Area of the Segment of a Circle CDF, leſs than the Semi-circle ACD, by its Diameter AC, ſuppos'd to be 125 Feet, and Arc CFD ſuppos'd to be 80 Degrees; the Area of the Sector CEDF will be found by *Probl.* 12. to be 2727 Feet and 9 Inches ſquare, from whence ſubtract the Area of the Iſoſceles Triangle DEC, which may be found thus. *Pl. XVIII Fig.* 154.

Having let fall from the Angle E, the Perpendicular EG, on the Baſe CD, which will biſect the Baſe CD, and Angle of the Segment CED, find the length of that Perpendicular EG, by making this Proportions in the two Right-angled Triangle EGC, EGD.

As the Radius	100000
To the Semi diameter BC or ED	$62\frac{1}{2}$
So is the Sine Complement of half the Angle CED	76604
To the Perpendicular EG	47. 10

Which will be found to be 47 Feet and about 10 Inches, multiply it by the Line DG or CG 40 Feet, 2 Inches, half the Line CD, which was found by *Problem* 13. to be 80 Feet, 4 Inches, and the Product 1921 Feet, 44 Inches ſquare, gives the Area of the Triangle CED.

Or let fall from one of the two Angles C, D, as the Angle C, the Perpendicular CH, to the oppoſite Side DE, whoſe length may be found by making this Proportion in the Right-angled Triangle CHE.

As the Radius	100000
To the Semi-diameter EC	$62\frac{1}{2}$
So is the Sine of the Angle CED	98480
To the Perpendicular CH	61. 6

Which being thus found to be about 61 Feet 6 Inches, and multiply'd by half the Radius ED $31\frac{1}{4}$, or a quarter of the Diameter AC, ſuppos'd to be 125 Feet, will give 1921 Feet 26 Inches for the Area of the Triangle CED.

The Area found by theſe two ways is not very exact, becauſe we have taken no notice of the Fractions of the Inches. To find it exactly follow this Rule, drawn from

the

the foregoing Proportion; *Multiply the eighth Part* 12310
Pl. XVIII. Fig. 154. *of the Sine* 98480 *of the Angle CED, by the Square* 15625 *of the Diameter* 125, *and divide the Product* 192343750 *by the Radius* 100000, *the Quotient* 1923 *Feet* 6 *Inches square, will give the Area of the Triangle CED,* which subtracted from that of the Sector CED, found to be 2727 Feet and 9 Inches square, leaves 803 Feet 90 Inches square for the Area of the Segment propos'd CDF.

SCHOLIUM.

If the Segment CDF were separated from the Circle, in which case the Diameter AC or FI is not known, measure the Base or Chord CD, and Height FG, call'd the *Versed Sine*, by which dividing the Square of half the Base CG, or DG, you will have the other Part GF of the Diameter FI; Because the Angle ICF being right *by* 31. 3. the Line CG, perpendicular to the Diameter FI, is a mean proportional between the two Parts GI, GF, *by* 8.6. Besides that *by* 25. 3. the Square CG is equal to the Rectangle under the two Parts GI, GF. Thus adding together the two Parts GI, GF, the Diameter FI will be known, and consequently the Semi-diameter EC, whose Square lessen'd by the Square CG, or Rectangle under the Parts CI, GF, gives the Square EG, &c.

PROBLEM XV.

To find the Area of a Segment of a Circle greater than a Semi-circle.

TO find the Area of the Segment of a Circle CDAB, greater than the Semi-circle ABC, whose Diameter AC, for instance, suppose to be 125 Feet, and Arc DABC 280 Degrees, in which Case the Arc CFD will be 80 Degrees; subtract from the Area of the whole Circle ABCD, which by *Probl.* 8. will be found to be 12271 Feet and 121 Inches Square, the Area of the little Segment CDF, which by *Probl.* 14. will be found to be 803 Feet 90 Inches square, and the Remainder 11468 Feet and 31 Inches Square, will give the Area of the greater Segment CDAB, which may be found also by adding the Area of the great Sector CEDAB, which by *Probl.* 13. will be found to be 9544 Feet and 112 Inches square, the Area of the Triangle CED, which by *Probl.* 14. will be found to be 1923 Feet and 67 Inches square.

PRO-

PROBLEM XVI.

To find the Area of a Space bounded by a Cycloid.

TO find the Area of the Space ABCD terminated by the Cycloid ABC, and right Line AC, equal to the Circumference of the generating Circle, whose Diameter is the Axe BD, which you may suppose to be 12 Inches, in which Case the Circumference of the generating Circle or Base AC will be found to be $37\frac{5}{7}$ Inches, by *Probl.* 6. and the Area of the generating Circle will be found by *Probl.* 8. to be $113\frac{7}{72}$ Inches square; triple this Area found, and you will have $339\frac{7}{24}$ Inches square for the Area of the Cycloid ABCD. The Demonstration is evident by *Theor.* 19. *Pl.* XVIII. *Fig.* 155.

Or, *Multiply the square* 144 *of the Area* 12, *by* 2356194, *and divide the Product* 339291936, *by* 1000000, and the Quotient will give, as before about $339\frac{7}{24}$ Feet square, for the Area sought.

SCHOLIUM.

Because by *Theor.* 19. the Area ABCD of the Cycloid is triple that of the generating Circle BGD, you easily see that the Space BGDCE is equal to the generating Circle, and consequently to $113\frac{7}{72}$ Inches square, half of which, $56\frac{79}{144}$ Inches square will give the Area of the Segment BCE, because the Segment BCE is equal to half the Space BCE, or half the Space BDAH, terminated by the Curve BHA describ'd by the help of the Tangents of the Cycloid BEC, by *Theor.* 8. Where we have demonstrated that the Segment BCE is half the Space BDAH, equal to the Space BGDCE, each being compos'd of an equal number of equal Lines, as 'tis easie to conclude from the Property of the Tangent EF, which is parallel to the corresponding Chord BG, as has been demonstrated in *Theor.* 19, where we have demonstrated also that the Line GE or BF or IH is equal to the corresponding Arc BOG.

PROBLEM XVII.

To find the Area of an Annulus, or Ring.

'TIS evident, that to find the Area of the Ring bounded by the two Circumferences of Concentric Circles ABCD, EFGH, whose Diameter AC, EF, are known, as AC 18 Fathoms, and EG 12 Fathoms, you have nothing

to do but to subtract the Area of the Circle EFGH,
Pl.XVIII which by *Probl.* 8. will be found to be 113 Fathoms and
Fig. 156. 3 Feet Square, from that of the Circle ABCD, which will be found to be 254 Fathoms and 17 Feet square, for the remainder 141 Fathoms and 14 Feet square will give the Area of the Annulus or Ring propos'd.

This Area may be found several other ways, but the easiest of all is this; *Multiply the Sum* 30 *of the two Diameters* 18, 12, *by their difference* 6, *and multiply the Product* 180 *by* 785398, *for a second Product* 141371640, *which divided by* 1000000, *gives* 141 *Fathoms and* 13 *Feet square for the Area sought.*

The Demonstration of this short Method will be evident to any one that knows that the Annulus or Ring AFC is equal to the Circle HIBD, whose Radius GB is perpendicular to the Diameter AC, and consequently a mean proportional between the Lines AG, GC, *by* 13. 6. for since the Square of that mean Proportional is by 17. 6. equal to the Rectangle of the Lines AG, GC, which are the Sum and Difference of the two Radius's OC, OG; that is to say by 52. to the difference of the Squares of the same two Radij OC, OG; and the Circles are to one another as the Squares of their Radij by 2. 12. it follows that the Circle HIBD, whose Radius is GB is equal to the difference of the Circles, whose Radij are OC, OG, that is to say, to the Annulus, or Ring, AFCH, &c.

PROBLEM XVIII.

To find the Area of an Ellipse.

To find the Area of the Ellipse ABCD, whose great Axe or Length AC suppose to be 20 Fathoms, and less Axe or Breadth BD to be 12 Fathoms, multiply the two
Fig. 157. Axes 20 and 12 together, and let the Product 240 be multiply'd by 785, and that Product 188400 divided by 1000 will give 188 Fathoms and 14 Feet square for the Area of the Ellipse propos'd; the Demonstration is evident by *Theor.* 17.

SCHOLIUM.

To find the Area more exactly, instead of the Ratio of 1000 to 785 use that of 1000000 to 785398, that is to say, multiply the Product 240 of the two Axes 20, 12, by 785398, and divide the Product 188495520 by 1000000 and

and the Quotient will give 188 Toises and 18 Feet square for the Area sought.

If a Segment of an Ellipse, as AEF, were to be measured, bounded by the right Line EF, perpendicular to the great Axe AC, measure the part AG, and divide the less Axe BD at the Point H, after the same manner as the great Axe AC is divided at the Point G, namely, by finding to the three Lines AC, AG, BD, a fourth proportional BH; then let fall from the Point H, the Line HI perpendicular to the Axe BD, and that will be terminated in I, by the Semi-circle BFD, describ'd on the Center O, about the little Axe BD. That being done, if by *Probl.* 14. you measure the Segment of the Circle, half of which is represented by BHI, and by *Problem* 8. the Circle whereof BDI is half, this Segment will be such a Part of its Circle as the Segment AEF is of the Ellipse ABCD by *Theor.* 18. Wherefore if to the Area of the Circle, and that of its Segment, and that of the Ellipse, you find a fourth Proportional, you will have that of the Segment propos'd AEF. The same way of Reasoning may be used for finding the Area of the Sector of the Ellipse OFA, but instead of the Area of the Circle and Ellipse you may substitute the less Axe BD, and great Axe AC, that are in the same Ratio by *Theor.* 16. *Pl.* XVIII. *Fig.* 157.

PROBLEM XIX.

To measure an Hyperbola.

TO find the Area of the Hyperbola ABC, whose Base AC is perpendicular to the Axe BD, and the two Asymptotes EF, EG, cutting one another in the Center E of the Hyperbola, at equal Angles on both Sides of the Axe ED; draw from the Point C, the right Line CF parallel to the Asymptote EG, and thro' the Vertex B, to the same Asymptote EG, the parallel BH, which will be equal to the Line EH. I imagine the Part FH divided into an infinite Number of equal Parts at the Points I, K, L, and draw thro' those Points the Lines IM, KN, LO, parallel to one another and to the Line BH or CF. *Plat.* XIX. *Fig.* 158.

This Preparation being thus made, substitute a for the Line EH or BH, b for the Line HF, and x for the Part HI, or IK, or KL, &c. then you will have HK$=2x$, HL$=3x$, and so on to the greatest HF, which we call'd b, and consequently EI$=a+x$, EK$=a+2x$, EL$=a+3x$ &c. and because by the Property of the Asymptotes the

Plat. XIX. Fig. 158. Rectangle of the Lines EH, BH, or the Square aa is equal to the Rectangle of the Lines EI, IM, and also to the Rectangle of the Lines EK, KN, and in like manner to the Rectangle of the Lines EL, LO, you will have,

$$IM = \frac{aa}{a+x} = a - x + \frac{1xx}{a} - \frac{x^3}{aa} + \frac{x^4}{a^3} - \frac{x^5}{a^4}, \text{ \&c.}$$

$$KN = \frac{aa}{a+2x} = a - 2x + \frac{4xx}{a} - \frac{8x^3}{aa} + \frac{16x^4}{a^3} - \frac{32x^5}{a^4} \text{ \&c.}$$

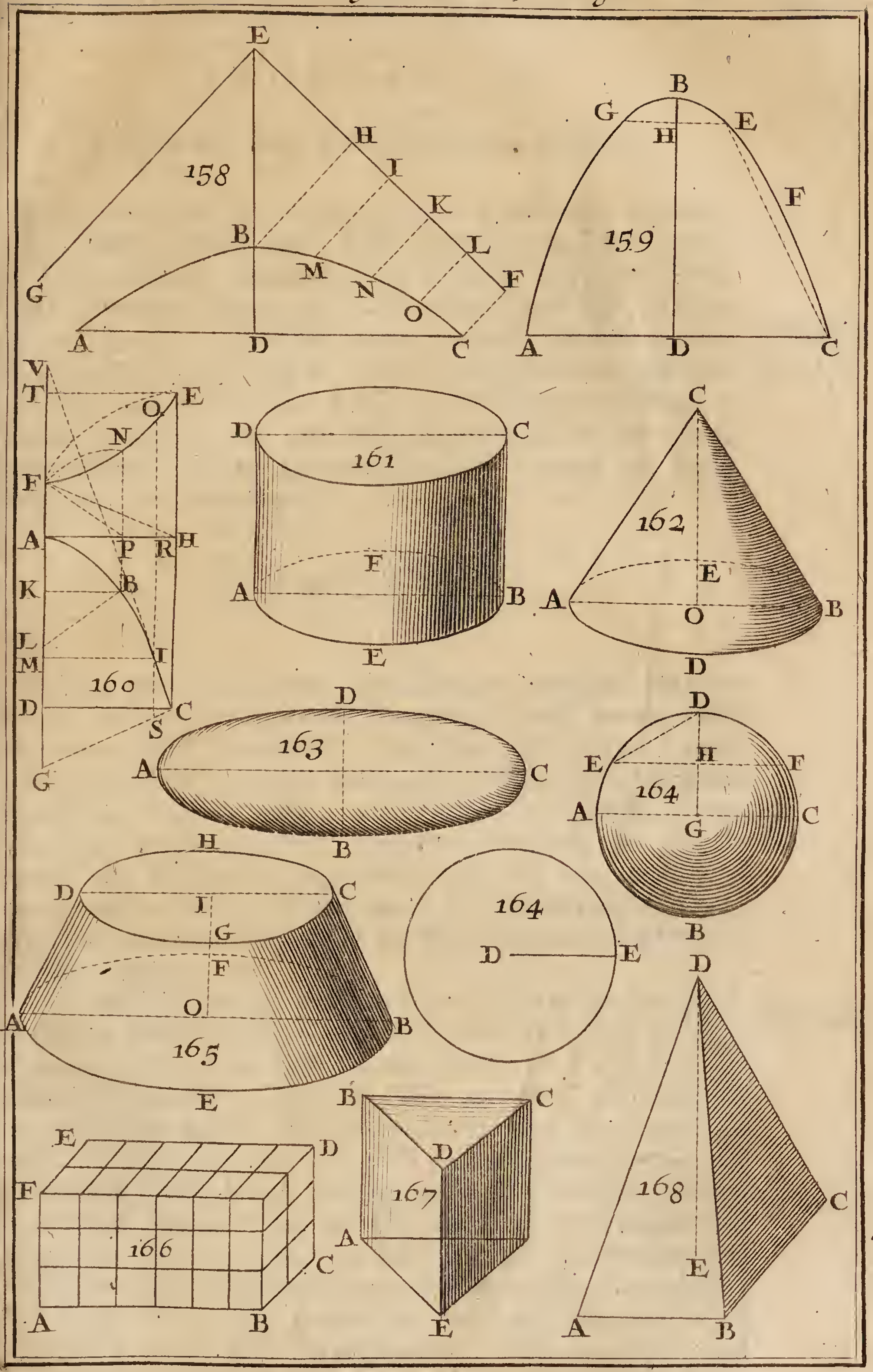
$$LO = \frac{aa}{a+3x} = a - 3x + \frac{9xx}{a} - \frac{27x^3}{aa} + \frac{81x^4}{a^3} - \frac{243x^5}{a^4} \text{ \&c.}$$

$$BCFH = ab - \tfrac{1}{2}bb + \frac{b^3}{3a} - \frac{b^4}{4aa} + \frac{b^5}{5a^3} - \frac{b^6}{6a^4}, \text{ \&c.}$$

Where you see that all a's are equivalent to ab, because the Letter b or the Line HF represents their Number: and by *Theor.* 10. that all the Terms x, $2x$, $3x$, &c. whose greatest is GK or b are equal to $\frac{1}{2}bb$; and by *Theor.* 11. that all the Squares xx, $4xx$, $9xx$, &c. whose greatest is bb, are equal to $\frac{1}{3}b^3$; Lastly, by *Theor.* 12. that all the Cubes in Infinitum x^3, $8x^3$, $27x^3$, &c. whose greatest is b^3, are equal to $\frac{1}{4}b^4$: In like manner all the Biquadrates in Infinitum x^4, $16x^4$, $81x^4$, &c. whose greatest is b^4, are equal to $\frac{1}{5}b^5$, and so on; and consequently the Sum of all the Parallels IM, KN, LO, &c. in Infinitum or Hyperbolic Space BCFH is equal to

$$ab - \frac{bb}{2} + \frac{b^3}{3a} - \frac{b^4}{4aa} + \frac{b^5}{5a^3} - \frac{b^6}{6a^4} + \frac{b^7}{7a^5}, \text{ \&c.}$$

If a be equal to 1, and b to $\frac{1}{10}$ you may conclude, that the Space BCFH is equal to $\frac{1}{10} - \frac{1}{200} + \frac{1}{3000} - \frac{1}{4000} + \frac{1}{50000} - \frac{1}{60000000}$, &c. Or, $\frac{571861}{6000000}$. Several Remarks might be made upon what has been said, but I shall only take Notice, that if the Space BCFH be known, the rest may easily be known.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PROBLEM XX.

To find the Area of the Quadratic Parabola.

TO find the the Area of the Quadratic Parabola ABC, whose Axe BD, suppose 124 Feet, bisects at right Angles, the Base AC, suppose 135 Feet: Multiply the values 124, 135, of the two Lines BD, AC together, for the Product 16740, whose double 33480, being divided by 3, gives 11160 Feet square for the Area of the Parabola propos'd ABC, as is evident by *Theor.* 9, which furnishes you also with the Method of measuring Parabola's of higher Degrees, so that I need not speak any more of them here. *Plat.XIX. Fig.* 159.

SCHOLIUM.

If you were to find the Area of the *Parabolic Segment* ECF, you must, besides the Parabola ABC, measure the Parabola GBE, bounded by the Line GE perpendicular to the Axe BD, and the Trapezoid DHEC; For subtract the Semi-parabola HBE, and Trapezoid DHEC, from the Semi-parabola DBC, and the remainder is the Area of the Parabolic Segment ECF. If you would find the length of the Circumference of the Parabola ABC, you may do it by the help of the Quadrature of the Hyperperbola, thus:

To find the Quantity of the Circumference of the Semi-parabola ABC, whose Axe is AD, and Base CD perpendicular to the Axe AD; draw thro' the Point C, the Line CE parallel to the Axe AD, and thro' the Vertex A, the right Line AH perpendicular to the same Axe AD. Take upon the Axe AD produced both ways the Lines AF, DG, equal each to half the *Parameter*, which is a third Proportional to AD and CD, and make HE equal to CG or HF. In like manner draw from the Point B taken at Discretion on the Parabolic Line ABC, the right Line BN parallel to the Axe AD, and after you have made BK perpendicular to the Axe AD, and KL equal to DG, or AF, that is to say, to half the Parameter, make PN equal to BL or PF. Lastly, describe thro' the Points F, N, E, and an infinity of others that may be drawn after the same manner, the Curve FNE and *Fig.* 160.

and that will be the Circumference of an Hyperbola, as

Plat. XIX. we shall afterwards demonstrate.

Fig. 160. This Preparation being made, let us imagine the Part CI, of the Parabolic Line ABC, to be infinitely small, and consequently a right Line, and that thro' the Point I, infinitely near the Point C, the Line IM passes parallel to the Base CD, and the Line SO Parallel to the Line CE or Axe AD; and then you may find *by* 4. 6. in the Triangles CIS, CDG, that are Similar, that the four Lines CI, CS, CG, DG, are proportional, and *by* 16. 6. that the Rectangle of the Extreams CI, DG, is equal to the Rectangle of the Means CS, CG, or HR, HE, that is to say, to the Figure RHEO, which may pass for a Rectangle, because the Line RH is suppos'd infinitely little. Whence 'tis easie to conclude, by the Method of Indivisibles, that the Hyperbolic Space AHEF, is equal to the Rectangle under the right Line CG, and Curve ABC. If therefore by *Probl.* 19. you measure the Area of the Hyperbolic Space AHEF, and divide by the right Line CG, you will have the length of the Parabolic Line ABC.

To demonstrate that the Curve FNE is an Hyperbolic Line, draw from the Point E, taken at discretion on that Curve, the right Line ET, perpendicular to the Axe AD produced, and then substituting a for AF, x for AT, or HE, or HF, and y for ET or AH, find in the right-angled Triangle AHF, this Equation $aa+yy=xx$, or $xx-aa=yy$, which is the Locus of an Equilateral Hyperbola, whose Center is A, &c. The two Triangles also that are right-angled, CIS, CDG, must be demonstrated to be Similar because 'tis not self-evident. To that end, take upon the Axe AD produced, the Line AV equal to the Part AD, and draw the right Line YC, touching the Parabolic Line ABC, in the Point C, by *Theor.* 6. Because by the nature of the Parabola the Square CD is equal to the Rectangle under AD, and Parameter, that is to say, the double of DG, or to the Rectangle under the single Line DG and the double of AD, that is to say the Line DV, you may find by 17. 6. that the three Lines DV, DC, DG, are proportional, and by 6. 6. that the two Right-angled Triangles CDV, CDG, are equiangular. Whence 'tis easie to conclude, that the Triangle GCV is right-angled in C, and by 8. 6. that the Triangle CDG as well as the Triangle CIS is similar to it, consequently the two Triangles CDG, CIS, are equiangular. *Which was to be Demonstrated.*

PRO-

PROBLEM XXI.

To measure the Convex Surface of a right Cylinder.

TO find the Area of the Convex Surface of the right Cylinder ABCD whose Height AD or BC, let be 24 Feet, and Circumference AEBF, of its Base 132 Feet, and their Product 3168 Feet square will give the Area of the Convex Surface of the Cylinder propos'd ABCD, whose Demonstration is evident by *Theor.* 20. *Plat.* XIX. *Fig.* 161.

SCHOLIUM.

If instead of knowing the Circumference of the Base AEBF, you knew its Diameter AB or CD, which seems easier, the Circumference AEBF may be found by *Probl.* 6. and then the Convex Surface of the Cylinder by what has been taught. But to avoid the Fractions that may happen in the Circumference, *Multiply the Diameter AB, by the Height AD, multiply the Product by* 314, *and divide this second Product by* 100, The Quotient will give the Surface sought, *by Theor.* 21. For Instance, if the Diameter AB be 12 Inches, and Height AB 9 Inches, the Product of these two Lines, 12, 9, will be 108, which multiplied by 314, and the Product 33912 divided by 100, gives 339 $\frac{17}{144}$ Inches Square for the Convex Surface of the Cylinder propos'd ABCD.

PROBLEM II.

To measure the Convex Surface of a Right Cone.

TO find the Area of the Convex Surface of the Right Cone ABC, whose Side AC or BC, is for Instance 42 Feet, and the Circumference of the Base ADBE 112 Feet; multiply the two Lines 42, 112 together, and half the Product 4704, will give 2352 Feet Square for the Convex Surface of the Cone propos'd ABC, *by Theor.* 22. *Fig.* 162.

SCHOLIUM.

If the Diameter AB be given, and not the Circumference of the Base ADBE, find the Circumference of the Base ADBE, *by Probl.* 6. and then the Surface of the Cone, by what has been taught here. But to avoid the Fractions that may happen in the Circumference, *mul-*

multiply the Diameter AB, by the Side AC, and then multiply that Product by 157, *and divide the second Product by* 100, the Quotient will give the Surface sought, by *Theor.* 23, Thus if the Diameter AB is 36 Feet, and Side AC 32. the Product of the two Lines 36, 32, will be 1152, which multiplied by 157, and its Product 180864 divided by 100, the Quotient will give 1808 Feet, and about 92 Inches Square for the Convex Surface of the Cone propos'd ABC.

PROBLEM XXIII.

To measure the Surface of the Frustum of a right Cone.

Plat. XIX. Fig. 165. TO find the Area of the Convex Surface of the Frustum of a right Cone ABCD, whose Side AD or BC is 21 Feet, and Circumference of its greatest Base AEBF 108 Feet, and the Circumference of its least Base DGCH 72 Feet. Add these two Circumferences 108, 72 together, and multiply their Sum 180 by the Side 12, and half the Product 2160 will give 1080 Feet square, for the Convex Surface of the Figure propos'd ABCD, by *Theor.* 24.

SCHOLIUM.

If instead of knowing the Circumferences AEBF, DGCH, you only know their Diameters AB, CD, the Circumferences AEBF, DGCH, may be found by *Probl.* 6. and then the Surface sought, as has been taught. But to avoid the Fractions that may happen in the Circumference, *Add together the two Diameters AB, CD, for their Sum, which multiply'd by* 157, *and the Product divided by* 100, *will give the Surface sought, by Theor.* 25. Thus if the greater Diameter AB be 24 Feet, and the less CD 18 Feet, their Sum 42 multiply'd by the Side AD, suppose 12 Feet, the Product 504 multiply'd by 157, gives 79128, which divided by 100 gives 791 Feet 40 Inches Square for the Convex Surface of the Frustum ABCD.

PROBLEM. XXIV.

To measure the Surface of a Sphere.

Fig. 166. TO find the Surface of a Sphere ABCD, whose Diameter AC is 18 Inches, multiply the Diameter 18 by its Circumference, which will be found by *Probl.* 6. to be about

56 Inches, and the Product 1077 Inches Square will give the Surface of the Sphere proposed ABCD, by *Theor.* 29.

Or which comes to the same, multiply by 4 the Area of a great Circle of the Sphere ABCD, which by *Prob.* 8. will be found to be 254 $\frac{49}{144}$ Inches square, and the Product 1017 $\frac{52}{144}$ Inches square gives the Area of the Surface propos'd ABCD. *Plat XIX. Fig.* 164.

Or, if you would avoid Fractions, multiply the Square 324 of the Diameter 18, by 314, and dividing the Product, 101734 by 100, the Quotient will give 1017 $\frac{49}{144}$ Inches square for the Surface sought, as is evident, by *Corol.* 3. *Theor.* 29.

SCHOLIUM.

Instead of using the Ratio of 100 to 314, make use of 100000 to 314159 in larger Calculations, if you would have the Surface sought with more exactness; Thus to find the Surface of the Earth, whose Diameter was found by *Probl.* 7. to be 2292 Leagues, multiply the Square 5253264 of the Diameter 2292 by 314159, and divide the Product 1650360164976 by 100000, the Quotient will give about 16503601 Leagues square for the Surface sought.

The Diameter of a Sphere may be found by taking it with a Spherical Compass, and applying it to a Line, divide into Feet and Inches, &c. But if you find any difficulty, apply a String round the Sphere, and that String extended will give the Circumference, by the means of which, the Diameter may be found by *Probl.* 7. and then the Surface by what has been taught: Or, find by *Probl.* 9. the Area of a great Circle of the Sphere, and its quadruple will be the Surface of the Sphere, by *Theor.* 28. Or, use this Rule, which has its Demonstration. *Multiply the Square of the Circumference by* 100, *and divide the Product by* 314. Thus if the Circumference be 56 Feet, its Square will be 3136, which multiply'd by 100, and the Product 313600 divided by 314, gives 998 Feet, and 104 Inches square, for the Surface of a Sphere whose Circumference is 56 Feet.

PROBLEM XXV.

To measure the Surface of a Segment of a Sphere.

TO find the Area of the Surface of the Segment, or Portion of a Sphere EFD, whose Arc, for instance,

EDE

EDF is 100 Degrees, its Diameter AC being known, suppose 18 Inches, in which case the Circumference of a great Circle ABCD, will be found by *Probl.* 6. to be $56\frac{1}{2}$ Inches, multiply this Circumference by the Versed Sine DH; which you will find to be about $3\frac{1}{4}$ Inches, by this Proportion.

Plat. XIX. Fig. 164.

As the Radius	1000000
To the versed Sine of half the Arc EDF	35721
So is the Semi-diameter AG	9
To the vers'd Sine DG	3. 3.

And the Product will give $185\frac{20}{144}$ Inches square for the Area of the Segment propos'd EFD. by *Corol.* 5. *Theorem* 18.

Or find the Chord DE of half the Arc EDF, by this Proportion.

As the Radius	100000
To the Sine of a Quarter of the Arc EDF	42262
So is the Diameter AC	18
To the Chord DE	7. $7\frac{1}{2}$

Which will be found to be $7\frac{7}{12}$ Inches, and that consider'd as a Radius of a Circle has for its Area, to be found by *Probl.* 8. $182\frac{92}{144}$ Inches square, which by *Theor.* 39. is equal to the Surface of the Portion of the Sphere EFD.

SCHOLIUM.

To avoid Fractions, which neglected will cause the Area found not to be exact, *Multiply the Square* 324, *of the Diameter* 18 *by the Versed Sine* 35721 *of half the Arc EDF, and multiply the Product* 11573604, *by* 157, *for a second Product* 1817055328, *which divided by* 100 *times the Radius, that is to say, by* 10000000, *gives* $181\frac{102}{144}$ *Inches square for a pretty exact Area of the Surface propos'd, EFD.*

This Rule is very fit for finding the Area of the Surface of that Part of the Earth, which is call'd the Frigid Zone, terminated by one of the Polar Circles, and whose Arc is 47 Degrees, being double the greatest Declination of the Sun, generally suppos'd to be 23 Degrees and 30 Minutes, because the Diameter of the Earth is known, namely, 2292 Leagues.

SCHO-

PROBLEM XXVI.

To measure the Surface of a Zone.

'TIS plain, that to find the Surface of the Zone AEFC, *Fig.* 164. bounded by the two Circumferences AC, EF, whose distances DADE, or DC, DF, from their common Pole D, are known; you have no more to do but to find by *Probl.* 25. the Surfaces of the two Portions of the Sphere ACD, EFD, and subtract the less from the greater; But 'tis more easily done by multiplying the Circumference of a great Circle of the Sphere by the Part GH of the common Axe GD, for then you will have the Surface of the Zone propos'd AEFC, by *Theor.* 31. This Part GH being equal to the difference of the Versed Sines of the two Distances DA, DE, may be easily found by this Proportion,

As the Radius
To the difference of the Versed Sines of the Distances, DA, DE,
So is the Semi-diameter of the Sphere,
To the Part GH.

Thus if the distance DE be 24 Degrees, and the distance DA 36, and Diameter of the Sphere ABCD, 18 Inches, the versed Sines of the Arcs DE, DA, will be 8646, 19099, and their Difference will be 10453, by the help of which and the Diameter of the Sphere, which which we suppos'd to be 18 Inches, the Part GH will be found by the foregoing Analogy to be 11 Twelfths of an Inch, by which multiplying the Circumference of a great Circle of the Sphere ABCD, which by *Probl.* 6. will be found to be 56 Inches and 6 Twelfths of an Inch, and you will have about 51 $\frac{114}{144}$ Inches square for the Area sought.

PROBLEM XXVII.

To measure the Surface of a Spheroid.

Some of the Modern Geometers have given us Methods, which they take to be Geometrical, for measuring the Surface of a Spheroid; but since these Methods are too speculative, and tedious in execution, I think it better to

to shew you the Method that is common among Surveyors, for finding the Area of the Surface of a Spheroid, though not perfectly exact, as I have remark'd in *Theor.* 31. and serves to measure Vaults and Oval Arches.

To measure therefore the Area of the Surface of the Spheroid ABCD, whose greater Axe AC, let be *the Axe of Circumvolution*, that is to say, that about which an Ellipse is made to revolve for generating the Spheroid. Suppose the greater Axe AC to be 45 Feet, and the lesser BD of 35, the Surface of a Sphere that has its less Axe 35 for the Diameter, will be found by *Probl.* 24. to be 3846 Feet, 72 Inches Square: And as this Surface is to that of the Spheroid, as the less Axe 35, to the greater 45, nearly, multiply the Surface of the Sphere thus found, that is to say, 3846 Feet, and 72 Inches Square by the number 45 of the great Axe, and divide the Product 173078 Feet, and 72 Inches Square, by the number 35 of the Diameter of the less Axe, you will find 4945 Feet, and 51 Inches Square for the Surface of the Spheroid propos'd ABCD.

Plat. XIX. *Fig.* 163.

SCHOLIUM.

Because I said this Problem is very useful for measuring Oval Arches, I shall add an easie Rule for finding the Concave Surface of an Oval Arch supposing that Surface is just half a Spheroid.

Multiply the two Axes AC, BD together, multiply that Product by 157, *and divide this Second Product by* 100, *and you will have the Surface sought.*

Thus here having suppos'd the greater Axe AC to be 45 Feet, and the lesser BD, 35, their Product 1575, multiply'd by 157, and the second Product 247275 divided by 100, gives 2472 Feet, 108 Inches square, for the Surface propos'd.

PROBLEM XXVIII.

To measure a Space bounded by a Spiral Line.

ARchimedes has left us a particular Treatise about the Spiral Line, which we shall not speak of here, for fear of swelling the Book to no purpose, because common Sur-

Surveyors, know nothing of it, much leſs how to deſcribe it. I ſhall therefore content my ſelf with ſhowing how to meaſure the Space bounded by the firſt Spiral, by the help of which, it will be eaſie to judge of the Content of the others, becauſe by *Prop.* 27. *Archim. of Spirals*, the Space of a Second Spiral contains ſix times that of the firſt, being double the Circle of the firſt Revolution, which is triple the firſt Spiral: the Space of a third Spiral contains twelve times that of the firſt, being double the ſecond, and quadruple the Circle of the firſt Revolution, or equal to the Circle of the ſecond Revolution: The ſpace of a fourth Spiral contain eighteen times that of the firſt, being triple that of the ſecond and ſextuple the Circle of the firſt Revolution, and ſo on. *Pla.* XX. *Fig.* 169.

Thus if the Line AD, anſwering to the firſt Spiral ABCD, and for that reaſon call'd a *Line of the firſt Revolution* be 12 Feet, in which caſe the Diameter of the Circle DIKL, call'd the *Circle of the firſt Revolution* will be 24 Feet; The Area of the Circle of the firſt Revolution will be found by *Probl.* 8. to be about 452 Feet ſquare, a third of which, 151 Feet Square, gives the Area of the firſt Spiral Plane ABCD, whoſe Sextuple 904 Feet ſquare gives the Content of the ſecond Spiral Space ABCDEFGH, &c.

N PART

PART IV.

OF

STEREOMETRY.

STEREOMETRY, teaches how to measure Solids or Bodies. And as in Planimetry Planes were measured by lesser Planes, so in Stereometry Solids are measured by lesser ones, generally Cubes, sometimes Rectangled Parallelopipeds whose Length exceeds their Breadth; But the right Angle is always retain'd, because 'tis always invariable, certain, and the only one in its kind, and besides most commodious and proper.

The Number of the Cubes contain'd in a Solid is found by Multiplication, because 'tis conceiv'd equal to a Rectangled Parallelopiped, whose Solidity is found by multiplying two of its Dimensions together for the Area of one of its Faces, which may be look'd upon as the Base of the Solid, and multiplying that Base by the Height, that is to say, by its third Dimension.

Pl. XIX. Fig. 166. Thus in the Right-angled Parallelopiped ABCDEF, you multiply its Length AB, suppose 6 Fathoms, by its Breadth BC 2 Fathoms, you will have 12 Square Fathoms for its Base ABC, which multiply'd by the Height CD 3 Fathoms, gives 36 Cubic Fathoms for the Solidity of this Parallelopiped, which are produc'd by the intersection of certain Planes drawn through the divisions of the opposite Sides.

Whence it follows, that since a Fathom in Length contains 6 Feet in Length, a Cubic Fathom must contain 216 Cubic Feet. For multiplying 6 by 6 you have 36 Feet Square, for a Square Fathom, which may be taken for the Base of a Cubic Fathom, and multiplying that Base 36 again by 6, the Height of a Cubic Fathom, you will have 216 Cubic Feet for the Value of a Cubic Fathom.

You may find by the same method, that a Foot long containing 12 Inches long, a Cubic Foot contains 1728 Cubic Inches, &c.

There-

Therefore when Cubic Fathoms are to be reduc'd into Cubic Feet, inſtead of multiplying them by 6, you muſt multiply them by 216. Thus if the Solidity of the Parallelopiped ABCDEF be 36 Cubic Fathoms, and you would know how many Cubic Feet they contain, multiply them by 216, and the Product gives 7776 Cubic Feet for the Solidity of the Parallelopiped ABCDEF.

On the contrary, when Cubic Feet are to be reduced into Cubic Fathoms, inſtead of dividing them by 6, divide by 216. And ſo by the reſt.

I ſaid Solids are ſometimes meaſured by right angled Parallelopipeds, whoſe Length exceeds their Breadth, this is done principally for the eaſe of Calculation, when different kinds are to be multiply'd: And then *a Foot of a Cubic Fathom* is a Right-angled Parallelopiped, containing 36 Cubic Feet, as a Fathom Square contains 36 Feet Square, this ſolid Rectangle is ſubſtituted in Practice inſtead of a Cubic Fathom, which is 216 Cubic Feet, and is call'd *a Foot of a Cubic Fathom*, becauſe it has a Foot for its Height, and a Square Fathom for its Baſe.

In like manner an *Inch of a Cubic Foot*, is a Right-angled Solid, that contains 144 Cubic Inches, as a Foot Square contains 144 ſquare Inches, this Solid being ſubſtituted in Practiſe, inſtead of a Cubic Inch containing 1728 Cubic Inches, and 'tis call'd an *Inch of a Cubic Foot*, becauſe it has an Inch for its Height, and a Foot Square for its Baſe.

Thus alſo a Right-angled Solid whoſe Height is an Inch, and Baſe a ſquare Fathom equal to 5184 Inches ſquare, is call'd an *Inch of a Cubic Fathom*, becauſe a Fathom long contains 72 Inches long, whoſe Solidity is conſequently 5184 Cubic Inches.

Hence in meaſuring 'tis uſually ſaid, that ſquare Fathoms multiply'd by long Fathoms, produce *Cubic Fathoms*: and that ſquare Feet multiply'd by long Feet produce Cubic Feet: And ſo of the reſt. But ſquare Fathoms multiply'd by long Feet produce *Feet of a Cubic Fathom*: and ſo Feet ſquare multiply'd by Inches long produce *Inches of a Cubic Foot*.

CHAPTER. I.

WE shall do here, as we did in Trigonometry and Planimetry, place the Theory first, and the Practical Part next, disengag'd from it, that it may be more easily taken up, and being founded on the Theorems that go before it, may please with its Evidence, both such as are more and such as are less expert.

THEOREM I.

The Solidity of a Sphere is the third of that of a Prism, having for its Base a Plane equal to the Surface of the Sphere, and its Height equal to the Radius of the same Sphere.

Plat. XX. *Fig.* 170. I Say the Sphere ABCD is equal to the third part of a Prism, for Instance of the Cylinder AEFG, whose Height AE or FG is equal to the Radius AE or EC of the Sphere, and Base AHGI is a Circle equal to the Surface of the Sphere, whose Diameter AG is by consequence double the Diameter AC of the same Sphere.

DEMONSTRATION.

Imagine the Surface of the Sphere divided into an infinite number of equal parts, and they may be taken for equal Planes, and Bases of as many Cones, whose Vertexes meet in the Center E of the Sphere, and common Height is consequently equal to the Radius EK or EL of the same Sphere.

Whence 'tis easy to conclude, that as *by* 10. 12. one of these Cones is equal to a third of a Prism of the same Base and Height, the Sphere ABCD also is equal to a third Part of a Prism, whose Base is the sum of all the Bases of the Cones, that is to say, to the Surface of the Sphere, and Height is the Radius of the same Sphere. *Which was to be demonstrated.*

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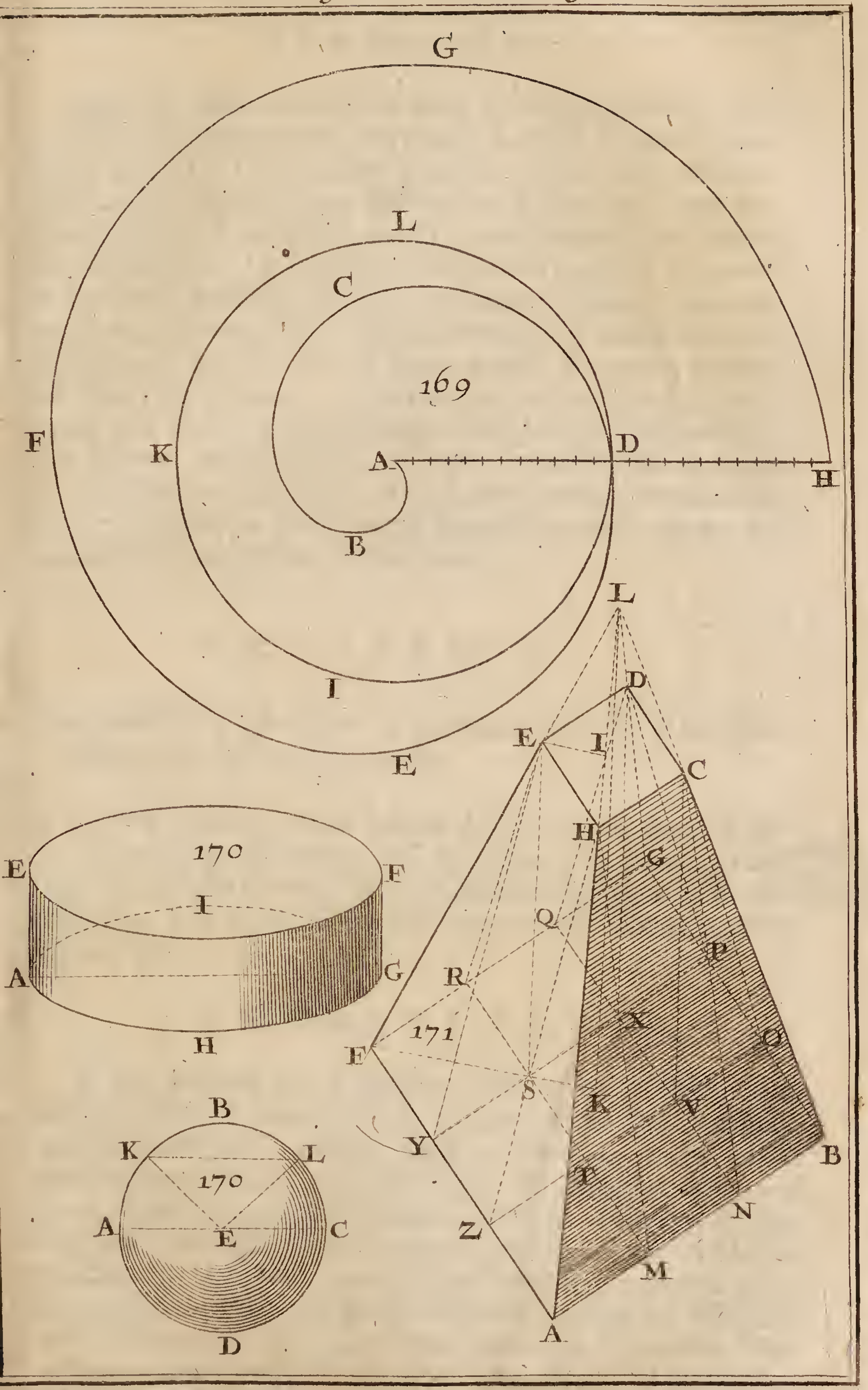
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SCHOLIUM.

After the ſame manner it may he demonſtrated, that a Sector of a Sphere, for inſtance, KELB is equal to the third part of a Priſm, that has for its Baſe the Surface KBL, that ſerves for a Baſe to the Priſm too, and the Radius of the Sphere EK or EL, the Side of the Sector, for its Height: And ſeeing a Priſm is triple a Pyramid of the ſame Baſe and Height, it follows, that a Sphere is equal to a Cone, whoſe Height is equal to the Radius of the Sphere, ane Baſe is a Circle, whoſe Diameter is double that of a Sphere: And that a Sector of a Sphere is equal to a Cone, whoſe Height is equal to the Radius of the Sphere, and the Baſe a Circle, whoſe Radius is equal to the Chord of half the Arc of the Sector, becauſe the Circle is equal to the Spheric Superficies that ſerves for the Baſe of the Sector of the Sphere.

THEOREM. II.

The Solidity of the Sphere is to that of the Cube of its Diameter, as 157 *to* 300 *nearly.*

I Say the Solidity of the Sphere ABCD is to that of the Cube of its Diameter AC as 157 to 300 nearly, that is to ſay, if the Cube of the Diameter AC contain'd, for inſtance, 300 Cubic Feet, the Sphere ABCD will contain about 157. *Plat.* XX. *Fig.* 170.

DEMONSTRATION.

If you ſuppoſe the Diameter AC to be 30 Feet, the Surface of the Sphere ABCD, or Area of the Baſe AHGI, will be found by *Probl.* 2. *Chap.* 2. *Part* 3. to be 2826 Feet ſquare, which multiply'd by the Height AE, 15 Feet, gives 42390 Cubic Feet, for the Solidity of the Cylinder AEFG, which being triple the Sphere ABCD, by *Theor.* I. the third part of this Product 42390, will give 14130 Cubic Feet, for the Solidity of the Sphere ABCD, whoſe Diameter AC is 30 Feet, and the Cube of this Diameter conſequently is 27000 Cubic Feet. The Sphere therefore ABCD is found to be to the Cube of its Diameter AC, as 14130 is to 27000 or dividing each Term by 90, as 157 is 300. *Which was to be demonſtrated.*

THEOREM III.

If a Plane cut a Sphere into two unequal Parts, one of them will be equal to a Cone, whose Base is the same with that of the Portion of the Sphere, and Height is compounded of that of the same Portion, and a Line that is a fourth proportional to three others, the Height of the other Portion, its own Height, and the Radius of the Sphere.

Pl. XXI. Fig. 172. I Say, if the Plane AECF cut the Sphere ABCD, whose Center is G, and Diameter BD, perpendicular to the cutting Plane AECF, and consequently to the Line AC, in the Plane, into two unequal Parts, AECB, AFCD, any one of those Portions, for Instance AECB is equal to the Cone, AECH, whose Base AECF, is the same with that of the Segment AECB, and Height HI compos'd of the Height BI, of the same Segment AECB, and of the Line BH, a fourth Proportional to these three, DI, BI, BG, or AG. Draw AB, AD, CG.

DEMONSTRATION.

Because, *by Construction*, these Four, BI, DI, BH, BG, are proportional, *by Composition*, BD, DI, GH, BG, will be proportional also; Instead of BD, DI, therefore you may substitute, when you please, GH, BG, that are in the same Ratio: And because the Angle BAD is right *by* 31. 3. and the Line AI perpendicular to the Line BD, the three Right-angled Triangles, DAI, DAB, BAI, *by* 8. 6. are equiangular, and *by* 4. 6. the three Lines BD, AD, DI, are proportional: Wherefore *by* 20. 6. the first BD will be to the third DI, as the Square of the first BD to the Square of the second AD, or as the Square AB to the Square AI, because BD, AD, AB, AI, are proportional, *by* 4. 6. and if you substitute GH, AG, instead of the two first Terms BD, DI, being in the same Ratio, you will find the Line GH is to the Line AG, as the Square AB is to the Square AI, or *by* 2. 12. as the Circle whose Radius is AB, is to the Circle whose Radius is AI, that is to say, to the Circle AECF. Consider therefore the Circle AB as the Base of a Cone whose Height is AG, and the Circle AEFC, as the Base of a Cone whose Height is GH, and you will find *by* 15. 11. that these two Cones having their Bases and Heights reciprocal, are equal, that is to say, the Cone whose Base is the Circle AECF,

AECF, and Height GH, is equal to the Cone, whose Base is the Circle AB, and Height AG, that is to say, by Theor. 1. to the Sector AGCB: And since the Cone whose Base is the Circle AECF, and Height GH, is equal to the Sum HAGC, of the two Cones AECH, AFCH, because their Heights HI, GI, are equal to the Height GH, when taken together, and the Base AECF is common to the three Cones; it follows, that the whole Solid HAGC is equal to the Sector AGCF, wherefore take away from each of the two equal Solids, the common Cone AFCG, and there will remain the Cone AECH, equal to the Segment AECB. *Which was to be Demonstrated.* *Pla. XXI. Fig. 172.*

THEOREM IV.

A Spheroid is to a Sphere, whose Diameter is equal to the Axe of Circumvolution, as the Square of the other Axe, to the Square of the same Axe of Circumvolution.

I Say the Spheroid ABCD, whose Axe of Circumvolution is AC, is to the circumscribed Sphere AFCG, whose Diameter is equal to the same Axe of Circumvolution AC, as the Square of the other Axe BD, is to the Square of the Axe of Circumvolution AC. *Fig. 173.*

PREPARATION.

Describe upon the Centre E of the Spheroid, in the Plane of the Ellipse ABCD, that generates the Spheroid by its Revolution about the Axe of Circumvolution AC, the Circle AFCG; and draw any Line HIK, in the Plane of that Circle, perpendicular to the Axe of Circumvolution AC.

DEMONSTRATION.

While the Semi-Ellipse revolving about the Axe AC generates the Spheroid ABCD, and the Semi-circle AFC, the Sphere AFCG, the Perpendiculars KH, KI, and an infinite number of others, that may be imagined, produce Circles, whose Centers are in the Axe of Revolution AC: So that the Spheroid ABCD, and Sphere AFCG, may be considered as the Sums of as many Infinite Circles, as the other, and as all the Infinite Circles are *by* 2. 12. as the Squares of their Radij KH, KI, proportional to EF, EB, and consequently to the two

Axes AC, BD, by *Theor.* 16. *Chap.* 1. *Part.* 3. it follows that all the Circles of the Spheroid ABCD, or the Spheroid ABCD, is to the Sum of all the Circles of the Sphere ACFG, or to the Sphere ACFG, as the Square of the Axe BD, to the Square of the Axe AC, *which was to be demonſtrated.*

Plat. XXI. *Fig.* 173.

THEOREM V.

A Spheroid is to the Solid under the Axe of Circumvolution, and Square of the other Axe, as 157 *to* 300 *nearly.*

Fig. 173. I ſay the Spheroid ABCD, whoſe Axe of Circumvolution is AC, is to a Parellelopiped, whoſe Height is equal to the Axe of Circumvolution AC, and Baſe the Square of the other Axe BD, as 157 to 300 nearly.

DEMONSTRATION.

Becauſe by *Theor.* 4. the Spheroid ABCD is to the Sphere AFCG, as the ſquare BD to the ſquare AC; make AC, the Axe of Revolution, the common Height of the two ſquares BD, AC conſider'd as Baſes, and you will find *by* 32.11. the Spheroid ABCD is to the Sphere ACFG, as the Solid contain'd under the Axe of Circumvolution AC, and ſquare of the other Axe BD, is to the Cube of the Axe of Circumvolution AC, and *by Permutation*, the Spheroid ABCD, is to the Solid under the Axe of Circumvolution AC, and the ſquare of the other Axe BD, as the Sphere ACFG, to the Cube AC, or *by Theor* 2. as 157 to 300. *which was to be demonſtrated.*

THEOREM VI.

A Segment of a Spheroid, whoſe Height is a part of the Axe of Circumvolution is to a Segment of the correſponding Sphere, as the Cone inſcribed in the Segment of the Spheroid is to the Cone inſcribed in the Segment of the Sphere.

Fig. 173. I ſay the Segment of the Spheroid LMA, whoſe Axe of Circumvolution is AC, is to the Segment of the correſponding Sphere NRA, as the Cone LOMA, is to the Cone NPRA, the Height AS being common to all the four Solids.

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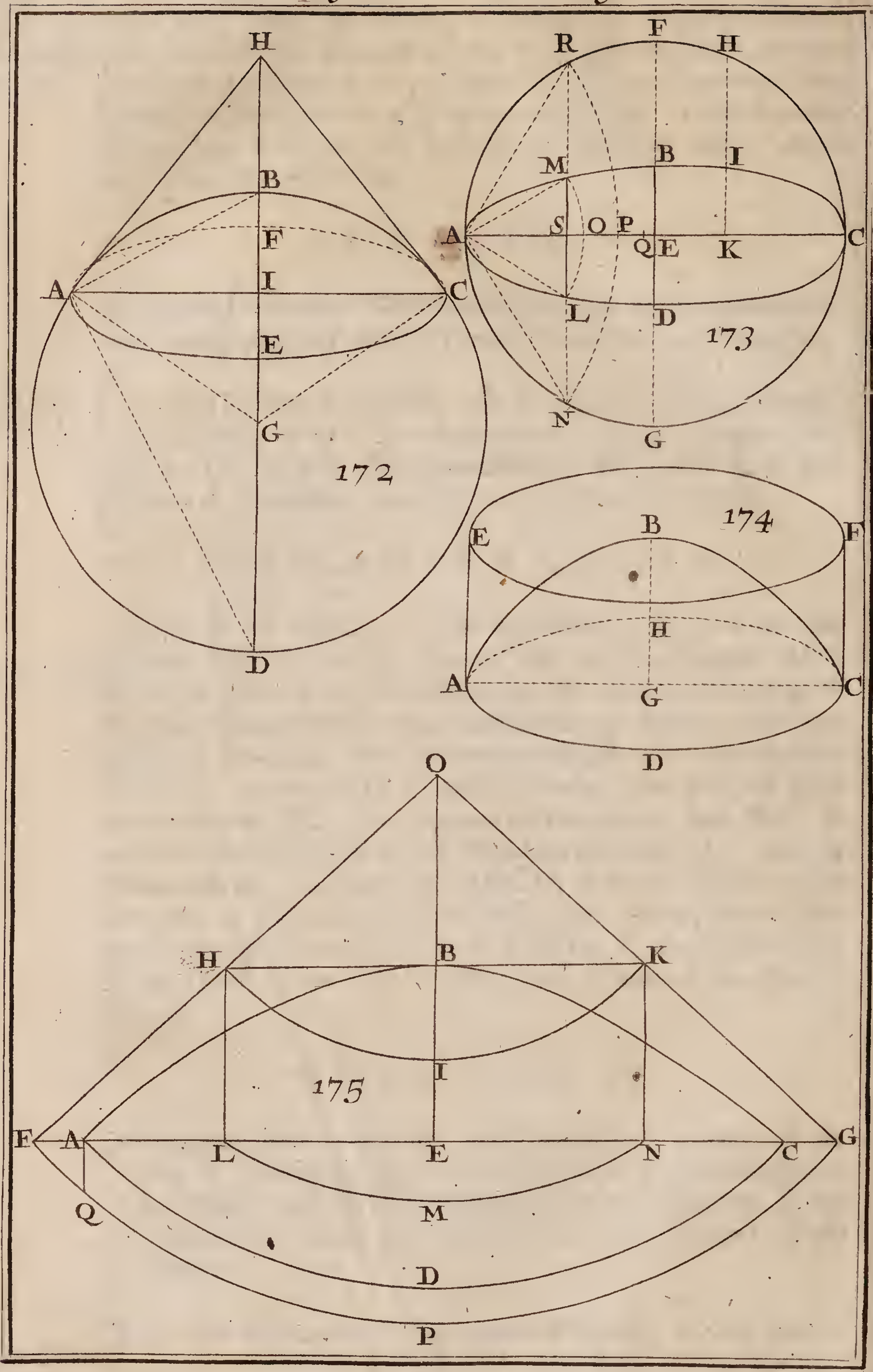
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DEMONSTRATION.

For we have demonſtrated in *Theor.* 4. that the Portion of the Spheroid LMA, is to the correſponding Portion of the Sphere NRA, as the ſquare of the Axe BD, is to the ſquare of the Axe of Circumvolution AC, or as the ſquare MS to the ſquare RS, or *by* 2. 12. as the Circle LOM, to the Circle NPR: Wherefore making AS, the common Height of theſe two Circles conſider'd as Baſes of the two Cones, you will find *by* 11. 12. the Segment LMA, is to the Segment NRA, as the Cone LOMA, to the Cone NPRA. *Which was to be demonſtrated.* *Plat. XXI. Fig.* 173.

COROLLARY.

It follows from this Propoſition, that the Segment of the Spheroid LMA is equal to a Cone, whoſe Baſe is the ſame with that of the Segment, namely the Circle LOM, and its Height is equal to the Line AQ made up of the Height AS of the Segment, and the Line SQ a fourth proportional to three others, the Height CS of the oppoſite Segment, the Height AS of the Segment, and the Semi-axe of Circumvolution AE, as in the Sphere, becauſe the Segment of the Sphere NRA is *by Theor.* 3. equal to a Cone, whoſe Height is the ſame Line AQ, and ſame Baſe with the Segment, namely the Circle NPR. Thus ſubſtituting the Cone in the place of the Segment NRA, you may make it evident by this

DEMONSTRATION.

Becauſe the Segment LMA is to the Segment NRA, as the Cone LOMA, is to the Cone NPRA, *by permutation*, the Segment LMA will be to the inſcribed Cone LOMA, as the Segment NRA is to its inſcribed Cone NPRA, ſubſtituting the preceding Cone whoſe Height is the compound Line AQ, and Baſe the ſame with the Cone's NPRA, namely the Circle NPR, inſtead of the Segment NRA, and inſtead of the two laſt Cones of equal Baſes, their common Heights AQ, AS, that are in the ſame Ratio, *by* 14. 12. you will find the Segment LMA, is to its inſcribed Cone LOMA, as the Height AQ, to the Height AS, and laſtly make the Circle LOM a Baſe common to thoſe two Lines conſider'd as the Heights of the two Cones, and you will find *by* 14. 12. the Segment LMA is to the Cone LOMA, as the Cone whoſe Height is

Pla. XXI. *Fig.* 173. is AQ, and Base is the Circle LOM, to the Cone LOMA, and consequently the Spheroid LMA is equal to a Cone, whose Height is that compound Line AQ, and Base the Circle LOM, namely the same with that of the Segment. *Which was to be demonstrated.*

THEOREM VII.

A Parabolic Conoid is equal to half a Cylinder of the same Base and Height.

Plat. XXI. *Fig.* 174. I say the Parabolic Conoid, or the Paraboloid ABCD, generated by the Revolution of the common Parabola ABC, about its Axe BG, whose Height is the Line BG, and Base the Circle ADCH, is equal to half the Cylinder AEFCD, whose Base is the same Circle ADCH, and Height the same Line BG.

DEMONSTRATION.

If you imagine right Lines drawn perpendicular to the Axe BG, or parallel to the Base AC of the Parabola ABC, thro' all the Points of the Height or Axe BG, which I suppose to be divided into an infinite number of equal Parts, all these infinite Parallels, being Ordinates to the Axe BG, will describe, by the Revolution of the Parabola ABC, about its Axe BG, as many parallel Circles, whose Centers will be in the Axe BG, and Diameters will be Parallels or Ordinates: And since the Squares of these Ordinates or Diameters of the Circles are by the nature of the common Parabola as the corresponding parts of the Axe BG, and consequently in a continued Arithmetic Proportion, the Circles that compose the Paraboloid, will also be in an Arithmetic Progression. Thus considering the Paraboloid ABCD, as the sum of an infinite number of Circles in continued Arithmetic Proportion, whose greatest is the Base ADCH, you may find *by Theor.* 11. *Chap.* 1. *Part* 3. that the sum of all the infinite Circles, or the Paraboloid ABCD, is equal to half the greatest ADCH multiplied by the Line BG, expressing their number, that is to say, to half the Cylinder AEFCD. *Which was to be demonstrated.*

THE-

THEOREM VIII.

An Hyperbolic Conoid generated by the Revolution of an Hyperbola about its Axe, is equal to the excess of the Frustum of a Cone, having for its Base that of the Asymptotic Cone, and for its Height that of the Conoid, above the Cylinder inscribed in the Asymptotic Cone, and of the same height with the Conoid.

I say that if the Hyperbola ABC, and Isosceles Triangle FOG, revolving about the Axe BE, of the Hyperbola ABC, whose Center is O, and Asymptotes OF, OG, and Base AC perpendicular to the Axe BE, generate the Hyperbolic Conoid ABCD, and Asymptotic Cone OFPG by that Revolution; the Conoid ABCD is equal to the excess of the Frustum of the Cone HFPGK, whose Height BE is the same with that of the Conoid ABCD, and whose Base FPG is the same with that of the Asymptotic Cone OFPG, above the Cylinder HLMNK of the same Height with the Conoid ABCD. *Plat. XXI. Fig.* 175.

PREPARATION.

Let fall from the Point A in the Plane of the Base FPG, a perpendicular AQ, to the Diameter FG, which being a Tangent to the Circle ADC in A, *by* 32. 3. will *by Prol.* 17. *Chap.* 2. *Part* 3. be the Radius of a Circle equal to the Ring FADCGP bounded by the Circumferences of the two Circles FPG, ADC, whose common Center is E, which is also the Center of the Circle LMN, serving for the Base of the Cylinder HLMNK. Draw in the Circle HIK, the Diameter HK, parallel to the Diameter FG, and it will be a Tangent to the Hyperbola in the Vertex B.

DEMONSTRATION.

Because the Square AQ is *by* 36. 3. equal to the Rectangle of the Lines AF, AG, that is to say by the nature of the Asymptotes, to the square of the Tangent BH, those two Lines AQ, BH will be equal, and their Circles also, that is to say the Circle HIK, or LMN be equal to the Circle whose Radius is AQ, or the Ring FADCGP: Wherefore the Bases and Heights being equal, the Cylinder HLMNK will be equal to the solid space BKGPFH, which encompasses the Conoid ABCD; and since taking away that solid Space from the Frustrum of the Cone

HFPGK,

HFPGK, you will have the Conoid ABCD for the excess, after the same manner taking away the Cylinder HLMNK equal to that solid Space BKGPFH of the same Frustrum of a Cone HFPGK, you will have the Hyperbolic Conoid ABCD for the excess. *Which was to be demonstrated.*

CHAPTER II.

THIS Chapter is a Corollary to the former, whose Theorems furnish us with short and easie Methods of measuring all sorts of Solids, as you will see in the following Problems.

PROBLEM I.

To find the Solidity of a Prism.

THE solidity of a right or oblique Prism is found by multiplying one of its two parallel and opposite Bases, by its Height which is a Line perpendicular to those Bases, which may be measur'd by the Principles of Planimetry, managing them as Triangles, if they be Triangular: As a Polygon, if Polygonal: As a Circle, if Circular; and lastly, as an Ellipse, if Elliptical, as are the Bases of the Elliptic Basons of several Fountains.

You have already seen the method of finding the solidity of a Right-angled Parallelopiped, and consequently of a Cube, in the Preface to this Fourth Part, so that there is no need of saying any more here: We shall give here first an Example of a Triangular Prism, as ABCDE, whose Height AB we will suppose to be 24 Feet, the side
Pl. XIX. BD of its Base BCD 10 Feet, the other side CD 17, and
Fig. 167. the third side BC 21; and then the Area of this Base or Triangle BCD will be found to be 84 Feet square, which multiplied by the Height 24, gives 2016 Cubic Feet for the solidity of the Prism propos'd ABCDE.

If you would find the Solidity of the Hexagonal Prism
Pl. XXII. ABCDE, whose Height ED is for instance 24 Feet, and
Fig. 176. side AB of its Base ABCD, that I suppose regular, 8 Feet; multiply the Base, which you will find to be 166 Feet square by the Height 24, and you will have 3984 Cubic Feet for the Solidity sought.

If you would find the Content of the Cylinder AEBCD, whose Height AD or BC is, for Example, 24 Feet, and Diameter AB of its Base AEBF 25 Feet; multiply the Base AEBF, which you find to be 490 Feet, and about 90 Inches square, by the Height AD or BC, supposed to be 24 Feet, and you will have 11775 Cubic Feet for the Content of the Cylinder proposed AEBCD. *Plat XIX. Fig. 161.*

Or to avoid Fractions happening in the Base, *multiply the Square* 625 *of the Diameter* 25 *by the Height of the Cylinder* 24, *and multiply the Product* 15000 *by* 785, *for a second Product* 11775000, *which divided by a* 1000, *will give as above,* 11775 *Cubic Feet for the Solidity sought.*

If you were to find the Content of the Elliptic Bason ABCD, whose Length AC is, for instance 124 Feet, Breadth BD 100, Depth AE 6, multiply the Area of the Base, which you will find to be 9734 Feet square, by the Depth, supposed to be 6 Feet, and you will have 58404 Cubic Feet of Water for the Content sought. *Pla. XXII. Fig. 177.*

Or to avoid Fractions in the Base, *multiply the Length* 124, *the Breadth* 100, *and the Depth* 6, *together for their solid Content* 74400, *which multiplied by* 785, *and the Product* 58404000 *divided by* 1000, will give in the Quotient as before, 58404 Cubic Feet for the Capacity sought.

SCHOLIUM.

I said in Planimetry, that when different kinds of Quantities were to be multiplied together, they were first to be reduced into their lowest Species, and the solid Content of the Body proposed to be found in them, and then by Division they may be reduced into their highest Denomination; as you will see in the following Examples.

To find the Solidity of the Rectangled Parallelopiped, or Wall ABCD, whose Length is CD 8 Fathoms, 4 Feet, or 52 Feet; Height BC 2 Fathoms 3 Feet, or 15 Feet; Breadth AB 2 Feet; multiply the 2 Feet of Breadth AB by the Height 15 Feet, and the Product 30 by the Length CD 52 Feet; and this second Product will give 1560 Cubic Feet, which divided by 216, because a Cubic Fathom contains 216 Cubic Feet, gives 7 Cubic Fathoms and 48 Cubic Feet for the Solidity of the Wall proposed ABCD. *Fig. 178.*

If the Length CD were 6 Fathoms, 3 Feet, 4 Inches, or 472 Inches, the Height BC 2 Fathoms, 4 Feet, 9 Inches, or 201 Inches, the Breadth AB, 2 Feet, 8 Inches,

Pla. XXII. Fig 178. or 32 Inches, multiply these three Lines together 472, 201, 32, and then divide the Product 3035904 Inches Cube by 1728, because a Cubic Foot contains 1728 Cubic Inches, and the Quotient gives 1756 Feet, and 1536 Inches Cube, for the Solidity of the Wall proposed ABCD. And if you divide the Cubic Feet 1756 by 216, the number of Cubic Feet contained in a Cubic Fathom, you will have the Solidity sought 8 Fathoms, 28 Feet, and 1536 Inches Cubic.

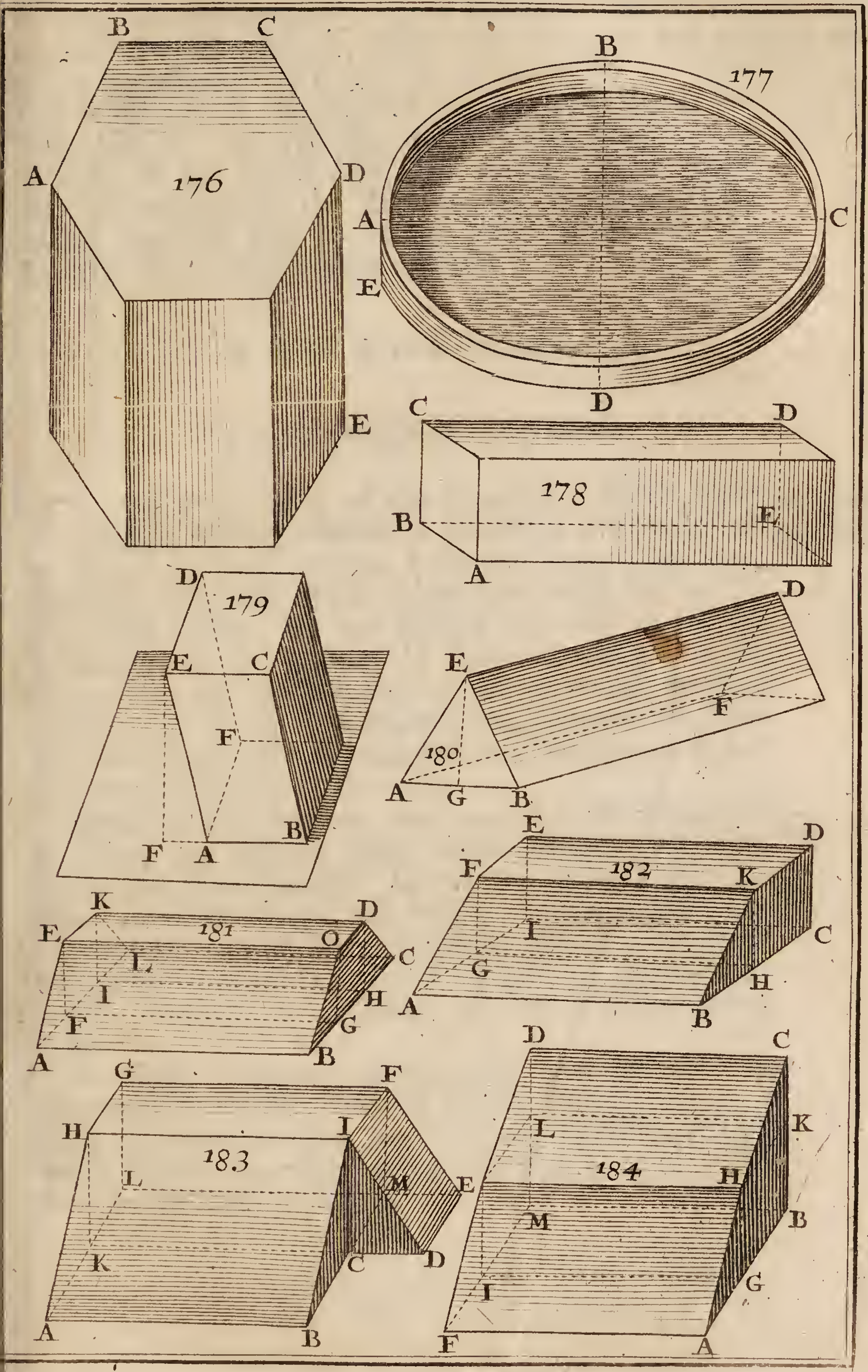
Or to come at the Fathoms at one Operation, divide the Cubic Inches found to be 3035904 by 373248, the value of a Cubic Fathom in Cubic Feet, and the Quotient will give 8 Cubic Fathoms as before, and the remainder 49920 divided by 1728, the value of a Cubic Foot in Cubic Inches, gives 28 Cubic Feet, 1536 Cubic Inches remaining as before.

There are in Stereometry, as well as Planimetry, shorter Methods of working; but I shall not speak any thing of them here, because they are most of them but Approximations to the Truth, and the rest too chargeable to the Memory.

When a right Prism is proposed to be measured, for instance the preceding one ABCD, 'tis evident the Side BC represents the Height, because Perpendicular to its Base ABE: But when an *oblique* one is proposed, that is to say, when its Sides are not parallel to the Base; as Fig. 179. ABCDE, whose Side AE is oblique to the Base ABF, then the Side AE can't be taken for the Height of the Prism, but a right Line EF must be taken Perpendicular to the Plane of the Base AEF. This remark will hold in all other Bodies, that are not right.

Fig. 180. In finding the Solidity of a Triangular Prism set upon one of its Parallelogrammic Faces, as ABCD, upon the Parallelogram ABCF, which in this Case is consider'd as its Base, in respect to which the Height of the Prism will be equal to the Height EG, of the Triangle ABE, representing the *Profile*, you need not multiply the Base ABCF, by the Height EG, but only by half that Height for its Solidity, because when you multiply it by the entire Height, you find the Solidity of a Prism of the same Base and Height, which *by* 40. 11. is double the Prism proposed ABCDE, which for that reason is call'd hereafter a *Semi-prism*, if its Base be a Parallelogram.

Suppose the Base ABCF to be a right-angled Parallelogram, its Length BC 24 Feet, Breadth AB 14, and consequently its Area 336 Feet square, found by multiplying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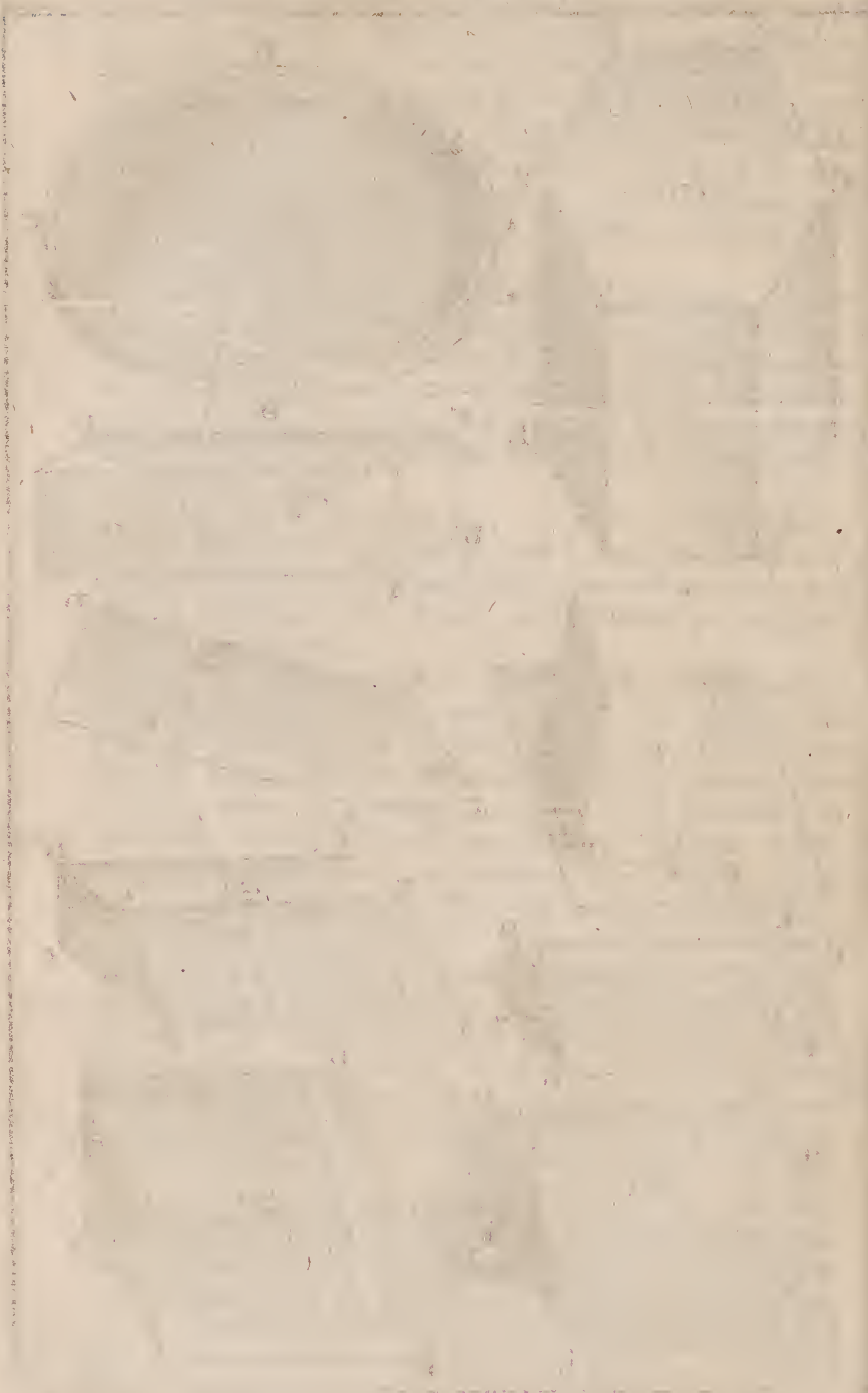


tiplying 24 its Length by 14 its Breadth: But if you suppose the Side BE 15 Feet, and the Side AE 13, the Perpendicular EG will be found to be 12 Feet, half which *viz.* 6, multiplied by the Area 336 of the Base ABCF, you will have 2016 Cubic Feet for the Solidity of the Prism ABCDE, which may be found also by multiplying the Area of the Triangle ABE, which you will find to be 84 Feet square, by the Length BC, supposed to be 24 Feet. *Pla. XXII. Fig.* 180.

PROBLEM II.

To find the Solidity of a Pyramid.

TO find the Solidity of the Pyramid ABCD, whose Height ED for instance is 36 Feet; and Side AB or BC of its Base, which I suppose Square, 18 Feet, multiply the Base ABC which you will find to be 324 Feet square, by the Height 36, and divide the Product 11664 by 3, and the Quotient will give 3888 Cubic Feet for the Solidity of the Pyramid proposed ABCD; for by that Multiplication, you find the Content of a Prism of the same Base and Altitude, and that is thrice the Pyramid, *by* 7. 12. *Pl. XIX. Fig.* 168.

After the same Manner you may find the Solidity of a Cone, for instance ABC, whose Height CO is 36 Feet, and the Diameter AB of the Base ADBE 30 Feet, in which Case, this Base or Circle ADBE will be found to be 706 Feet, 72 Inches square, which multiplied by a third 12 of the Height 36, gives 8478 Cubic Feet for its Solidity of the Cone proposed ABC. *Fig.* 162.

To avoid Fractions in the Base, *multiply* 900 *the Square of* 30 *by* 785, *and the Product* 706500 *by* 12 *a third of* 36 *the Height, for a second Product* 8478000, *which divided by* 1000, *gives, as before,* 8478 *Cubic Feet for the Solidity sought.*

SCHO-

SCHOLIUM.

Plat. XIX. Fig. 162. The Cones used in Practice are generally right, and then the Line representing the Height, falls on the Center of the Base; and since it sometimes happens that that Height can't easily be measured, in this Case find the Length of the Side AC, or BC, which we will suppose 25 Inches, and of the Semi-diameter AO, or BO, 15 Inches, whose Square 225 substracted from 625 the square of the Side AB, leaves 400, whose square Root 20, is the Height CO in Feet, the Demonstration is evident *from* 47. 1.

PROBLEM III.

To find the Solidity of the Frustum of a Pyramid.

Pla. XX. Fig. 171. TO find the Solidity of the Frustum of a Pyramid ABCDEF, imagine the Sides produced to L, and you will have an intire Pyramid ABLF, from which take away the Pyramid HLCE that was added, and there will remain the Frustum. But to find the Solidities of these two Pyramids that are right, suppose the Base ABGF to be a square, whose Side AB is for instance 36 Feet, and the Side CD of the little Base HCDE, a Square also, 8 Feet, in which Case the little Base HCDE will be 64 Feet square, and the great one ABGF 1296 Feet square. Suppose the Height of the Frustum IK to be 42 Feet, by it, and the given Sides of each Base you may find the Height LI of the little Pyramid HCLE, by making the following Analogy, drawn from the Similitude of the two Triangles LIE, LKF, &c.

As the difference of the Sides AB, CD.	28
To the little Side CD.	8
So is the Height IK.	42
To the Height LI.	12

To which, being found to be 12 Feet, adding 42 Feet the Height IK, you will have 54 Feet the greater's Height LK, which may also be found at once by this Analogy.

As the difference of the Sides AB, CD	28	*Pla. XX.*
To the great Side AB	36	*Fig.* 171.
So is the Height IK	42	
To the Height LK in Feet	54	

Multiply the great Base 1296 by a third of the Height LK 18, and you will have 23328 Cubic Feet for the Solidity of the great Pyramid ABLF; multiply the little Base 64 by a third of its Height 4, and you will have 256 Cubic Feet for the Solidity of the little Pyramid HCLE, which taken from the Solidity of the greater ABLF 23328, will leave 23072 Cubic Feet for the Frustum of the Pyramid ABCDEF.

Or to avoid Fractions in the Heights LI, LK, *add together the two Bases* 1296, 64, *for their Sum* 1360, *and multiply them together for their Product* 82944, *add its square Root* 288 *to the foregoing Sum* 1360, *this second Sum* 1648, *multiplied by* 14, *the third part of the Height IK, gives* 20312 *Cubic Feet for the Solidity sought, as before.*

SCHOLIUM.

This Frustum of a Pyramid ABCDEF being right, as they commonly are, its Solidity may be found, by reducing it into a Prism whose Base is the Square STVX, equal to the little Base CDEH; into four equal Semi-prisms, whose Bases are the equal Rectangles YZTS, MNVT, VOPX, QRSX, in each of which the Breadth is equal to the Side of the little Base CDHE, and the Length is equal to 14, half 28 the excess of 36 the Side of the great Base ABCD, above 8 the Side of the less CDHE; and into four equal Pyramids, whose Vertices are at the four Points, C, D, E, H, and whose Bases are the four equal Squares BV, PQ, RY, AT, each of which is 196 Feet square, because its Side is 14 Feet, namely half the difference of the two Sides AB, CD.

If therefore you multiply 64 the Area of the Base STVX of the Prism, by 42 the Height, you will have 2688 Cubic Feet for the Solidity of the Prism. If you multiply the Base ZTSY, whose Area you will find to be 112 Feet square, by 21 half the Height 42, you will have 2352 Cubic Feet for the Semi-prism, whose Base is ZTSY, and multiplying this Solidity found 2352 by 4, you will have 9408 Cubic Feet for the four Semi-

 prisms,

Plat. XX. prisms. Lastly, multiply the Base of one of these four
Fig. 171. Pyramids which is 196 Feet square, by 14 a third of the Height 42, and you will have 2744 Cubic Feet for one Pyramid, its Quadruple will give 10976 Cubic Feet for the four Pyramids, thus adding together the Solidities found, 2688, 9408, 11976, of the Prism, of the four Semi-prisms, and four Pyramids, you will have 23072 Cubic Feet for the Solidity of the Frustum of the Pyramid ABCDEF as before.

PROBLEM IV.

To find the Solidity of the Frustum of a Cone.

Plat. XIX. TO find the Solidity of the Frustum of a Cone
Fig. 165. ABCD, whose Height IO is, for instance, 15 Feet, and Diameter AB of its great Base AEBF, 24 Feet, and Diameter CD of the little Base DGCH, 18 Feet, in which Case the great Base will be 452 Feet square, and the little one 254; you may produce the Sides of the Frustum of the Cone, as you did in the former Problem, and so form an intire Cone; but the foregoing Rule which I shall repeat here, is generally used, namely,

Add together the two Bases 452, 254, *for their Sum* 706, *multiply them together for their Product* 114808, *whose square Root* 339 *added to the preceding Sum* 706, *gives a second Sum* 1045, *which multiplied by* 5, *the third of the Height IO, gives* 5225 *Cubic Feet for the Solidity of the Frustum of the Cone ABCD.*

Or to avoid the Fractions in the Bases, and Square Root of the Product, follow this other Rule, which, as well as the other, is easily Demonstrated.

Multiply the two Diameters 24, 18, *together for their Product* 432, *add it to* 900 *the Sum of the squares* 576, 324 *of the same Diameters* 24, 18, *for a second Sum* 1332, *multiply this by* 157, *for the Product* 209124, *which multiplied by* 5 *a third of the Height* IO, *and* 522810, *half the Product* 1045620 *divided by* 100, *gives with great accuracy, the Solidity sought,* 5228 *Cubic Feet.*

PRO-

PROBLEM V.

To measure a Taludated or sloaping Body.

TO find the Solidity of a *Taludated Body*, that is to say, a Body broader on one Side than the other; as the Wall ABCDEF, which is broader above than below, its Base ABCI being larger than the upper Face DEFK, or Rectangle GHCI, its *Orthographic Plane* by the whole Rectangle ABHG, the Base of the Semi-prism or *Talud* ABHKF, whose Profile is the Right-angled Triangle BHK, as the Profile of the Wall is the Trapezoid BCDK; multiply the Area of the Right-angled Triangle BHK, by AB the Length of the Wall, for the Solidity of the Prism ABHKF; found also by multiplying the Base ABHG, by half the Height HK. In like manner multiply the Area of the Rectangle GHCI, by the same Height HK, or CD, for the Solidity of the Parallelopiped GHCDEF, to which adding that of the Talud ABHKF, you will have the intire Solidity of the Wall propos'd ABCDEF; easily found also by multiplying the Area of its Profile BCDK by its Length AB. *Pla.XXII. Fig.* 182.

Suppose the Length AB to be 48 Feet, the Breadth BH of the Talud 6 Feet, the Height HK 12 Feet, and DK 4 Feet, consequently the Depth BC 10 Feet; the Solidity of the Semi-prism ABHKF, will be found to be 1728 Cubic Feet, and that of the Prism GHCDE 2304 Cubic Feet, and the whole Wall ABCDEF 4032 Cubic Feet. Or the Area of the Trapezoid BCDK, which you will find to be 84 Feet square, multiply'd by the Length AB, suppos'd to be 48 Feet, gives, as before, 4032 for the Solidity sought.

After the same manner you may find the Solidity of the Wall ABCDK, sloap'd on both Sides, namely, by adding to the Solidity of the Prism FGHDKI, whose Orthographic Plan is the Rectangle FGHI, the Solidity of the Talud ABGOEF, whose Base is the Rectangle ABGF, and the Solidity also of the other Talud IHCDKL, whose Base is the Rectangle IHCL, and Profile the Triangle DHC, Right-angled in H. Or multiplying the Profile BCDO of the Wall by its Length AB. *Fig.* 181.

Thus if AB the Length of the Wall be 48 Feet, BG the Breadth of the Talud ABGOEF 6 Feet, the Breadth CH of the other Talud IHCDKL 5 Feet, GO or HD the Height of the Wall 12 Feet, and OD 3 Feet, consequently BC 14 Feet, the Solidity of the Talud or Semi-

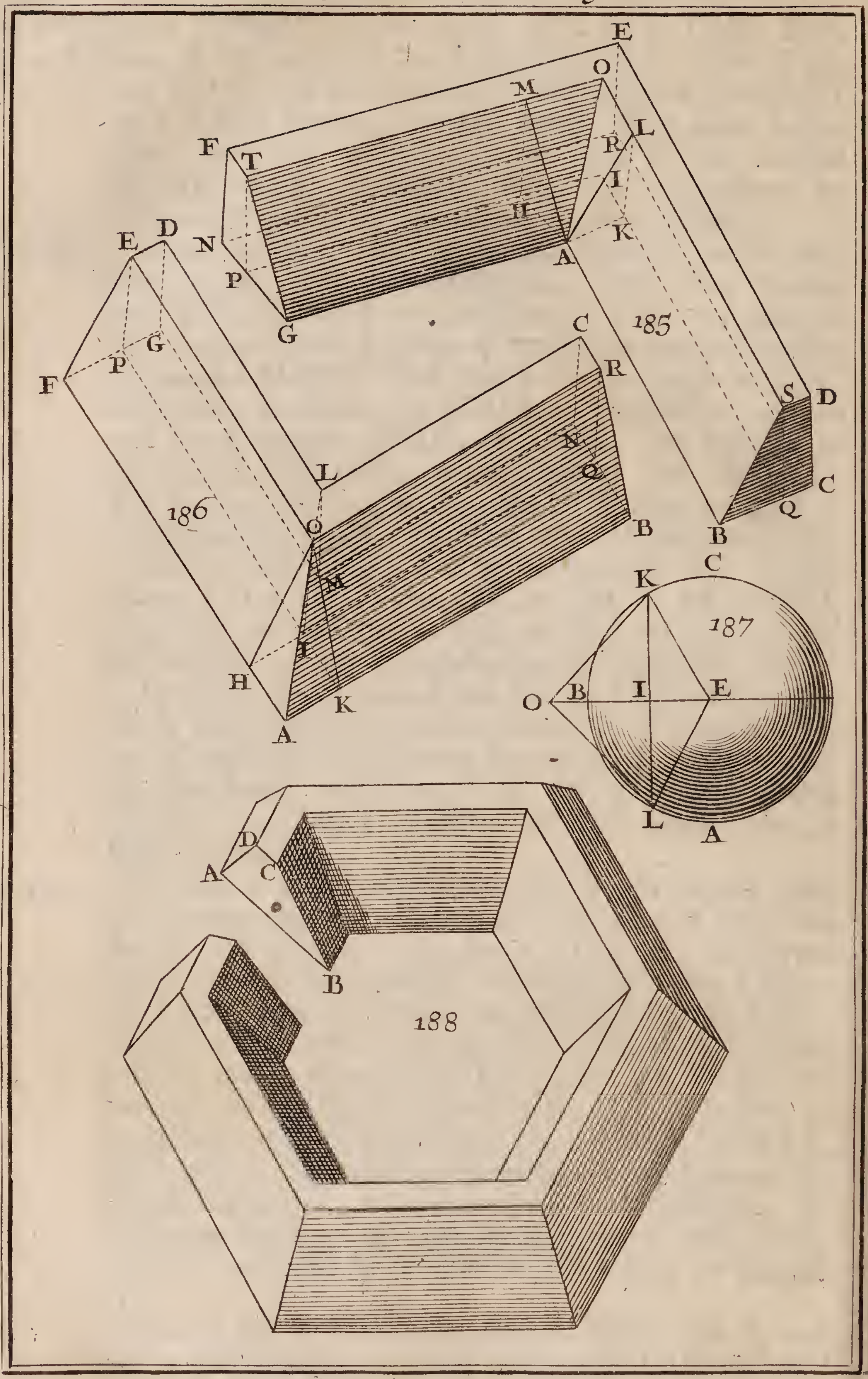
Pla. XXII. Fig. 181. prism ABGOF will be found to be 1728 Cubic Feet, that of the other Talud IHCDKL 1440 Cubic Feet, and that of the Right-angled Parallelopiped FGHDKI 1728 Cubic Inches, and the whole Wall 4846 Cubic Feet, found by multiplying the Area of the Profile BCDO, which you will find to be 102 Feet square, by the Length AB, which we suppos'd to be 48 Feet.

Fig. 183. When the two Taluds are joined together, as it happens in the Wall ABCDEFGH, whose two Taluds ABCIHK, CDEFI are join'd in the Point I; add in like manner to the Solidity of the Right-angled Parallelopiped KCMFGH, whose Base is the Rectangle KCML, and Profile the Rectangle CMFI, or KLGH, or if you will the Rectangle KCIH, or LMFG, the Solidity of the Talud ABCIHK, whose Base is the Rectangle ABCK, and Profile the Triangle BCI, Right-angled in C, for the Solidity of the Wall propos'd ABCDEFGH.

Thus if AB, or KC, or LM be 24 Feet, BC, or AK 6 Feet, CD or ME 5 Feet, DE, or CM, or KL, or GH, or FI 4 Feet, and CI, or FM, or KH, or LG 12 Feet, the Solidity of the Parallelopiped KCMFGH will be 1152 Cubic Feet, that of the Talud ABCIHK of 864 Cubic Feet, and the Solidity of the other Talud CDEFI, 120 Cubic Feet, so that the Sum of these three Solidities thus found 1152, 864, 120, will give 3136 Cubic Feet for the intire Solidity of the Wall propos'd ABCDEFGH.

Fig. 184. If of these Taluds thus joined, the one is on one Side, and the other above, which in this Situation is called the *Glacis*, as it happens in the Body ABCDEF, which has before the Talud AGHEIF, whose Base is the Rectangle FAGI, and the Profile the Triangle AGH Right-angled in G; and above the Glacis EHKCDL, whose Base is the Rect-angle EHKL, and the Profile the Triangle HKC Right-angled in K; add as before, to the Solidity of the Right-angled Parallelopiped IGBKLE, whose Base is the Rectangle IGBM, the Solidities of the Talud FAGHEI, and of the Glacis EHKCDLE, for the intire Solidity of the Body propos'd ABCDEF, found by multiplying its Profile ABCH by its Length AF.

Thus if AF be a 24 Feet, AG 4, BG 6, BK 8, and KC 5, the Solidity of the Parallelopiped IGBKLE will be 1152 Cubic Feet, that of the Talud AGHEIF 384 Cubic Feet, and that of the Glacis EHKCDL 360 Cubic Feet, and these three Solidities 1152, 384, 360,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360, added together give 1896 Cubic Feet for the Solidity sought.

If the two Taluds that join, belong to two Walls sloaping outwards, as the two Walls ABCL, AFDL are, that I suppose to be of an equal Height, and whose Bases are the two Trapezoids BMNB, AMGF, the Profiles are the two Trapezoids ENCR, DEFG, and the two external Taluds are AIORQB, AIOEPF, whose Bases are the Trapezeids AIQB, AIPF, and the two Right-angled Triangles BQR, EPF are their Profils, let fall from the Angle at the Base of each Talud, the two Lines IK, IH, Perpendicular to AB, AF, and join the right Lines OK, OH, and then each Wall will be reduc'd into a Prism and Semi-prism, namely, the Wall ABCL, into a Prism, IQRCLO, whose Base is the Trapezoid IQNM, and Profile the Rectangle QRCN, or IOLM, and into the Semi-prism KIORQB, whose Base is the Rectangle KIQB, and Profile the Triangle BQR right-angled in Q, or the Triangle KIO, right angled in I: And in like manner the Wall AFDL, into the Prism IMLDEP, whose Base is the Trapezoid IMGP, and Profile the Rectangle PGDE, or IMLO; and into the Semi-prism HIOEPF, whose Base is the Rectangle HIPF, and Profile is the Triangle FPE, right-angled in P, or the Triangle HIO, right-angled in I. If therefore you add together the Solidities of the two Prisms and Semi-prisms, and besides, the Solidity of the Pyramid AHOK, whose Base is the Quadrilateral Figure AHIK, and Vertex at the Angle O, called the *Saillant Angle*, because it jutts or terminates outwards, and you will have the Solidity of the two Walls propos'd ABCL, AFDL. *Pl. XXIII. Fig. 186.*

Suppose the two Walls ABCL, AFDL equal, and the Length AB 40 Feet, IQ 30, MN 25, QR 8, and BQ 6; in which case AK will be 10 Feet, and QN 3: The Area of the Trapezoid IMNQ will be 82 Feet and 72 Inches square, and the Solidity of the Prism IQRCLO 660 Cubic Feet, and the Area of the Rectangle KIQR 180 Feet square, and the Solidity of the Semi-prism KBQROI 720 Cubic Feet, and the Sum of these two Solidities found 660, 720, is 1380, whose double will give 2760 Cubic Feet for the Sum of the Prism, and Semi-prisms found in the two Walls, to which add the Solidity of the Pyramid OKAH, which you will find to be 160 Cubic Feet, its Base AHIK being 60 Feet square, and you will have in all 2920 Cubic Feet for the Solidities of the two Walls ABCL, AFDL.

Pl. XXIII. Fig. 185. If the two Taluds that are join'd to one another, belong to two Walls sloap'd inwards, as it happens in the two Walls ABCDER, AGNFER, that I suppose of equal Heights, whose Bases are the two Trapezoids ABCR, AGNR, the common Profile the Trapezoid BCDS; because I suppose the Walls are equally broad at Top and at Bottom, and the two interior Taluds are AOSQB, AOTPG, whose Bases are the Trapezoids ABQI, AGPI, and the common Profile the Triangle BQS right-angled in Q; let fall from the Angle A, called the *Entrant Angle*, because it enters within, on the Base of each Talud, the two Lines AH, AK, perpendicular to IP, IQ, and on the Vertical Face of each Prism, thro' the Points H, K, the two Lines HM, KL, perpendicular to IP, IQ, or AH, AK, and draw the right Lines AL, AM, and they will form two Pyramids, whose Heights are the Perpendiculars AH, AK, and Bases the Rectangles IOMH, IOLK, and common Vertex the Point A. Thus the two Walls propos'd ABCDE, AGNFE, are found reduc'd into two Prisms QDSLK, PFTMOEL, or a single one whose Base is the Plane PNRCQIP, and Height CD, or QS; into two Semi-prisms, namely, BQSLKA, whose Base is the Rectangle AKQB, and Profile the Triangle BQS right-angled in Q, and the Plane PFTMHA, whose Base is the rectangle AGPH, and Profil the Triangle AHM right-angled in M; and into two Pyramids AIOLKA, AIOMHA: consequently add together all these Solidities, and you will have the Solidity of the two Walls propos'd ABCDE, AGNFE.

Suppose, as before, the two Walls ABCDE, AGNFE equal, and the length AB 40 Feet, IQ 50, RC 55, BQ 6, CD 8, in which case QC will be 3, and IK 10: The Plane PNRCQI 315 Feet square, and its Prism PIQCDEF 2520 Cubic Feet; the Semi-prism ABQSLK 960 Cubic Feet, and the Pyramid AIOLKA 160 Cubic Feet, whose double 320 added to 1920, the double of the former Solid 960, and the Sum 2240 added to the former Solid 2520, will give in all 4760 Cubic Feet for the Solidity of the two Walls propos'd ABCDE, AGNEF.

SCHOLIUM.

Fig. 188. From what has been said, you may easily find the Solidity of two or more Walls join'd together, and sloaping within and without: But when they have the Height, and internal and external Talud equal, and make equal Angles, their Solidity may be found at once, multiply-

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ing their Profile as ABCD, repreſenting the Baſe by a Line drawn thro' the middle of the Plane that ſerves for a Baſe, that right Line being conſider'd as the Height of a Priſm compos'd of all thoſe Walls.

'Tis eaſie by the ſame Principles to find the Solidity of a Rampart, and other Parts of a Fortification: For inſtance, to find the Solidity of the Rampart of a Fortified Polygon, where AB repreſents half the Courtine, BC the Flank of the Baſtion, ſuppoſed Perpendicular to the Courtine for the caſe of Calculation, and CD the Face of the Baſtion, terminated at the Point D, by the Capital Line HM, terminating the Line GH of the internal Talud, parallel to the Line EF of the Rampart, and the Line IKLM of the external Talud following parallelwiſe the Line ABCD of the former Draught: So that the Figure GIKLMH repreſents the Plan of a Semi-courtine, and Semi-baſtion, or the Line AE repreſents the Breadth of the Rampart above, that is to ſay, the Breadth of the Terre-plain, ſuppoſe 12 Fathom or 72 Feet: The Line EG repreſents the quantity of the internal Talud 18 Feet; and the Line AI repreſents the quantity of the external Talud, for inſtance 12 Feet. The Height of the Rampart we ſhall ſuppoſe 18 Feet, as you ſee in the Profile, which ſhews the Rampart is above 102 Feet broad below, without troubling our ſelves at preſent whether thoſe Suppoſitions are intirely agreeable to the Maxims of a good Fortification. *Pla.XXIV. Fig. 189.*

To find the Solidity of this Quantity of Earth, and firſt that of the Priſm contain'd between the two Taluds, and whoſe Baſe is the Plane ABCDEF; firſt find the Surface of this Plane, reducing it into a Trapezoid AEFT, by the Line; BA produc'd into the Right-angled Triangle BCT; and Oblique-angled Triangle CTD, by the Diagonal CT.

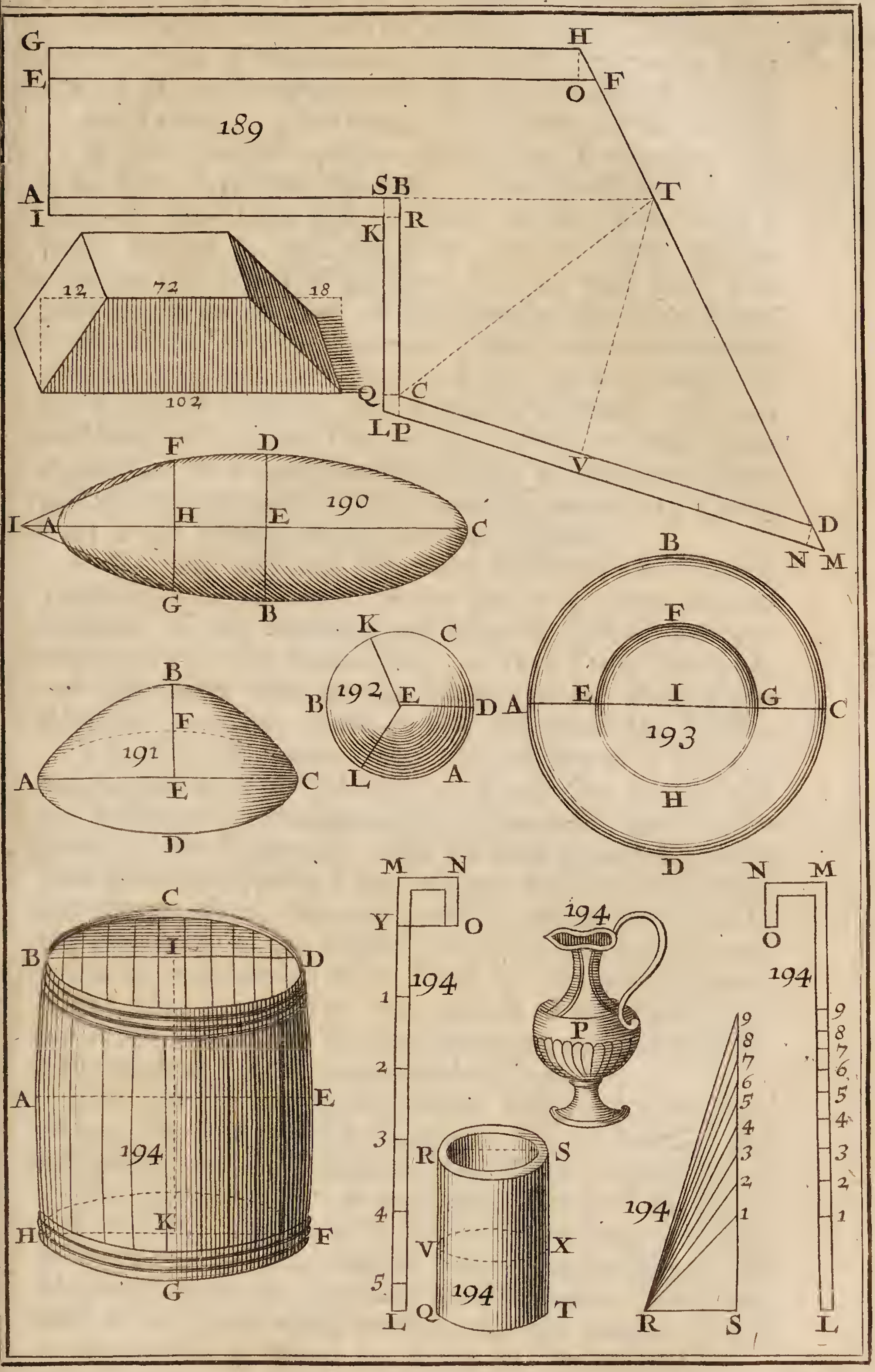
Multiply the Sum 720 of the Line EF, found to be about 344 Feet, and the Line AT 376 Feet, by the Line AE ſuppoſed to be 72 Feet, and half the Product 51840 will give 25920 Feet ſquare for the Area of the Trapezoid AEFT. Multiply the Line BT, found about 160 Feet, by the Line BC, ſuppoſed to be 120 Feet, and half the Product 192000 will give 9600 Feet ſquare for the Area of the Right-angled Triangle BCT. Then multiply the Face CD, which will be found to be 270 Feet, by its perpendicular TV 160 Feet, and half the Product 43200 will give 21600 Feet ſquare for the Area of the Oblique-angled Triangle CTD. Add theſe three Areas thus found 25920, 19200, 21600, and you will have 66700 Feet

Pl. XXIV. *Fig.* 189. square for the Plane ABCDEF, which multiplied by the Height of the Rampart, suppos'd to be 18 Feet, will give 1200600 Cubic Feet for the Solidity of the internal Prism bounded by the two Taluds, and plac'd on the Base ABCDFE.

To find the Solidity of the Taluds, reduce them to Semi-prisms and Pyramids, by letting fall from the Angles of their Bases perpendiculars between the two parallel Sides, namely HO, DN, CP, CQ, KR, KS: And the internal Talud, whose Base is the Trapezoid GEFH, will be found divided into a Prism, whose Base is the Rectangle GEOH, and Height the same with that of the Rampart, and in to a Pyramid whose Vertex is H, and Height the perpendicular HO, and Base a Rectangle, whose breadth is the Line OF, and length equal to the Height of the Rampart: And the external Talud you will find divided into three Semi-prisms, whose common Height is the same as the Ramparts, and the Bases the three Rectangles AK, CK, CN, and into three Pyramids, two of which have their Vertices upwards, consequently have the Height of the Rampart for the common Height, and their Bases the Right-angled Triangle DNM, and the Quadrilateral Figure CPLQ; and the third is taken for one, that has the same Height as the Rampart, and the Square KRBS for a Base, because so small a part is inconsiderable: But in Geometrical rigor, instead of this Pyramid you ought to take the double of another, that has its Point below at the Point K, the Perpendicular KR for its Height; and a Rectangle for its Base, whose Breadth is the Line BR, and Length equal to the Height of the Rampart.

Multiply the Length EO, or GH, which you will find about 330 Feet, by the Breadth EG, suppos'd to be 18 Feet, and you will have 5940 Feet square for the Area of the Rectangle GO, which being multiply'd by 9 half the Height of the Rampart, gives 53460 Cubic Feet for the Solidity of the Semi-prism, whose Base is the Rectangle GO: Multiply the Line OF found to be about 14 Feet, by the Height of the Rampart, suppos'd to be 18 Feet, and the Product 252 by 6 a third Part of the Line HO, gives 1512 Cubic Feet for the Solidity of the Pyramid, whose Vertex is H, and Height HO, to which add the Solidity of the foregoing Semi-prism 53460, and you will have 54972 Cubic Feet for the Solidity of the internal Talud.

Multiply the Sum 580 of the Line IK, that you will find about 206 Feet, of the Line KQ 109 Feet, and of the



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the Line PN 265 Feet, by the common Breadth AI, ſup- *Pl. XXIV.* poſed to be 12 Feet, and you will have 6960 Feet ſquare *Fig.* 189. for the Sum of the Areas of the three Rectangles AK, CK, CN, which being multiply'd by 9 half the Height of the Rampart, gives 62640 Cubic Feet for the Solidity of the three Semi-priſms, whoſe three Rectangles AK, CK, CN, repreſent the Baſes: Then multiply 26 the Sum of the Line KR 12 Feet, of the Line QL 8 Feet, and of 6, half the Line MN 12 Feet, by the common Breadth DN 12 Feet, and you will have 312 Feet ſquare for the Sum of the Baſes of the three Pyramids contain'd in the external Talud, wherefore multiply this Sum 312 by 6 the third Part of the Height of the Rampart common to the three Pyramids, and you will have 1872 Cubic Feet for the Solidity of theſe three Pyramids, to which add the Solidity of the three Semi-priſms aforemention'd 62640, and the Sum 64512 Cubic Feet is the Solidity of the external Talud.

Laſtly, add together the three Solidities thus found, 1200600, 54972, 64512, of the Priſm between the two Taluds, of the external, and internal Talud, and you will have 1320084 Cubic Feet, or 6111 Cubic Fathoms, and 108 Cubic Feet for the Solidity of the Part of the Rampart propoſed, whoſe Baſe is the Plane IKLMHG.

'Tis after this manner that the Solidity of a Ditch is found, that is of the Quantity of Earth dug up: As alſo the Solidity of a Parapet and its Banquette, and the Solidity of the Eſplanade: But for theſe three laſt Bodies, that have the oppoſite Lines of their Baſes parallel, there are ſhorter ways, but not perfectly exact, by reaſon of the inequality of the Angles, yet not to be ſlighted; and one is by multiplying their Profil by the Quantity of a Right Line drawn thro' the middle of the Plane, that ſerves for a Baſe, for by this means you will have the Solidity at once, without any conſiderable Error.

This Method is of good Uſe in finding, by one Operation, the Solidity of a Rampart, with it Parapet and Banquette, when the Baſtions are Hollow, which one is obliged to know, that one may judge nearly of the Quantity of Earth, that muſt be dug up for making the Rampart and its Parapet; And for finding the Solidity of the Maiſonry made for ſuſtaining the Terraſſes, which one muſt be acquinted with, to determine the Expence, and in what ſpace of Time the Work may be done by a certain Number of Men, or what Number of Men will finiſh it in a determin'd Time, &c.

PROB-

PROBLEM VI.

To find the Solidity of a Sphere, the Diameter being given.

Pl. XXIV. Fig. 192. TO find the Solidity of a Sphere ABCD, its Diameter BD being given, for instance 18 Inches; its Surface being found about 1017 $\frac{49}{144}$ Inches Square, multiply it by 3 the sixth Part of its Diameter 18, and the Product will give 3052 $\frac{36}{1728}$ Cubic Inches for the Solidity of the Sphere proposed ABCD, as is evident *from Theor.* 1.

Or to avoid the Fractions that may happen in finding the Surface of the Sphere, *multiply* 5832 *the Cube of the Diameter* 18, *always by* 157, *and divide* 305208, *the third Part of the Product* 915624 *by* 100, *and the Quotient will give* 3052 $\frac{138}{1728}$ *Cubic Inches for the Solidity sought*, the Demonstration is evident *from Theor.* 2.

PROBLEM VII.

To find the Solidity of a Sphere, its Circumference being given.

Fig. 192. TO find the Solidity of the Sphere ABCD, its Circumference for instance 56 Feet, being given: find its Diameter 17 Feet 10 Inches, and its Surface 998 Feet, 104 Inches square, and that multiply'd by the Diameter, and then divided by 6, will give 2968 Cubic Feet, and about 733 Cubic Inches for the Solidity of the Sphere proposed ABCD.

Or to avoid the Fractions that commonly happen in finding the Diameter, and Surface of the Sphere, and hinder the exactness of the Answer, *Multiply* 175616 *the Cube of* 56 *the Circumference, by* 10000, *and divide the Product* 17560000 *by* 592175, *and the Quotient will give* 2965 *Cubic Feet, and* 636 *Cubic Inches for the Solidity sought.*

PRO-

PROBLEM VIII.

To find the Solidity of the Sector of a Sphere.

TO find the Solidity of the Sector of a Sphere LEKB, whose Arc KBL is for instance 100 Degrees, the Diameter BD being given, suppose 18 Inches; the Surface that serves for a Base to the Sector of the Sphere, you will find to be 181 $\frac{102}{144}$ Inches square, that multiplied by 3 the sixth part of the Diameter 18, will give 545 $\frac{216}{1728}$ Cubic Inches for the Solidity of the Sector proposed KBLE, the Demonstration is evident *by Theor.* 1. *Pl. XXIV. Fig.* 192.

Or to avoid the Fractions in the Base of the Sector of the Sphere, *Multiply* 35721 *the versed Sine of the half BK or BL of the Arc KBL, by* 5832 *the Cube of the Diameter* 18, *and multiply the Product* 208324872 *always by* 157, *for a second Product* 32707004904, *and its sixth part* 5451167484, *being divided by* 100 *times the Radius* 100000, viz. 10000000, *will give* 545 $\frac{201}{1728}$ *Cubic Inches, for the Solidity sought.*

PROBLEM IX.

To find the Solidity of the Segment of a Sphere.

TO find the Solidity of the Segment or Portion of the Sphere KLB, whose Arc KBL is for instance 100 Degrees, the Diameter BD being given, 18 Inches for instance, subtract from the Sector KELB, found by *Probl.* 8. to be 545 $\frac{201}{1728}$ Cubic Inches, the Cone KLE, that you will find to be 287 $\frac{1402}{1728}$ Cubic Inches, and the remainder 257 $\frac{527}{1728}$ Cubic Inches is the Solidity of the Segment proposed KLB. *Pl. XXIII. Fig.* 187.

SCHOLIUM.

THE Height EI of the Cone KEL will be found to be 5 $\frac{3}{4}$ Inches, in the Triangle KIE, right-angled at I, and the Diameter KL of the Base of the same Cone KEL, will be found 13$\frac{3}{4}$ Inches, in the Isosceles Triangle KEL, then 'tis easie to find the Solidity of the Cone KEL, *by Probl.* 2. But to avoid Fractions in the Height EI, or Diameter KL, follow this Rule that has a Demonstration.

Mul-

Pl. XXIII. Fig. 187. *Multiply the Square* 5868172816 *of the Sine* 76604 *of the half BK, or BL, of the Arc KBL, by the Sine* 64279 *Complement of that half, and the Product* 377200280439664 *multiply by the Cube* 5832 *of the Diameter* 18, *for a ſecond Product* 2199832035524120448, *which you muſt multiply by* 157, *and divide the twelfth part* 28781135798107242528, *of the third Product* 345373629577286910336, *by the Centuple of* 1000000000000000, *the Cube of the Radius* 100000, viz. 100000000000000000, *and the Quotient will give* 287 *Cubic Inches for the Solidity of the Cone KEL*, which may be very commodiouſly found by the help of Logarithms, that will ſave you ſeveral long Multiplications.

The ſame Segment KLB may be found independantly from the Sector, adding to its Height BI, the Line BO a fourth proportional to DI, BI, BE, and find the Solidity of the Cone KOL, *by Theor.* 3. equal to the Sector BKL. But for Practice the other Rule is better.

PROBLEM X.

To find the Solidity of a Spheroid.

Pla. XXIV. Fig. 190. TO find the Solidity of the Spheroid ABCD, whoſe Axe of Rotation AC is for inſtance 32 Inches, and the other Axe BD is 18 Inches; find by *Prob.* 6. the Solidity of a Sphere, whoſe Diameter is the Axe of Rotation AC, and it will be 17148 $\frac{1019}{1728}$ Cubic Inches, multiply it by 324, the Square of the Diameter BD, and the Product 5556143 $\frac{108}{1728}$ Cubic Inches divided by the Number 1024, the Square of the Axe of Rotation AC, gives 5425 $\frac{1591}{1728}$ Cubic Inches for the Solidity of the Spheroid propoſed ABCD, as is evident *by Theor.* 4.

To avoid Fractions in finding the Solidity of the Sphere: *Multiply* 324 *the Square of the Axe BD, by the Axe of Rotation AC, ſuppoſed to be* 32 *Inches, and multiply the Product* 10368 *by* 157, *for a ſecond Product* 1627776, *and its third part* 542592 *divided by* 100, *gives* 5425 *Solid Inches for the Solidity ſought.* The Demonſtration is evident *from Theor.* 5.

PROB-

PROBLEM XI.

To find the Solidity of a Segment of a Spheroid.

TO find the Solidity of the Segment of a Spheroid AGF, whose Base is a Circle whose Diameter FG is 12 Feet, and perpendicular to the Axe of Rotation AC, which suppose 36 Feet, and whose Height is AH 9 Feet, in which case CH will be 27 Feet: add to the Height AH, the Line AI 6 Feet, namely a fourth proportional to CH, AH, AE, and then the Line HI will be found to be 15 Feet, the Height of the Cone FIG equal to the Segment AFG, *by Corol. Theor.* 6. The Base of this Cone is the same as that of the Segment, namely the Circle whose Diameter FG was supposed to be 12 Feet, will be found to be 103 Feet 6 Inches square, that multiply'd by 5 the third of the Height HI, gives 565 Cubic Feet, and 360 Cubic Inches for the Solidity of the Cone IFG, or the Segment proposed AFG. *Pl. XXIV. Fig.* 190.

SCHOLIUM.

A Segment of a Spheroid may be measur'd another way, because *by Theor.* 6. it is to a Cone inscribed, as the corresponding Sphere is to its inscrib'd Cone: But the Operation drawn from this Theorem being longer than the former, it does not deserve to be insisted on.

PROBLEM XII.

To find the Solidity of a Parabolic Conoid.

TO find the Solidity of the Parabolic Conoid ABCD, whose Axe is BE 8 Feet, and Diameter AC of its Base ADCF 12 Feet, this Base ADCF will be found to be 113 Feet, 6 Inches square, that multiply'd by 4, half the Axe BE, gives 452 Cubic Feet, and 288 Cubic Inches for the Solidity of the Paraboloid proposed ABCD. The Demonstration is evident *from Theor.* 7. *Fig.* 191.

To avoid Fractions in finding the Base of the Paraboloid, *Multiply* 144 *the Square of the Diameter AC, suppos'd to be* 12 *Feet, by the Axe BE, supposed to be* 8, *and*

Pla. XXIV. Fig. 191. *and multiply the Product* 1152 *always by* 785, *for a Second Product* 904320, *and its half* 452160 *divided by* 1000, *gives* 452 *Cubic Feet, and* 23 *Cubic Inches for the Solidity sought.*

PROBLEM XIII.

To find the Solidity of an Hyperbolic Conoid.

Pla. XXI. Fig. 175. TO find the Solidity of the Hyperbolic Conoid ABCD, whose Asymptotic Cone is FOGP, and Frustum is FHKGP, as has been shewn in *Theor.* 8. from whence this Method of finding the Solidity of this Conoid is drawn, tho' an easier one may be had, as shall be shewn, suppose the Transvers Semi-Diameter OB 36 Feet, the second Diameter HK, or LN 48, and the Axe BE 9, in which case the Radius AE will be 18 Feet, and the other Radius FE 30, the Axe OE 45, the Diameter AC 36, and the other Diameter FG 60 Feet: The Cylinder HLMNK will be found to be about 16278 Cubic Feet, from whence taking the Frustum HFPGK, which you will find to be 20686 Cubic Feet, there will remain 4408 Cubic Feet for the Solidity of the Conoid supposed ABCD.

To avoid Fractions in finding the Solidity of the Cylinder and Frustum, *Multiply* 108 *the Sum of the two Diameters AC, LN, by the greater AC, and then the Product* 6480 *always by* 157, *for a second Product* 1017360, *from whence subtract* 723456 *the Product of the invariable Number* 314, *by* 2304 *the Square of the less Diameter LN, and the Remainder* 293904 *multiply'd by the Axe BE, supposed to be* 9 *Feet, and the sixth part* 440856 *of that Product* 2645136 *divided by* 100, *will give as before* 4408 *Cubic Feet for the Solidity sought.*

Or multiply the Cone ABCD, that you will find to be 3052 Cubic Feet, by 117 the Sum of the Line OE, and double the Line OB, and divide the Product 357084 by 81, the Sum of the Lines OE, OB, and the Quotient will give as before 4408 Cubic Feet for the Solidity of the Conoid proposed ABCD.

But because Fractions may happen in finding the Solidity of the Cone ABCD, you may avoid them by this other Rule, that may be demonstrated by *Prop.* 27. *of* Archimedes's *of Conoids and Spheroids.*

Multiply the Product 405 *of the two Lines OE, BE, by* 471 *always, and subtract from the Product* 190755, *the Product*

25434

25434 of the invariable Number 314, by the Square 81 of the Axe BE, and the Remainder 165321, multiplied by the Square 1296 of the Diameter AC, and the Product 214256016 divided by the excess 48600 of two Hundred times the Line OE above six Hundred times the Line BE, that is, of 54000 above 5400, and you will have as before 4408 Cubic Feet for the Solidity of the Hyperbolic Conoid ABCD. *Plat. XXI. Fig. 175.*

PROBLEM XIV.

To find the Solidity of an Orb.

TO find the Solidity of the Orb ABCDH, bounded by the little Sphere EFGH contain'd in the great one ABCD; take the less from the greater, and the remainder is the Solidity sought. Thus if the Diameter AC be for instance 24 Feet, and the Diameter EG 18 Feet, the Sphere EFGH will be 3052 Cubic Feet, and 138 Cubic Inches, which subtracted from the Sphere ABCD, that you will find *by Probl.* 6. to be 7234 Cubic Feet, and 967 Cubic Inches, there will remain 4182 Cubic Feet, and 829 Cubic Inches for the Solidity of the Orb ABCDH. *Pla. XXIV. Fig. 193.*

To avoid Fractions in finding the Solidity of each Sphere, *Multiply* 7992 *the difference of the Cubes* 13824, 5832, *of the Diameters AC, EG, always by* 157, *and divide* 418248, *a third part of the Product* 1254744, *by* 100, *and the Quotient will give as before* 4182 *Cubic Feet, and* 829 *Cubic Inches for the Solidity sought.*

PROBLEM XV.

To find the Solidity of the Five regular Bodies.

I said in *Def.* 59. there were but Five regular Bodies, *viz.* The Tetraedrum, Hexaedrum, Octaedrum, Dodecaedrum, and Icosaedrum. The Tetraedrum being a sort of a Pyramid may be measur'd *by Probl.* 2. the Hexaedrum or Cube being a sort of a Prism may be measur'd *by Probl.* 1. and the Solidities of the three others may be found by reducing them into equal Pyramids, having their common Vertex in the Center of the regular Solid, and their Bases the same as the Faces of the Polyedrum. Consequently since there are as many equal Pyramids as the regular Body has Faces, 'tis only multiplying the Area of one of its Faces by the number of

of the Faces, and that Product by a third part of the common Height of these Pyramids.

For instance, to measure an Octaedrum, multiply the Area of one of the Equilateral Triangles by Eight, and the Product by a third part of the Perpendicular drawn from the Center of the Octaedrum thro' the Center of the Equilateral Triangle, and the Product of that second Multiplication will give the Solidity of the Octaedrum proposed.

To find the Solidity of a Dodecaedrum, multiply the Area of one of the regular Pentagons by twelve, and the Product by a third of the Perpendicular drawn from the Center of the Dodecaedrum thro' the Center of the Pentagon: In like manner to find the Solidity of the Icosaedrum, multiply the Area of one of the equilateral Triangles by twenty, and the Product by a third of the Perpendicular.

SCHOLIUM.

If you have a Polyedrum before you, you may easily measure the Side of one of its Faces, and the Height of one of its Pyramids: But if it be an imaginary one, the Method following will shew you how to find the common Height of the Pyramids that compose it, *viz.* by supposing the Side of one of its Faces to be of a certain Magnitude, as 100000 Parts representing Feet, Inches, or any other measures you please, this number above others being pitched upon, because more Commodious in Practice, and the Solidity of such a Body being once found for this Number 100000, it may easily be found by the Golden Rule for any other Number you please, since *by* 33.11. similar Bodies are as the Cubes of their Homologous Sides.

First, to find the Height of one of the equal Pyramids of an Octaedrum, whose Side is supposed to be 100000 parts, the same with the Whole Sine in the Table of Sines, in regard of which the Secant of 45 Degrees, namely 141421 is the Diameter of the Octaedrum, or circumscribed Sphere, and the Tangent of 30 Degrees, namely, 57735 is the Radius of a Circle circumscribed about one of the equilateral Triangles, that serves for a Base to the Octaedrum; subtract 3333330225 the Square of this Radius or Tangent 57735 from 5000000000, half the Square 10000000000, of the whole Sine or Side 100000, and the Square Root of the remainder 1666669775, will give 40824 for the Height of the Pyramid sought, whose Base is found by multiplying 86602 the Sine of 60 Degrees,

grees, by 50000 half the whole Sine, or Side 100000, for the Product 4330100000, will be the Area of the Base of the Pyramid, which multiply'd by 8, and the Product 34640800000 multiply'd by 13608 a third of the Height found 40824, will give 471392006400000 for the Solidity of the Octaedrum proposed.

The Solidity may with greater ease and exactness be found, by multiplying the Diameter found, or Secant of 45 Degrees 141421 by 10000000000 the Square of the whole Sine or Side 100000, for the third part 471403333333333 of the Product 1414210000000000 will be the exact Solidity of the Polyedrum proposed. From whence one may easily conclude, that to find the Solidity of an Octaedrum, whose Side is any other Number than 100000, for instance 80, you must multiply 512000 the Cube of the Side 80 by 4714, and divide the Product 241356000 by 100000, and the Quotient will give 241356$\frac{4}{5}$ for the Solidity sought.

Secondly, To find the Height of one of the Pyramids equal to the Dodecaedrum, one of whose Sides is supposed to be 100000 parts, as the Radius is in the Tables or Sines, in regard of which 85065 half the Secant 170130 of an Arc of 54 Degrees, is the Radius of a Circle circumscrib'd about a Pentagon, that serves for a Base to the Dodecaedrum, subtract 7236054225 the Square of that Radius 85065, from the Square of the Radius of the Dodecaedrum, that is from 19625400812 the triple of 654133604 the Square of 80902, the Sine of the same Arc of 54 Degrees, and the Square Root of the remainder 12399346587 will give 111352 for the Height of one of the twelve equal Pyramids, that the Dodecaedrum is composed of, by which multiply the Product 6881900000 of the Radius or Side 100000 by 688190 the quintuple of the Tangent 137638, of the same Arc of 54 Degrees, and you will have 766313328800000 for the Solidity of the Dodecaedrum proposed. Thence 'tis easie to conclude, that to find the Solidity of a Dodecaedrum whose Side is any other Number besides 10000, as 80, you must multiply 512000 the Cube of 80, by 7663, and divide the Product 3923456000 by 1000, and the Quotient will give 3923456 for the Solidity sought.

But not to insist any longer on those regular Bodies, that seldom are used, I shall finish this Problem, by telling you, that to find the Solidity of an Icosaedrum, the Side being given, for instance 8 Feet, you must multiply 512 the Cube of the Side 8 by 218,

and divide the Product 111616 by 100, and the Quotient will give 1116 Cubic Feet, 276 Cubic Inches for the Solidity of an Icosaedrum, whose Side is 8 Feet.

PROBLEM XVI.

To find the Solidity of an irregular Body.

THE Solution of the 5 *Probl.* and some others, make it evident, that to find the Solidity of an irregular Body, you must reduce it, if you can, into Prisms, Semiprisms, and other Bodies, that are measurable by the help of the preceeding Problems: For the Solidities of all those Bodies added together, will give the Solidity of the Body proposed.

But if the Body proposed be too big, and so irregular, as that you can't easily reduce it into others, whose Solid Content may be found by the foregoing Problems, you must have a Vessel in the form of a Prism, whose Base is exactly known, and pour some Water into it, and then sink the Body proposed; by that means a Prism of Water equal in Bulk to the Body will ascend, multiply the Base therefore by the Height of the Water that ascended, and you will have the Solidity of the Prism of Water, and consequently of the Body proposed.

SCHOLIUM.

Perhaps the Vessel is not large enough to contain the Body proposed, in this case you must get a less Body of the same Matter, and find its Solidity, and by that you may find that of the Body proposed tho' greater, by weighing the two Bodies exactly, their Solidities being as their Weights, because supposed to be of the same Homogeneous Matter.

PROBLEM XVII.

To find the Solidity of an empty Body.

'TIS evident that to find the Solidity of an empty Body, you need but find the Solidity as if it were full, and ſubſtract from thence the Capacity of the vacuity for the remainder, which is the Solidity of the Body propoſed.

SCHOLIUM.

The finding of the Solid Content of Similar Bodies, ſerves principally to find the Solidity of ſeveral Walls join'd together, that is, that make an Incloſure, which we have ſpoke of in *Probl.* 5. and finding the Capacity of a Vacuity ſerves for Ditches, that we have ſpoke of, and Pits, and Caves; but there are particular Methods for meaſuring theſe ſorts of Bodies, and ſeveral others, as Vaults, Arches, Stair-caſes, *&c.* that we ſhall not explain here leſt we deviate from our Subject, they not being exact, Notwithſtanding I ſhall ſay ſomething concerning the finding the Solid Content of Casks that are of frequent Uſe.

PROBLEM XVIII.

To Gauge a Cask.

IF all Casks were Similar, then the Solidity of one being known, that of the reſt might eaſily be found, becauſe ſimilar Bodies are as the Cubes of their Homologous Sides. But all not being made after the ſame manner, nor any certain Rule, nor ſingle certain Method of Gauging them can be eſtabliſh'd, tho' they have circular Bottoms, and commonly equal ones, but the Staves that form the Sides are ſometimes flat, ſometimes Convex.

When the Staves are flat, the Cask may be conſider'd as an aggregate of two Fruſtums of a Cone, that may be meaſur'd by *Probl.* 4. and if the oppoſite Heads are equal Circles, as they commonly are, the Content of the Cask is found by multiplying the exceſs of the Square of the Sum of the two Diameters BD, AE, above their Product, by the Height IK of the Cask, and multiplying the Product by 157 for a ſecond Product, whoſe ſixth part muſt be divided by 100. *Pl.* XXIV. *Fig.* 194.

Pl. XXIV. *Fig.* 194. Thus if the great Diameter AE be 24 Inches, and the less BD, or HF 18, their Product will be 432, and their Sum 42, whose Square 1764 multiply'd by the Length IK, suppose 36 Inches, produces 47952, which multiply'd by 157, gives a second Product 7528464, whose Sixth part 1254744, divided by 100, gives 12547 Cubic Inches for the Content of the Cask sought ACEG.

You may for dispatch, multiply half the Sum of the two Circles, whose Diameters are AE, BD, by IK, the Length of the Cask: And then to avoid Fractions in each Circle, multiply 900 the Sum of the Squares 576, 324, of the Diameters 24, 18, by the length IK, suppos'd to be 36 Inches, and multiply the Product 32400 by 785 for a second Product 25434000, half of which 12717000 divided by 1000, gives 12717 Cubic Inches for the Solidity sought, which you see being thus found, is something greater than it should be, which is an error for the Advantage of the Gauger.

The error will be less considerable, if instead of taking a mean Circle, you take a mean Diameter between AE, BD, that you will find to be 21 Inches, being half the Sum of AE, BD. The Area of the Circle answering this mean Diameter 21 is 346 Inches square, that multiply'd by 36, the length IK, will give 12456 Cubic Inches for the Solidity sought, less than the true one 12547; but 'tis not so much less as the foregoing 12717 is above.

When the Staves are Convex, as BAH, DEF, the Cask may be consider'd as an aggregate of the two Parts of a Spheroid, and may be measur'd by the Rules in *Probl.* 11. But this Speculation is of little Use in the Practice, where the *Capacity* is sought only in the gross, which Artists call the *Content*, in the Measures of that Place, as Gallons, Quarts, &c.

The Solidity of the Cask ACEG, being found to be 12547 Cubic Inches, 'tis easie to find how many Pints it contains, namely, by dividing the Content 12547 by 49, which is the Number of Cubic Inches nearly, that are contain'd in a *Paris* Pint, and the Quotient will give 256 Pints for the Content or Capacity of the Cask propos'd ACEG.

But to find the Capacity more exactly, you must take Notice that a *Muid* is suppos'd to contain 8 Cubic Feet, or 280 Pints, or 13824 Cubic Inches, which shews that 280 Pints are equal to 13824 Cubic Inches; wherefore, to find how many Pints

Pints there are in 12547 Inches, which is the Content of the Cask ACEG, say by the Rule of Three, if 13824 Cubic Inches make 280 Pints, how many will 12547 Cubic Inches make? So that multiplying the Solidity 12547 by 280, and dividing the Product 3513160 by 13824, or which is easier, multiplying the Solidity 12547 by 35, and dividing the Product 439145 by 1728, the Quotient will give about 254 Pints for the Capacity sought. *Pl. XXIV. Fig. 194.*

So much for the Theory, what follows is the usual Practice of Gaugers that Gage, that is, readily to find the Content of a Cask by the help of a Rod, divided on one Side into a certain Number of equal Parts, and on the other into unequal Parts. This Rod, called commonly the Gauging Rod, is made like LMNO, and divided as that is.

Having settled upon the Measure, you will use in Gauging Casks, as a Pint P, whose Figure being irregular, must be reduc'd into a regular one, filling it with Water, or some other Liquor, and pouring it out into another regular Vessel, as the Concave Cylinder QRST; and supposing the Liquor takes up the Cylindric Prism QVXT, whose Base is a Circle, having for a Diameter a Line equal to RS the internal Diameter of the Concave Cylinder; transfer the Height QV, or TX of this Cylindric Prism upon the Gage, on one of its Faces from the Point Y, that answers to the Point O, towards L, as many times as it will bear, and mark the Places with the Figures 1, 2, 3, 4, 5, &c. and this Face call'd *the Side of the equal Parts.*

Then mark on the opposite Face of the same Gauging Rod, that I suppose Square, and about Four or Five Foot long, the Diameters of a Base double, triple, quadruple, &c. that of the Cylindric Prism QVXT, which may be done in this manner.

Having drawn the indefinite Line S9 perpendicular to the Diameter RS, take in that Line S9 the part S, 1 equal to RS, and the part S2, that *by* 47. 1. will be the Diameter of a double Base, equal to R1, and after the same manner the part S3, which will be the Diameter of a triple Base equal to R2, and so on: And transfer the unequal Divisions of the Line S9 upon the Gauging Rod from L towards M, by Points, to which annex the Figures 1, 2, 3, 4, &c. and this Face thus unequally divided, is call'd *the Side of unequal Parts*, which may be also divided after this manner.

Divide

Pl. XXIV. Fig. 194. Divide the Diameter RS, into a certain Number of pretty small equal Parts, as into 100, whose Square 10000 doubled, tripled, quadrupled, &c. gives 20000, 30000, 40000, &c. and the square Roots give 141, 173, 200, &c. for the quantity of the Diameters of the Bases, the doubles, triples, quadruples, &c. Having made then as before, the Line L1 equal to RS, or 100 Parts, make the Line S2 of 141 Parts, the Line S3 of 173 Parts, the Line S4 200 Parts, and so on, till you compleat the Division.

The Gauging Rod being divided after this manner, is to be used thus. Apply the Rod along the Cask, whose Content you are to find, so that the Point O of the Hook NO touch one of the Heads, for finding on the Side of the equal Parts the Distance IK of the two Heads, allowing something for their thickness: We will suppose the Distance IK 14 equal Parts or Heights. Then if the Cask be empty, apply the Rod to the Bung, to find on the Side of the unequal Parts the Quantity of the great Diameter AE, that we will suppose 24 unequal Parts, or Diameters: Find after the same manner the quantity of the less Diameter BD, or HF, which suppose to be 16 unequal Parts; then add these two Diameters thus found together 24, 16, and multiply their Sum 40 by the Length IK, that we supposed 14 equal Parts, and the Product 560, halved gives 280 Pints for the Content of the Cask proposed ACEG.

Here the two little Diameters BD, HF, are suppos'd equal, but if they are unequal, take half the Sum for one of the two Diameters, and work as has been taught.

The Rod may be made otherwise, and easier thus. Because a Cylinder, that has for its Height 3 Feet, 3 Inches $\frac{6}{12}$, and as much consequently for the Diameter of its Base, contains a thousand *Paris* Pints, make the Rod LM 3 Feet, 3 Inches, $\frac{6}{12}$ long, and divide it first into ten equal Parts, each of which will be the Diameter, and Height of a Cylinder, containing a Pint, and divide, for greater exactness, each of these ten Parts, into ten other less Parts, and each of these new Parts will be the Height, and Diameter of a Cylinder, containing the thousandth part of a Pint. Lastly, annex to these Divisions the Figures from 5 to 5, or from 10 to 10 and the Rod will be finish'd, and is thus to be used. Having found as before, how many of the little Parts of the Rod are contain'd in the Length of

of the Veſſel, and the Diameters at the Heads and Bung; add as before, the great Diameter AE to one of the little ones HF, BD, if they are equal; otherwiſe inſtead of one take half their Sum, as was ſaid before, and multiply it by it ſelf for its Square, which multiply'd by the Length IK of the Cask within, and divide the Product by 4000, for the number of Pints contain'd in the Veſſel ſuppoſed. *Pl. XXIV. Fig.* 194.

This way gives the Content of the Cask leſs than Truth; but if you would have it more exact, do thus; Subtract from the great Diameter AE, one of the leſs HF, BD, if they are equal, or half their Sum if they be unequal, and multiply the Sum by it ſelf for its Square; that multiply'd by IK the Length of the Cask within, and the Product divided by 1000, gives as before, the Number of Pints ſought.

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With an APPENDIX, concerning that Manner of *FORTIFYING* which is Truly Mr. *Vauban*'s: Never before in *Engliſh*.

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TO

The HONOURABLE

JOHN RICHMOND,

alias

WEBB, Esq;

One of the *Lieutenants-General* of Her MAJESTY's Forces in the *Low-Countries*. Colonel of the QUEEN's Regiment of Foot: And Governour of the *Isle of Wight*.

SIR,

A Treatise upon any Part of Military Affairs can never be Recommended to the World with so much Advantage, as when it is Honour'd with a Name so Great in War, as that of *General* WEBB. And an Author appearing under so Powerful a Protector is as secure of Success, as

as a Souldier marching under ſo Invincible a Commander. Upon this account it is, that I preſume to offer the following Tranſlation to You, not as worthy of your Acceptance, but as wanting your Patronage.

The Subject of this Book is *Fortification*, an Art in it ſelf highly Valuable, and which no One better underſtands than You; tho' your Courage has always made it, both Uſeleſs to your Enemies, and Unneceſſary to Your-ſelf.

I dare not attempt the Character of ſo Great a Hero, whoſe Actions ſhall be Recorded by the *Britiſh* and *French* Annals, and whoſe Fame ſhall be Celebrated as long as *Liſle* and *Winnendale* have Names in Hiſtory. I am,

SIR,

Your moſt Obedient

Humble Servant,

J. T. DESAGULIERS.

THE BOOKSELLER TO THE READER.

MR. *Ozanam*'s Treatiſes, eſpecially his *Courſe* of the *Mathematicks*, have been ſo well Received, that I have, by the Direction and Approbation of ſeveral Ingenious Gentlemen in that Science, had them Done into *Engliſh* from the *Paris* Edition. I muſt own, that a great many Books of *Fortification* have been publiſh'd of late; but as what our Author has wrote on divers Parts of the Mathematicks, has not been leſs Valued for being taken out of other Treatiſes, becauſe of his Own large Additions and Emendations, I am perſwaded that This will meet with the like Succeſs. For, beſides the Author's known Merit, all good Judges will grant, that, how often ſoever this Matter has been treated of, during the Laſt and Theſe preſent Wars; yet, it is not ſo Eaſy as ſome People imagine, to Write well concerning *Fortification*, and eſpecially that of *Irregular Places:* There being ſo many different Caſes, by reaſon of the Situation and Quality of the Ground, which it is hard thoroughly to Examine, and reduce to Particular Rules. Without doubt this Art may be daily improv'd; and

and Mr. *Ozanam*'s Notions, added to thoſe of other Authors, muſt needs contribute a great deal to its Perfection. But if that Part of *Fortification* which teaches to *Defend Places* is difficult, that which teaches to *Attack them* is no leſs ſo. And Maſters of this Art affirm, That this Laſt Part, however Important, has been but lightly handled. This Mr. *Ozanam* well knew, and therefore dwelt longer upon it than any One before Him ever did.

Here are very Uſeful Inſtructions, which may be Acceptable to a great many; not only Military Men, but ſuch as would learn to ſpeak Judiciouſly upon this Subject, and underſtand what is to be met with in Publick Papers, or ſaid in Converſation about it.

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A TREATISE OF *FORTIFICATION.*

FORTIFICATION, which is alſo called *Military Architecture*, is an Art which with the help of Geometry and Arithmetick, teaches how to *Fortify*; that is, to encloſe a Place, or City, with ſuch Works, as may ſecure it from, or enable it to reſiſt, the Invaſions of the Enemy; which is done by drawing a continued Line, inclin'd to the Angles of the Polygon, that includes the Place to be Fortify'd: This Line is call'd the *firſt Draught*, or *Maſter-line* (in French) *Premier Trait*, and *Ligne de Cordon*, upon which are laid the Foundations of that encloſure of Walls, which encompaſſeth the *Fortreſs*, or Fortify'd Place, and is made up of Curtains and Baſtions, ſo built and diſpos'd, that one may eaſily Defend one's ſelf therein, when Attack'd; that from thence the Enemy may be ſeen and repell'd, which Way ſoever he Approaches, and be hinder'd from making himſelf Maſter of the Place; and that a ſmall Number of Men may with Advantage reſiſt a greater that wou'd force 'em.

N.B. *In this Tranſlation, the French Terms will be retain'd, as they are in Mr.* Harris's Lexicon Technicum, *whenever he ſpeaks of* Fortification: *But ſuch as are leaſt obvious ſhall be explain'd by a Marginal Note, the Firſt Time they are Mention'd.*

Because this Polygon may be regular or irregular; Fortification also may be regular and irregular: Both which depend upon the following Maxims, not to be well understood till we have first explain'd some Terms peculiar to Fortification. Wherefore we shall begin by Definitions.

An Explanation of the Ichnographical Terms of a Fortified Place.

Plate 2. *Fig.* 6. THE 6th *Figure* represents a Demi-Hexagon Fortified, where the Polygon ABCD is call'd the *inward Polygon*, and the Polygon GKVX the *outward Polygon*. Every side of the inward Polygon is call'd inward side, as AB; and every side of the outward Polygon, as GK, is call'd outward side. When the inward and outward Polygons are regular, as here in the Figure, they have the same *Center*, as S, which is the same with the Center of the circumscrib'd Circle: And then we shall call *little Radius*, the Radius of the Circle circumscrib'd about the inward Polygon, as SA; and *great Radius*, the Radius of the Circle circumscrib'd about the outward Polygon, as SG.

To *Fortify outwards*, is when we Fortify upon Paper, beginning by the inward side; and when we Fortify upon Paper, beginning by the outward side, that way is call'd *Fortifying inwards*.

The Figure AEFG represents the Plan of a *Demi-Bastion*, which is compounded of the *Demi-Gorge* AE, of the *Capital* AG, of the Flank EF, and of the *Face* FG, which is also call'd *Pan of the Bastion*. This Flank EF, or HI, is call'd *Right-flank*, when it is Perpendicular to the *Curtain* EH; and *Oblique-flank*, when it makes an obtuse Angle with the Curtain EF; which is now practis'd by all Engineers, for such reasons as you will see in *Maxim* 7.

The Figure HIKLMPON is a whole *Bastion*, call'd also *Bulwark*, which is made up of such parts as the foregoing; and besides, a *Gorge*, or *Gorge-line*, which is here represented by the Line HN, drawn from the end

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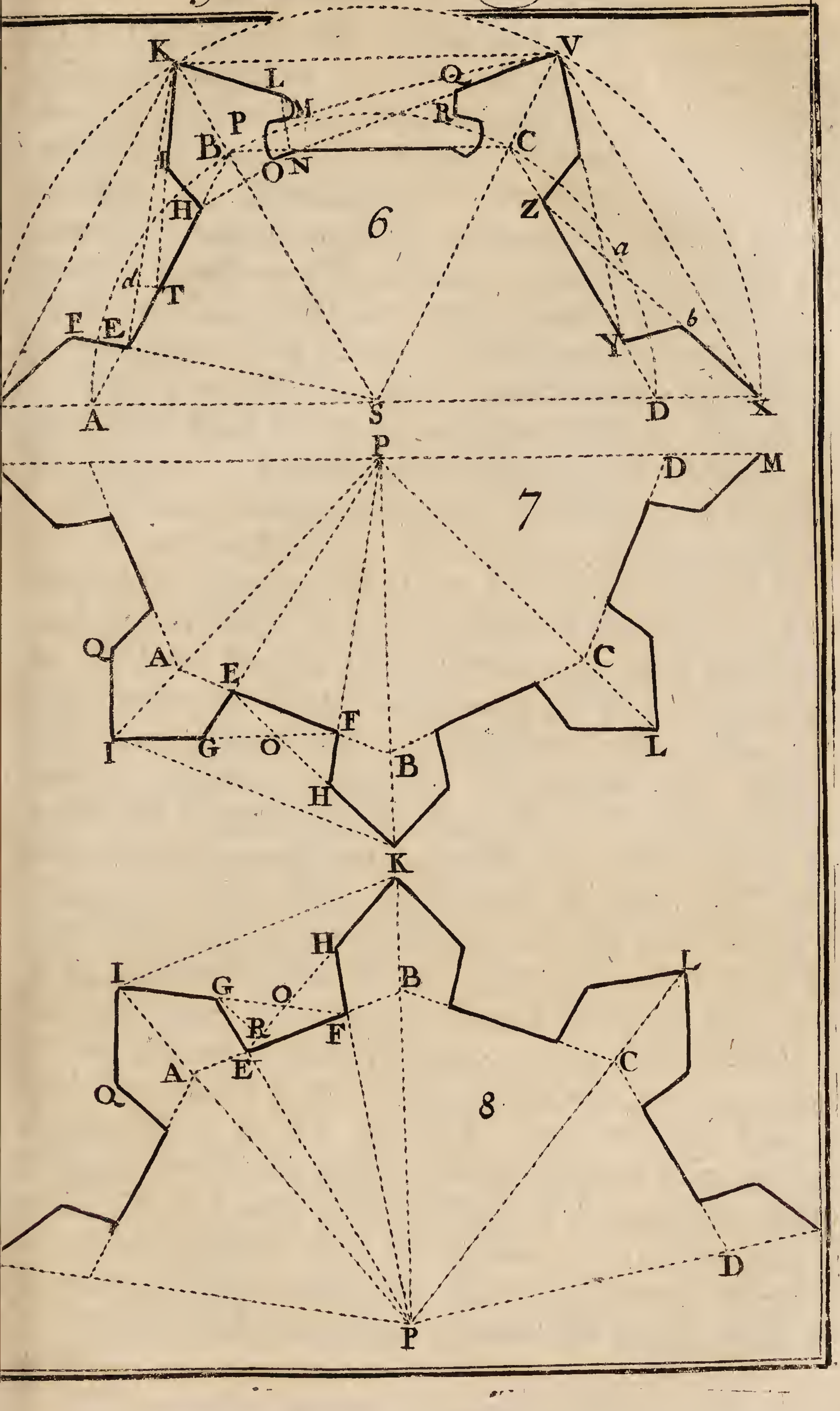
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H of the Flank HI, to the end N of the other Flank NL, which Flank is here cut by the *Epaule*, Shoulder, or *Orillon* LM, which is call'd a round *Orillon*, because it is round; and a *square Orillon* when it is square, as QR. It covers part of the Flank, which for that reason is taken inwards; whence it is call'd *retir'd* or *cover'd Flank*, as also *Casemate*, and *Place-Basse*, as OP: And the Line NO, or MP, is call'd *Retirade*, or *Retrenchment of the Flank*, or *Platform of the Casemate*. *Plate* 2. *Fig.* 6.

The Line EF has been call'd *Flank*, because it defends the Face IK of the opposite Bastion; for to *Flank*, in Terms of Fortification, signifies to Defend. It is call'd *Flank fichant*, because a Musketeer, at its end E, can shoot (or *ficher*, as the French call it,) against the Face IK, whence the Line EK, drawn from E, the end of the Flank to K, the *Point* of the oppos'd Bastion is call'd the *Fichant-line of Defence*, or only the *Fichant-line*, and also the *Great-line of Defence*; the *Rasant-line*, or *Rasant-line of Defence*, and also *Little-line of Defence*, being the Right-line KT, which is made by lengthning the Face IK to T, from whence a Musketeer cannot shoot against, but only rase or shoot along the Face IK: And then the part ET of the Curtain is call'd *Second-flank*, and also *Fire in the Curtain*; because from all the parts of that Second-flank ET, one may shoot or fire against the Face IK. There is not always a Second-flank, as in the Curtain YZ, and then there is no Fichant-line neither, but only a Rasant-line, as YV and ZX; and in that Case the Flank is call'd the *Rasant-flank*, because from its end Y or Z, one can only rase the Face of the opposite Bastion. Hence it appears, that the Flanks are as the Arms of a Place Fortified; this part also the Enemy endeavours to destroy first, in order to pass the Ditch with more ease.

We have said in our Geometry, that the Angle ASB is call'd the *Angle of the Center*, and that the Angle ABC is call'd the *Angle of the Polygon*, which may be call'd the *Angle of the inward Polygon*, to distinguish it from

Plate 2. Fig. 6. the Angle GKV, which may be call'd the *Angle of the outward Polygon:* And we ſhall here call the Angle FEH, which the Flank EF makes with the Curtain EH, *Angle of the Flank*, and the Angle EFG, which the ſame Flank EF makes with the Face FG, is call'd *Angle of the Epaule*, or ſimply the *Epaule*, whence comes the Word *Epauler*, which is to cover the Flank, or the Shoulder of the Beſieged, ſo that he may not be diſcover'd on that ſide. Thus one may ſee that thoſe which defend the Place in the Flank EF are Cover'd and Epauled, or Shoulder'd by the Face FG, which in ſuch a Caſe is call'd *Epaulement:* The Point, or Angle of the inward Polygon, is call'd the *Center of the Baſtion.*

The ſame Flank EF being lengthned, makes in the Center S, with the Radius AS, the Angle ASE, which we ſhall call the *Flank-forming Angle.* The Angle IKL, which the Two Faces KI, KL, make at the Point of their Baſtion, is call'd the *Angle of the Baſtion*, and alſo the *Flank'd-angle*, becauſe it is flank'd and defended by the Two oppoſite Flanks. The Right-line drawn from Plate 2. Fig. 6. one Flank'd-angle to another, and which marks the Diſtance of 'em, as GK, is call'd the *Baſe-line*, the ſame as we have before call'd outward ſide; and the part EB, made up of the Curtain EH, and the Demi-gorge HB, is call'd the *Lengthned-curtain.* We have alſo ſaid in our Geometry, that a *Re-entrant angle* is that which retires inwards, and we call ſuch an Angle, in Terms of Fortification, a *Tenaille-angle*, or only *Tenaille*; and ſuch of 'em as are made by the meeting of Two Raſant-lines and are obtuſe, are call'd *Flanking angles*, as *a*, the Two Lines of which *a*V and *a*X are alſo call'd *Tenailles:* And it is call'd *outward Flanking-angle*, to diſtinguiſh it from the *inward Flanking-angle*, made by the concurrence of the Flank and the Raſant-line, when the Flank is Raſant; that is, when there is no Second-flank, as UY*b*; or when there is a Second-flank, by the concurrence of the raſant Line, and the Second-flank, as KTE.

The

The said Rasant-line makes with the Curtain an acute Angle, which is call'd the * *Angle diminué*, as KTH, which is equal to the Angle GKI, made by the Face KI, and the Base-line GK, because it is Parallel to the inward side AB, at least in a Regular Polygon; And the Capital AG with the Demi-gorge AE, make the Angle GAE, which is call'd the *Gorge-angle*, and is always equal to as many Degrees as half the Angle of the Polygon wants of 180; which half Angle of the Polygon (*viz.*) SAB, is call'd the *Base-angle*, and is equal to the Angle SGK, by reason of the Two Parallels AB, GK; at least when the Polygon is Regular. The other Terms we shall explain in their proper Places.

Plate 2. Fig. 6.

* or, *Angle diminish'd.*

General Maxims of Fortification.

I.

*There ought to be no Place in the whole * Enceinte of the Fortress, but what is seen, or flank'd by the Besieged.*

* *Enclosure, Compass, or Circumference.*

WE shall begin by this Fundamental Maxim, because 'tis the universal end of Fortification, in order to fight the Enemy with Advantage; it being certain, that if there was any part of the Enceinte of the Place, which was not well flank'd, the Enemy cou'd there be cover'd, and take the Place so much the more easily, as he shou'd meet with less Resistance.

II.

The length of the great Line of Defence ought rather to be within Musket-shot, than within Cannon-shot.

THO' in the defence of Places both Muskets and Cannons are us'd, nevertheless Reason and Experience teaches us to proportion the great Line of Defence rather to the reach of a Musket, than that of a Cannon; because a Cannon requires a great deal of trouble to be Charg'd and Levell'd, which hinders it from being often fir'd; consumes a vast deal of Ammunition; makes

most commonly random or uncertain ſhots, and as it is very much expos'd to the Enemies Battery, may by one ſhot of the Enemy be render'd uſeleſs; may take wind or burſt, or be diſmounted; and the time loſt, in re-placing it, or putting a freſh one in its room, is very precious, eſpecially in an Aſſault: now this wont happen to Musketeers, who may ſhoot an hundred, nay two hundred ſhots, whilſt one Cannon is firing; beſides the defence by Musketeers is cheaper, as well as more certain and eaſy; and this way both Cannons and Muskets may be uſeful, whereas if it were otherwiſe we ſhou'd be depriv'd of the Musket, which is the better of the two.

III.

The great Line of Defence ought to be in length about 120 * *Toiſes.*

* *A Meaſure of 6 feet, a* Fathom.

SINCE the great Line of defence ought, by the foregoing Maxim, to be proportion'd to the reach of Musket-ſhot, we muſt now determine the length of that Line, which is as a Standard or Rule for all the others, that all theſe lines may be of a length fit and convenient to defend the main Body of the Place: and this cannot be better done, than by determining how far a Musket will kill, which at fartheſt is but 150 Toiſes. Therefore to Defend from the Flank the Face of the oppoſite Baſtion, and repel the Enemy that attacks the Angle or Point of it, the Line of Defence ought not to be above 150 Toiſes, tho' it ſhou'd be as long as poſſible, to have the feweſt Baſtions one can in the ſame Enceinte, by reaſon of the great Expences that they wou'd put the Beſieged to. Nevertheleſs, to have a more vigorous reſiſtance, we will ſuppoſe the Line of Defence of 120 Toiſes, as being a convenient diſtance for practice; becauſe we are free to chooſe any number leſs than 150, and more than 100; for were the Line but 100 Toiſes, the Baſtions wou'd be too near and too ſmall; which wou'd increaſe the number, and leſſen the ſtrength of the Baſtions in the ſame compaſs.

IV.

IV.

The Line of Defence must end in the Angle of the Flank-rasant, when there is no Second-flank.

IT must be so, that the Face of the opposite Bastion may be defended from the whole Flank; for when there is no Second-flank, if that Line of defence, that is, the Face lengthen'd, shou'd cut the Rasant-flank, part of that Flank, namely, that between the Angle of the Flank and the Rasant-line, wou'd be useless in defending the Face of the opposite Bastion, and so the length of that Flank wou'd increase the charge to little purpose; and there can be no greater fault, than not to employ the whole length of the Flank in defending the Face of the opposite Bastion, which being the most expos'd to the Enemy, ought to be the best defended, and the Flank only can defend it, since a Line cannot Flank it self.

V.

The greatest Flanks are the best, as also the greatest Demi-gorges, and the greatest Second Flanks.

IT is plain from the foregoing Maxim, that the greater the Flanks are, the fitter they are for defence; because they can hold more Defenders, and when they are to be cover'd, they may have stronger Orillons, and Casemates fit to receive more pieces of Cannon, without having occasion for any more than two Batteries, a high one, and a low one. It is plain also, that the greatest Demi-gorges are the best, because they shorten the Line of Defence, are capable of containing more Batteries, and that jointly with the Flank they make the Bastion larger and fitter for Retrenchments, which the besieg'd may have ready as soon as the Enemy's Mine has been sprung. And lastly 'tis plain, the more fire is in the Curtain, the better is its defence, at least when a Second-flank may be had without any prejudice to the other parts; because 'tis always so much fire got, whose obliqueness also is the less, the more

ſides there are in the Polygon: and becauſe generally the length of this Second-flank increaſeth proportionably with the Number of the Baſtions; that is, the more open the Angle of the Polygon is, when the Flank-angle is not to be obtuſe; it is eaſy to conclude that the *greater the Polygons are the better they are, becauſe even according to our method of Fortifying, which we ſhall explain in *Max.*8. they may have both greater Flanks, and greater Demi-gorges.

* *That is, The more ſides they have.*

This makes me wonder why ſeveral Authors ſhou'd neglect a Second-flank, which to me ſeems very advantageous, for the reaſons already given; beſides when there is no Second-flank, it ſeems almoſt impoſſible in great Polygons, where the *Angle of the Figure*, that is, the Angle of the Polygon, which is alſo call'd the *Angle of the Circumference,* is very great or open, to hinder the Flank'd-angle from becoming obtuſe, which I think a conſiderable fault; becauſe when the Flank'd-angle is a right, or ſomewhat under a Right-angle, the Tenaille is the cloſer, the Raſant-line the ſhorter, the fire of the Curtain multiplied, and the Capital of the Baſtion increas'd, which renders it fitter for Retrenchments, and expoſes the Faces of the Baſtion leſs to the Enemy: moreover a right Flank'd-angle has all the ſtrength poſſible; for at its point or vertex it oppoſes all its ſolidity to right * Batteries.

* *That is, ſuch Batteries as ſhoot Perpendicular to the Face.*

We ſhall add to all theſe reaſons this following, which to me ſeems a very ſtrong one, (*viz.*) that when there is a Second-flank, one may from the *Flank-fichant* ſhoot more eaſily and farther into a breach which ſhou'd be made in the Face of the oppoſite Baſtion, and ſo be very troubleſome to the Beſiegers when they have a mind to lodge themſelves in the breach, and one may hurt the Enemy more in their paſſing the Ditch, becauſe from a *Flank-fichant* one may diſcover more of the Ditch than from a *Flank-raſant.*

Plate 2. *Fig* 8. To judge how fit a *Flank-raſant* as GE, is to defend the Ditch, and the Face of the oppoſite Baſtion KH, we ought not to conſider its length, but that of the line

GR,

GR, which is drawn from the Angle of the *Epaule* G, Perpendicular to the *Rasant-line* EK; it being certain, that the *Flank-rasant* EG, can hold no more Musketeers to shoot obliquely, than the Perpendicular GR can hold that will shoot streight forwards.

Likewise to judge how capable a Second-flank, as ET, is to defend the opposite Face KI, and to oppose the passage of the Ditch, its length must not be consider'd, by reason of its too great obliquity, but the length of the line T*d*, which is drawn from T, the end of that Second-flank, Perpendicular to the rasant Line KT, for the reason before alledg'd. *Fig. 6.*

VI.

The Flank'd-Angle ought to be of 70 degrees at least.

THE *Dutch* suffer it to be of 60 degrees, because in the Square they cannot make it greater, according to their manner of Fortifying, which is to add 15 degrees to the Base-angle for the angle of the Bastion, as we shall say in its place: but, as by our Method, which we shall explain in *Max.* 8. we can make the Flank'd-angle of about 70 degrees in the Square, which is the first Figure that can be useful, the Triangle being of too small a Compass, we have fix'd the aperture of the Flank'd-angle to that number of 70 degrees, that it may the better be able to resist the effort of Batteries, if the Enemy had a mind to blunt and batter its Point. Whence it is easy to conclude, that no angle of a Polygon, less than a right one, can be well Fortified, and therefore that a Triangle is always imperfect.

VII.

The Flank ought to have one part cover'd.

AS the Flank is intended only for the defence of the Place, and is the chief part that fights for its safety, so we ought to neglect nothing that may contribute to its preservation, and hinder it from being destroy'd. Tho' the *Dutch* make no Orillons to cover their

their Flanks, thinking perhaps that ſimple Flanks diſcover'd the Field better, and that they were enough cover'd by the *Outworks*, or *detach'd Pieces*, which they generally make before the Curtains: yet Experience has ſhew'd in many Sieges, that the defence of a ſimple Flank was quickly ruin'd, and that ſuch a Place has been forc'd to capitulate, as ſoon as a Lodgment was made on the Counterſcarp.

An Orillon, and eſpecially a round one, is very uſeful, becauſe it moleſts the Enemy very much whilſt he is paſſing the Ditch, and alſo the Miner, what place ſoever he works in; and can cover at leaſt Two pieces of Cannon, which tho' they can't be diſmounted by the Enemy's Cannon, yet diſcover the Miner, the Breach, and a good part of the Ditch. Since there are ſo many Advantages, we ought to prefer a cover'd Flank to a ſimple Flank, which is the more expos'd to the Enemy the greater it is, as it wou'd be in our Method, becauſe its Front towards the Enemy is greater.

VIII.

The Flank ought neither to be Perpendicular to the Curtain, nor to the Line of Defence, nor to the Face of the Baſtion.

E*Rrard* has drawn the Flank perpendicular to the Face of the Baſtion, the better to cover it without need of Orillons; but by covering it ſo much, he made it too ſmall and uſeleſs, the Gorge too narrow, the *Embraſures too oblique, and the Ditch not enough defended. The Chevalier *de Ville*, and ſeveral others have made it Perpendicular to the Curtain, the better to defend from thence the Gates and Bridges which are commonly made about the middle of the Curtain; but it has been obſerv'd, that in Figures of many ſides the angles of the Merlons oppos'd to the Batteries were yet too acute, and the Ditch ill defended. Laſtly, Count *Pagan* has made the Flank Perpendicular to the raſant Line, that is, to the lengthned Face of the oppoſite Baſtion, the better to defend it, but a Flank thus drawn, to

*Port-holes.

to me seems too oblique, too small, and too much expos'd to the Enemy's Cannon, especially when it is not cover'd. Wherefore we shall take a Medium between the two last, as the most proper, by drawing the Flank from the Center of the Place.

Our way of Fortifying therefore is, first to draw the Flanks from the Center of the Place, which seems impossible to be done in irregular Figures that have no Center; but yet we shall find means to Fortify irregular places by that Rule: and we shou'd not adhere so firmly to this way, if it had not very considerable advantages.

Tho' 'tis impossible to establish any general Rule of Fortifying, but what will be defective at length, by reason of the difference of the Angles in different Polygons; neverthelefs, we shall here establish a Rule for a regular Polygon, which will be as general as possible, by assigning to the Flanks and Demi-gorges such proportions as may suit with the nature of every Polygon: giving the inward side of the Polygon 120 Toises, that the great Line of Defence may be about that length, according to *Max.* 3. the Demi-gorge, as many Toises more than 20, as the Polygon to be Fortify'd has sides; so that in the square the Demi-gorge will be of 24 Toises, of 25 in the Pentagon, of 26 in the Hexagon, and so on as far as the Decagon, in which the Demi-gorge being of 30 Toises is to continue so in greater Polygons: and the Flank an equal number of Toises to four times the number of the sides of the propos'd Polygon; that is, 16 Toises in the Square, 20 in the Pentagon, 24 in the Hexagon, and so on as far as the Decagon, where the Flank being of 40 Toises, remains of that bignefs in greater Polygons; that is, in such as have more sides.

The reason of this increase of Demi-gorges and Flanks is drawn from the nature of Polygons, which as the number of their sides increases, have the Angles more open, which consequently must be capable of receiving greater Bastions; and least they shou'd be vastly large, we have fix'd the Demi-gorges at 30 Toises in the Decagon, and the Flanks at 40; and if any one thinks

thinks 'em too large, they make 'em less than that. We shall in the beginning of the 3d *Part*, give another more general Method by which the Flanks are not made so great as here; but then the Demi-gorges become greater, still increasing as the Polygons; without being oblig'd to fix 'em at such a bigness, as in the foregoing Method: for when the Polygons become great, the Demi-gorges increase so little according to that Method, that they cannot exceed 36 Toises upon an inward side of 120 Toises, in a Polygon of 30 sides.

IX.

A Fortress ought to be equally strong every where; and to command all the places round about.

THE first part of this Maxim is self-evident; because an inequality of strength shews the Enemy the weakest side of the place, which consequently he must attack: the Second is also evident, because otherwise the Enemy wou'd cover his designs, his approaches wou'd be favour'd, and he might with ease batter and ruin the place.

X.

Such Works as are nearest to the Center of the Place ought always to be higher than those that are farther off.

FOR thus such Works as are farthest off, lowest, and most expos'd to the Enemy, who may easily take 'em, will be the better discover'd from the nearest and highest; from whence consequently they may be defended, and the Enemy repell'd, being hindred from covering himself in such pieces as he has taken.

XI.

The foregoing Maxims must be made to agree with one another, as much as possible.

I Say as much as possible, because when we wou'd too strictly observe one Maxim, we often deviate from another more considerable: for example, if we have a mind

mind to increaſe the Second-flank, either the Flank or Flank'd-Angle will be diminiſh'd, and the Face of the Baſtion increas'd to no purpoſe: if we have a mind to have the Angle of the Baſtion very much open, the good defence that it may receive from the Curtain is taken from it, and it becomes more expos'd to the Enemy's Cannon: if we wou'd have a Gorge ſomewhat great, the Face of the Baſtion becomes alſo great which is a fault; becauſe the longer a Face is, the weaker it is, the Enemy attacking it from a larger Front; and they want only ſuch a length as may make 'em uſeful for the defence of Out-works, when there are any. In fine, there is every where advantage and diſadvantage; and our reaſon is to judge whether it be more advantageous to adhere to one Maxim, than it is prejudicial to recede from another; moderating the thing in ſuch manner, that the Fortification may not greatly differ from the principal Maxims.

THE

The First Part.

OF
Regular Fortification.

A Place is ſeldom regular of it ſelf; but when there is Field-room about it, from an irregular one may eaſily make it a regular one, which ought to be done as often as one can conveniently, that the Fortreſs may be equally Defended in every Place. One might by *Max.* 3. give the ſide of the regular Figure 150 Toiſes, to the intent that the line of Defence ſhou'd be about that length, and that ſo the Place ſhou'd have the feweſt Baſtions that may be, to avoid expence; but if we wou'd have a more vigorous Defence, and fire upon the Enemy from a leſs diſtance and ſurer, we muſt give only 120 Toiſes to the inward ſide of the Polygon.

To find out the number of the ſides of the regular Figure, that the irregular one propos'd, or ſuch a regular one, whoſe ſides are either too long or too ſhort, may be turn'd into; we muſt divide the whole Circumference or Perimeter of the Figure to be reduc'd by 120, if the great Line of Defence is to be of about 120 Toiſes, and by 140, if we wou'd give the great Line of Defence 140 Toiſes; and the Quotient will be the number of the ſides of that regular Figure, to which the given Figure may be reduc'd.

Plate 1. *Fig.* 1. This reduction is eaſy to put in Practice, when the Place to be Fortified is not extremely Irregular, as the irregular Hexagon ABCDEF, whoſe ſides are ſuppos'd

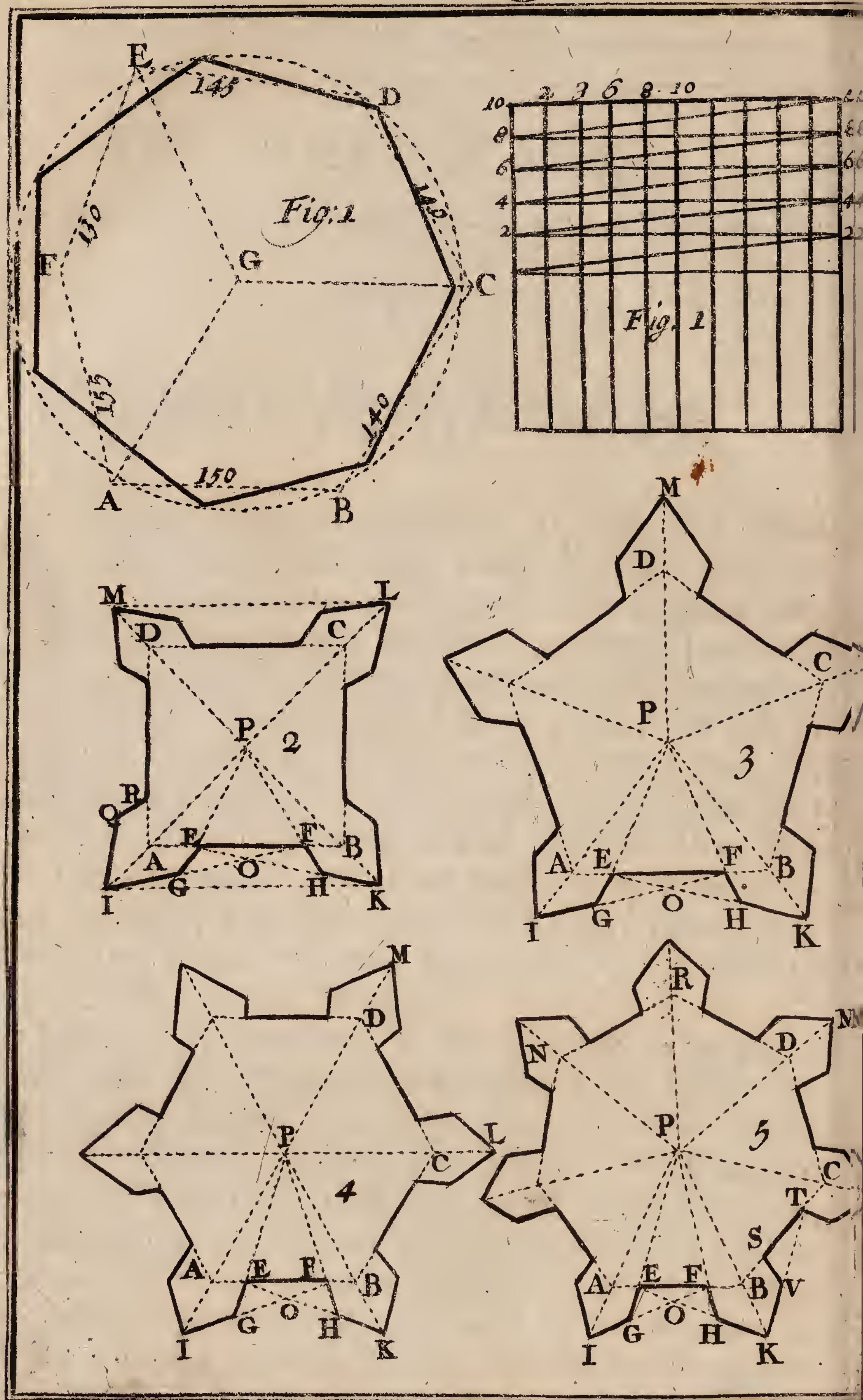


Fig:1
Fig. 1
2
3
4
5

to be of as many Toiſes as you ſee mark'd within the Figure; for tho' of its Nature it has no Center, one may eaſily find its Center by approximation, (*viz.*) by finding the center of a Circle, which paſſes thro' the Three moſt diſtant Angles of the Figure, as A, C, E, and that center G muſt be taken for the Center requir'd, without any conſiderable error; for as Fortification is but an Art, it would be ridiculous to go to obſerve in it the Rules of Geometry in their greateſt ſtrictneſs.

Having thus found the center G of the irregular Figure ABCDE, deſcribe a Circle whoſe Radius muſt be the ſame with that of the regular Figure, which has been found by dividing the Perimeter of the irregular one by 120. This Radius will be found in the following Table, which ſhews the quantity of Radii of regular Polygons from the Square to the Dodecagon, the inward ſide being ſuppos'd of 120 Toiſes; and likewiſe the quantity of the chief Lines and Angles of all the Polygons that are Fortified our way, which ſuits pretty well with the *Maxims* of good Fortification.

A Ta-

A Table of the Angles and Lines of a fortified Polygon, from the Square to the Dodecagon, the inward side being of 120 Toises.

Polygons.	IV.		V.		VI.		VII.		VIII.		IX.		X.		XI.		XII.	
Angle of the Center	90.	0.	72.	0.	60.	0.	51.	26.	45.	0.	40.	0.	36.	0.	32.	44.	30.	0.
Angle of the Polygon	90.	0.	108.	0	120.	0	128.	34.	135.	0	140.	0.	144.	0	147.	16.	150.	0.
Flank-forming Angle	14.	2.	13.	1.	11.	53.	10.	36.	10.	2.	9.	21.	8.	44.	7.	50.	7.	22.
Angle of the Flank	120.	58.	112.	59.	108.	7.	105.	7.	102.	28.	100.	39.	99.	16.	98.	32	97.	38.
Angle diminué	9.	42.	13.	19	16.	49	20.	14.	23.	46.	27.	16.	30.	43.	30.	49.	31.	16
Angle of the Epaule	130.	40.	126.	18.	124.	56.	125.	21	126.	14.	127.	55.	129.	59.	129.	21.	128.	54.
Flanking Angle	160.	36.	153.	22.	146.	22.	139.	32.	132.	28.	125.	28.	118.	34.	118.	22	117.	28
Flank'd Angle	70.	36.	81.	22.	86.	22.	88.	6.	87.	28.	85.	28.	82.	34.	85.	38.	87.	28.
Demi-gorge	24.	0.	25.	0.	26.	0.	27.	0.	28.	0.	29.	0.	30	0.	30.	0	30.	0.
Curtain	72.	0.	70.	0.	68.	0.	66.	0.	64.	0.	62.	0.	60.	0.	60.	0.	60.	0
Flank	16.	0.	20.	0.	24.	0.	28.	0	32.	0.	36.	0	40.	0.	40.	0.	40.	0.
Great Line of Defence	117.	3.	117.	5.	119.	0.	120.	3.	122.	5.	126.	0.	129.	0.	127.	0.	125.	4.
Face of the Bastion	36.	1.	37.	5.	40.	1.	42.	2.	45.	2.	48.	5.	52.	3.	50.	0.	49.	1.
Capital	28.	0.	33.	3.	39.	4.	46.	1.	53.	4.	61.	2.	69.	4.	67.	5.	67.	3
Little Radius.	84.	5.	102	1.	120.	0.	138.	2.	156.	5.	175.	2.	194.	1.	217.	5.	231.	5.

If then the circumference of a Circle be defcrib'd from the center G, with the Radius that will be found in the foregoing Table, in the Column under the Polygon, which has been found by the divifion, the diftance of 120 Toifes will divide that circumference into as many equal Parts as the Polygon has fides: If therefore the points of the divifion be join'd by Right-lines, one fhall have a regular Polygon, whofe Perimeter will be equal to that of the Polygon to be Fortified, and whofe Area will be much the fame with that of the faid Polygon. I have faid *much the fame*; becaufe it does not follow that diffimilar Figures, which are *Ifoperimeter*, that is, of the fame outward Compafs or Perimeter, fhou'd be equal. *Plate 1. Fig. 1.*

In this Example, by adding together all the fides of the given irregular Hexagon ABCDE, we fhall find its Perimeter to be of 840 Toifes, which being divided by 120, the quotient 7 fhews that the irregular Hexagon ABCDE may be reduc'd to a regular Heptagon, whofe fides are 120 Toifes each, and whofe Perimeter will be the fame with that of A B C D E the propos'd Hexagon, namely of 840 Toifes. And becaufe the Radius of a Heptagon in the foregoing Table is found to be of about 138 Toifes, if with the diftance of 138 Toifes a Circle be drawn about the center G, the diftance of 120 Toifes will divide it into Seven equal parts, and we fhall have a regular Heptagon, inftead of the irregular Hexagon ABCDE. What remains is to teach the manner of Fortifying that Heptagon, and all other Polygons.

How to Fortify a Square.

THO' a fquare Place be not very fit to be Fortified, having too fmall an *Area*, and Angles too acute to receive Baftions of confiderable Strength, by reafon of the obliquenefs of their Flanks; yet I won't omit to fhew the Manner of Fortifying it, becaufe it may ferve for a Citadel, and to begin with, in our Method; it being the Firft, as the Hexagon is the Second, and the Hep- *Plate 1. Fig. 1.*

tagon the Third, *&c.* of the Figures; for the Triangle is too imperfect to be reckon'd in the number of such Figures as may be well Fortified and Defended.

Plate 1. *Fig.* 2. Let ABCD be a Square, which we wou'd Fortify; whose center P is found by the intersection of the Two Diagonals AC, BD, each side of which, is suppos'd to be of 120 Toises: Take upon each of these sides the Demi-gorges AE, BF, *&c.* each of 24 Toises, and from the center P thro' the Points EF, draw the Flanks EG, FH, of 16 Toises, after which from the same Points E, F, thro' the Points H, G, draw the Rasant-lines EHK, FGI, which will at the Points I, K, cut the lengthned Diameters, and those intersections will make the Points, or Angles of the Bastions; and this is the whole Operation. What remains now, is to explain the Terms, tho' they have been explain'd elsewhere.

APB	*Angle of the Center.*
ABC	*Angle of the Polygon.*
GIQ	*Flank'd-Angle.*
GOH	*Flanking-Angle.*
IGE	*Angle of the Epaule.*
GEF	*Angle of the Flank.*
EFG	*Angle diminué.*
APE	*Flank-forming Angle.*
ABCD	*Polygon.*
EGIQR	*Bastion.*

AB	*Inward side.*
LM	*Outward side.*
AP	*Radius.*
EG	*Flank.*
EF	*Curtain.*
EB	*Lengthned Curtain.*
GI	*Face of the Bastion.*
FI	*Line of Defence.*
AI	*Capital Line.*

Every

Every Plan ought to have its particular Scale, as the First Figure has its own, to take upon it the necessary Measures; but we have not made any for the other Figures, because it is easy to make one, by dividing a Line equal to the inward side AB, into 120 equal Parts, which will represent Toises, or by applying the length of the inward side AB, which is 120 Toises, to 120 upon the equal parts of the Compasses of Proportion, *&c.*

How to Fortify a Pentagon.

A Pentagon is fit to make a Fort of sufficient Strength, because its Flanks are greater and less inclin'd than in the Square; the Figure being of such a Shape, that in Fortifying it, one may observe most of the Maxims which are requir'd in good Fortification. *Plate* 1. *Fig.* 3.

To Fortify it, take upon the inward sides AB, BC, CD, *&c.* which we shall almost always suppose of 120 Toises, the Demi-gorges, AE, BF, *&c.* of 25 Toises each, and draw as before, from the center P, thro' the points E, F, the Flanks EG, FH, of 20 Toises each, to draw also as before, from the same points E, F, thro' the points H, G, the Rasant-lines EK, FI, which will by their intersection upon the lengthned Radii PB, PA, give the Points of the Bastions at K, and I, *&c.*

How to Fortify an Hexagon.

THE Hexagon is yet fitter than the Pentagon, to make a Fort well Defended, because its flank'd Angle will be more open, the Flank greater, and less oblique. *Fig.* 4.

To Fortify it, take upon the inward sides AB, BC, CD, *&c.* which are always suppos'd of 120 Toises, the Demi-gorges AE, BF, *&c.* of 26 Toises each; and having, as before, drawn from the center P, thro' the ends E, F, of the Curtain, the Flanks EG, FH, of 24 Toises each, finish the rest as in the Two foregoing Polygons.

How to Fortify an Heptagon.

Plate 1. Fig. 5. THE Heptagon is yet better than the Hexagon, as well becauſe the Flank becomes greater, and leſs inclin'd, as becauſe both the Flank'd-angle and the Demi-gorge become alſo greater.

To Fortify it, make the Demi-gorges AE, BF, of 27 Toiſes each, and the Flanks EG, FH, of 28 Toiſes each, which muſt always be drawn from P the Center of the Place; after which I, K, the Points of the Baſtions will be found, by means of the Raſant-lines FGI, EHK, as in the foregoing Polygons.

Plate 2. Fig. 7, 8. In the Octogon, give each Demi-gorge 28 Toiſes, and each Flank 32. In the Enneagon 29 for the Demi-gorge, and 36 for the Flank, and in the Decagon, and all the other Polygons that follow it, make the Demi-gorge of 30, and the Flank of the Baſtion of 40 Toiſes; the Point of the Baſtion will be every where found by drawing a Raſant-line of Defence, at leaſt when the Angle of the Baſtion does not happen to be obtuſe; for when it becomes obtuſe, it muſt be rendred right by means of a Semi-circle deſcrib'd upon both the Flanks, to have a Second-flank upon the Curtain, as we ſhall ſay more particularly in the Chevalier de *Ville*'s manner of Fortification.

How to know the Angles and Lines of a Polygon Fortified, by the foregoing Method.

THE parts of a Polygon Fortified by any Method whatſoever, may be eaſily known by the particular Scale of the Plan, or by the Compaſſes of Proportion, which may ſerve as a Scale for all manner of Plans; or rather by Trigonometry, without having occaſion for a Plan exactly Fortified, provided the Method be known according to which we have a mind to Fortify it, as you will ſee in the Two following Examples, which will ſerve proportionably for all the other Polygons that may be Fortified by our Method.

The

The Practice of it upon an Octogon.

THE inward ſide AB being ſuppos'd of 120 Toiſes, the Demi-gorges AE, BF, will be of 28 Toiſes each, the Flanks EG, FH, of 32 Toiſes each, and the reſt will be found by computation after the ſame manner. Plate 2. Fig. 7.

If the Demi-gorge AE, 28, be taken from the inward ſide AB, 120, the remainder will be 92 Toiſes for the lengthned Curtain EB, from which if the Demi-gorge BF, 28, be alſo taken, the remainder will be 64 Toiſes for the Curtain EF.

If the whole Circle or 360 Degrees be divided by the number of the ſides of the Figure, as here by 8, the Quotient will be 45 Degrees for the Angle of the Center APB, which being taken from 180 Degrees, there will be left 135 for the Angle of the Polygon ABC, half of which will be 67 Degrees and 30 Minutes for the Angle of the Baſe ABP.

By means of thoſe Lines and Angles thus known, the other Lines, and the other Angles will be eaſily known by Trigonometry, as you will ſee.

Firſt to find out the Radius AP, in the Iſoſceles Triangle APB, in which beſides the Angles, the Baſe AB of 120 Toiſes is known; one muſt make this Analogy,

As the Sine of the Angle of the Center APB	70711
To its oppoſite ſide AB	120
So the Sine of the Baſe-angle ABP	92388
To its oppoſite ſide AP	156. 5.

and the Radius will be found of 156 Toiſes and about 5 Foot.

If you wou'd have an Analogy, which begins by the whole Sine, and is more fit for Practice, let it be thus,

As the whole Sine	100000
To the Secant of the Baſe-angle ABP	261313
So half the inward ſide AB	60
To the Radius AP, BP	156. 5

By

Plate 2. Fig. 7. By the means of that Radius AP, thus known to be of 156 Toises and 5 Foot, one may know the Angle of the Flank GEF, or its equal AEP, in the oblique-angled Triangle APE, in which the Two sides AE of 28 Toises, and AP of 156 Toises and 5 Foot are known, as also the Angle EAP which they make of 67 Degrees and 30 Minutes, in the following manner:

Add together the Two sides AE, AP, to have their sum of 184 Toises and 5 Foot, and take the least AE from the greatest AP, to have 128 Toises 5 Foot, their difference. Take out from 180 Degrees the known Angle EAP, 67. 30'. and 112. 30'. will remain for the sum of the unknown Angles AEP, APE, half of which is 56. 15'. Then make this Analogy,

As the sum of the sides AE, AP	184. 5.
To the difference of the same sides AE, AP	128. 5.
So the Tangent of half the sum of the Angles AEP, APE	149661
To the Tangent of half their difference	104317.

to which about 46. 13'. answers in the Tables for half the difference of the same Angles, AEP, APE, which being added to half their sum 56. 15'. we shall have 102. 28'. for the greatest Angle AEP, or for the Angle of the Flank GEF, which is the Angle requir'd.

By means of the Angle of the Flank GEF thus known to be of 102 Degrees and 28 Minutes, one may in the same manner know the Angle *diminué* EFG in the oblique-angled Triangle GEF, where the Two sides are known (*viz.*) EG of 32, and EF of 64 Toises, as also GEF the Angle which they make of 102. 28'. after this manner.

Add together the Two sides EF, EG, to have 96 Toises their sum; then take the least EG, 32, from the greatest EF, 64, to have their difference of 32 Toises. Take the known Angle GEF, 102. 28'. from 180 Degrees, and there will remain 77 Degrees and 32 Minutes for the sum of the Two unknown Angles EFG, EGF, then make this Analogy,

As

As the ſum of the ſides EG, EF 96 *Plate 2.*
To the difference of the ſame ſides EG, EF 32 *Fig. 7.*
So the Tangent of half the ſum of the Angles EFG, EGF 80306
To the Tangent of half their difference 26768

to which about 14. 59′ anſwers in the Tables for half the difference of the ſaid Angles EFG, EGF, which being taken from half their ſum 38. 46′. the remainder will be 23 Degrees and 47 Minutes for the Angle *diminuë* EFG, which is the Angle requir'd.

If to this Angle *diminuë* thus found to be of 23. 47′. we add GEF the Angle of the Flank, which we have found to be of 120. 28′. we ſhall have 126. 15′, for the Angle of the Epaule, EGI, or FHK: And if 47. 34′. which is double the Angle *diminuë*, be taken from 120 Degrees, the remainder will be 132. 26′. for the outward Flanking-angle GOH, from which, if APB the Angle of the Center of 45 Degrees be taken, the remainder will be 87. 26′. for the Flank'd-angle GIQ, which will be alſo found by taking from the Angle of the Polygon, which is of 135 Degrees, 47. 34′. being double the Angle *diminuë* EFG, which has been found to be 23. 47′. and which being taken from the Angle of the Flank GEF, that we have found to be of 102. 28′. the remainder will be 78 Degrees and 41 Minutes for the inward Flanking-angle GEO.

In the oblique-angled Triangle AFI, in which beſides all the Angles, we know, the ſide, or lengthen'd Curtain AF of 92 Toiſes, we may find the Line of Defence FI, and the Capital AI, by the Two following Analogies,

As the Sine of AIF, which is half of the Flank'd-angle QIG 69109
To the lengthned Curtain AF 92
So the Sine of the Angle of the Gorge IAF 92388
To its oppoſite ſide, or Line of Defence FI 122.5

which we ſhall find to be of 122 Toiſes, and about 5 Foot.

Plate 2. Fig. 7.

As the Sine of AIF which is half of the Flank'd-angle QIG 69109
To its opposite side or lengthned Curtain AF 92
So the Sine of the Angle diminué IFA 40328
To the Capital AI 53. 4.

which we shall find to be of 53 Toises, and about 4 Foot, to which, if we add the little Radius AP, which we have found to be of 156 Toises and 5 Foot, we shall have 210 Toises and 3 Foot for the great Radius PI, or PK, by means of which, one may find the outward side IK, by making the following Analogy in the Isosceles Triangle IPK,

As the Sine of the Angle of the Base PIK 92388
To the great Radius PK 210. 3.
So the Sine of the Angle of the Center IPK 70711
To the outward side IK 161

which we shall find to be of about 161 Toises.

Or else by making in the Two similar Triangles APB, IPK, this other Analogy,

As the little Radius AP 156. 5.
To the inward side AB 120
So the great Radius IP 210. 3.
To the outward side IK 161.

But the readiest of all the Analogies, is the following, which begins by the whole Sine,

As the whole Sine 100000
To double the great Radius IP 421
So the Sine of half the Angle of the Center IPK 38268
To the outward side IK 161.

To find out the Face GI, which is not the side of a Triangle, the line FG must first be found in the oblique-angled Triangle EFG, by this Analogy,

As

As the Sine of the Angle diminuë EFG	40328
To its opposite side EG	32
So the Sine of the Angle of the Flank GEF	97642
To its opposite side FG	77. 2.

which we ſhall find to be of about 77 Toiſes and 2 Foot, which if we take from the Line of Defence FI, which has been found to be of 122 Toiſes and 3 Foot, for the Face GI, which was required.

The Practice of it upon an Enneagon.

THE inward ſide AB being always ſuppos'd of 120 Toiſes, the Demi-gorge AE, or BF will be of 29 Toiſes, and the Flank EG, or FH of 36. The reſt will be thus known by computation. *Plate 2. Fig. 8.*

If from the inward ſide AB, 120, the Demi-gorge BF, 29, be taken, the remainder will be 91 Toiſes for the lengthned Curtain AF, from which, taking the other Demi-gorge AE, 29, we ſhall have 62 Toiſes for the Curtain EF.

If the whole Circle be divided by the number of the ſides of the Figure, that is 360 by 9, the Quotient will be 40 Degrees for the Angle of the Center APB, which being taken from 180 Degrees, the remainder will be 140 Degrees for the Angle ABC of the Polygon, half of which is 70 Degrees for the Angle of the Baſe ABP.

By the means of theſe Lines and Angles thus known, 'twill be eaſy to find out the other Lines and Angles by Trigonometry, in this manner.

Firſt, to know the Radius AP or BP, we muſt make the following Analogy in the Iſoſceles Triangle APB,

As the Sine of the Angle of the Center APB	64279
To the oppoſite inward ſide AB	120
So the Sine of the Angle of the Baſe ABP	93969
To its oppoſite ſide, or the Radius AP	175. 2.

that we ſhall find to be of 175 Toiſes, and about 2 Foot.

The Analogy beginning with the whole Sine, is thus,

As

Plate 2. Fig. 8.

As the whole Sine	100000
To the Secant of the Angle of the Base ABP	292380
So half the inward side AB	60
To the Radius AP, or BP	175. 2.

By means of this Radius AP thus known of 175 Toises and 2 Foot, one may know the Angle of the Flank GEF, or its equal AEP, in the oblique-angled Triangle APE, in this manner:

Add together the sides AE, AP, to have their sum of 204 Toises and 2 Foot, and take the least AE from the greatest AP, to have their difference of 146 Toises and 2 Foot. Take the known Angle PAE from 180 Degrees to have for the remainder 110 the sum of the unknown Angles AEP, APE, half of which is 55 Degrees: and make this Analogy,

As the sum of the sides AE, AP	204. 2.
To the difference of the same sides AE, AP	146. 2.
So the Tangent of half the sum of the Angles AEP, APE	142815
To the Tangent of half their difference	102269

to which about 45. 39'. answers in the Tables, being half that difference, which being added to 55 half the sum 110 of the said Angles AEP, APE, we shall have 100. 39'. for the greater Angle AEP, or for GEF the Angle of the Flank required.

By means of the Angle of the Flank thus found to be of 100 Degrees and 39 Minutes, one may in the same manner know the aperture of the Angle *diminué* EFG, in the Triangle GEF, as you will see.

Add together the Two sides EF, 62, EG, 36, to have their sum 98, and take the least EG, 36, from the greatest EF, 62, to have their difference 26. From 180 Degrees take out the known Angle GEF, 100. 39'. and half the remainder will be 39. 40'. for half the sum of the unknown Angles EFG, EGF. Then make this Analogy,

As

As the ſum of the ſides EF, EG 98 *Plate 2. Fig. 8.*
To the difference of the ſame ſides EF, EG 26
So the Tangent of half the ſum of the Angles EFG, EGF 82923
To the Tangent of half their difference 21999

to which about 12. 24′. anſwers in the Tables, which taken from 39. 40′. half the ſum of 79. 20′. of the Angles EFG, EGF, the remainder will be 27. 16′. for the Angle *diminué* EFG, which is requir'd.

If to this Angle *diminué* thus found of 27. 16′. the Angle of the Flank GEF, 100. 39′. be added, we ſhall have 127. 55′. for the Angle of the *Epaule* EGI, or FHI: and if 54. 32′. double the ſaid Angle *diminué* EFG, 27. 16′. be taken from 180 Degrees, the remainder will be 125. 28′. for the outward Flanking-angle GOH, or EOK, from which taking the Angle of the Center APB, which is of 40 Degrees, the remainder will be 85. 28′. for the Flank'd-angle QIG, which will be alſo found, by taking from the Angle of the Polygon which is of 140 Degrees, 54. 32′. being double the Angle *diminué* GFE, which we have found to be of 27. 16′. and which being taken from the Angle of the Flank GEF of 100. 39′. the remainder will be 73. 23′. for the inward Flanking-angle GEO.

In the oblique-angled Triangle AFI, in which beſides the Angles, the ſide or lengthned Curtain AF of 91 Toiſes is known, one may by the Two following Analogies know the Line of Defence FI, and the Capital AI.

As the Sine of the Angle AIF 67965
To its oppoſite ſide AF 91
So the Sine of the Gorge-angle IAF 93969
To the Line of Defence FI 126

which will be of about 126 Toiſes.

As

Plate 2. Fig. 8.

As the Sine of the Angle AIF	67965
To its opposite side AF	91
So the Sine of the Angle diminué AFI	45813
To the Capital AI its opposite side	61. 2.

which we ſhall find to be of 61 Toiſes, and about 2 Foot, to which adding the little Radius AP, which has been found of 175 Toiſes and 2 Foot, we ſhall have 236 Toiſes and 4 Foot for the great Radius PI, or PK, by means of which the outward ſide IK may be found by the following Analogy in the Iſoſceles Triangle IPK.

As the Sine of the Angle of the Baſe PIK	93969
To the great Radius PK	236. 4.
So the Sine of the Angle of the Center IPK	64279
To the outward ſide IK	161. 5.

which is of 161 Toiſes, and about 5 Foot.

Or by this Analogy in the two ſimilar Iſoſceles Triangles APB, IPK,

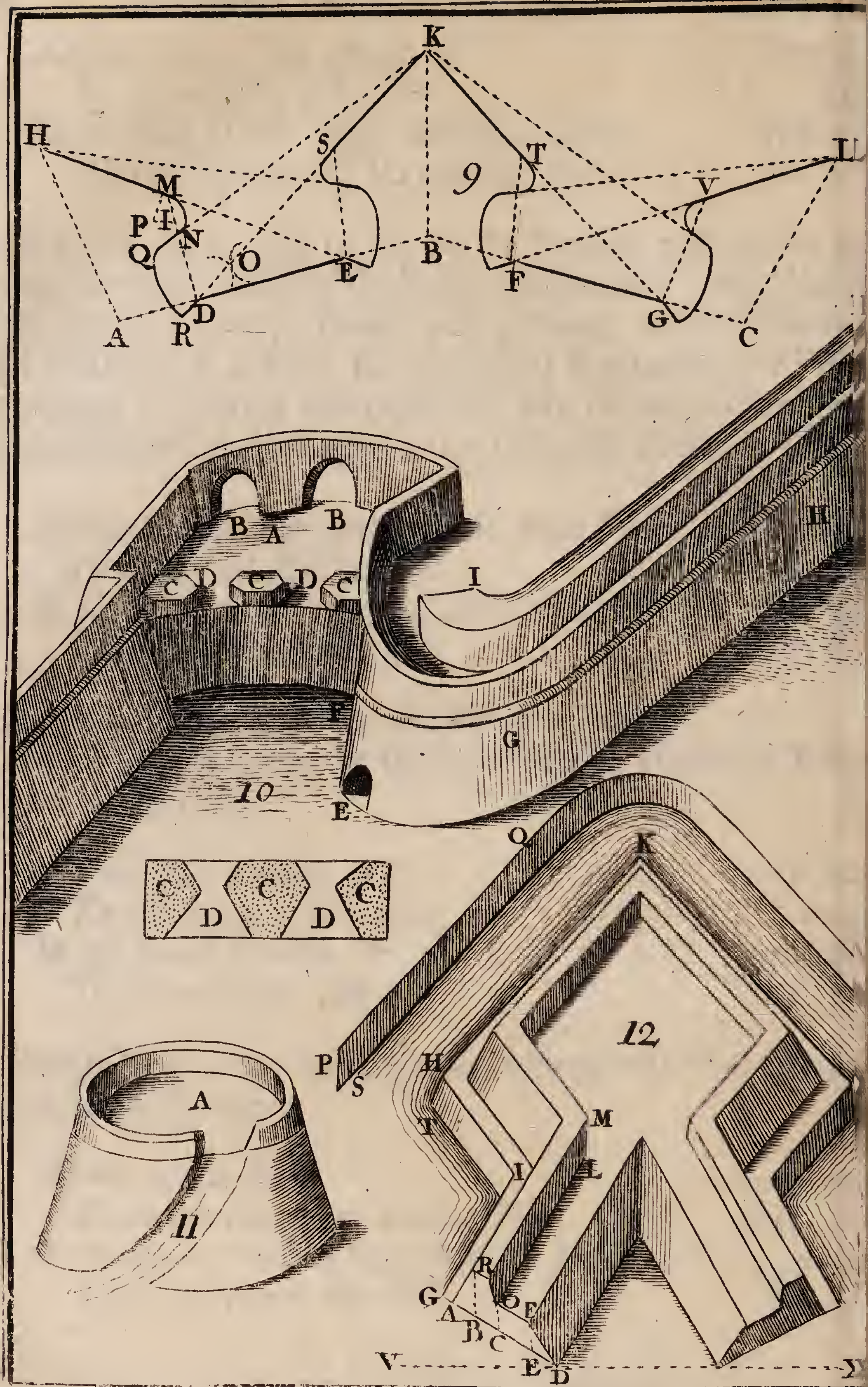
As the little Radius AP	175. 2.
To the inward ſide AB	120
So the great Radius IP	236. 4.
To the outward ſide IK	161. 5.

But the following Analogy beginning with the whole Sine, is the fitteſt.

As the whole Sine	100000
To double the great Radius PI	473. 2.
So the Sine of half the Angle of the Center IPK	34202
To the outward ſide IK	161. 5.

To find the Face GI, which is not the ſide of a Triangle, the Line FG muſt firſt be found in the oblique-angled Triangle EFG, by this Analogy,

As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As the Sine of the Angle diminuè EFG	45813	*Plate* 2.
To its opposite side EG	36	*Fig.* 8.
So the Sine of the Angle of the Flank GEF	98277	
To its opposite side FG	77. 1.	

which will be of 77 Toises and about one foot, and which being taken from the Line of Defence FI, which we have found to be of 126 Toises, the remainder will be 48 Toises, and 5 Foot for the Face GI requir'd.

The foregoing Table is computed after this manner, where you see that the Angle of the Flank grows less as the Polygon has a greater number of sides; so that it will be right in a Polygon that has an infinite number of sides, that is in a *Streight line.*

How to Fortify a Place by help of the computed Tables.

SOme times it happens that we have a mind to represent only part of a Fortified Polygon, ABC, whose two sides, AB, BC, are (for example) sides of a Decagon, whose Center cannot conveniently be had; in such a case we must make use of the said foregoing Table, by its assistance and the help of a particular scale of Toises to Fortify the two sides AB, BC, which we suppose of 120 Toises, in the following manner. *Plate* 3. *Fig.* 9.

Having taken the Demi-gorges AD, BE, BF, CG, of 30 Toises each, as they are mark'd in the Table for the Decagon, with the distance 69 Toises and 4 Foot which is the length of the Capital of the Decagon, describe from the three points A, B, C, arches of a circle, and from the ends D, E, F, G, of the Curtains DE, FG, describe other Arches with the distance of 129 Toises, which is the length of the Line of Defence in the Decagon, to have by the intersection of these Arches the * Points of the Bastions at H, K, L, from which draw to the said ends of the Curtains D, E, F, G, the rasant-lines EH, DK, GK, FL, to have upon them the Faces HM, KS, KT, LV, and joyn the Flanks DM, ES, FT, GV, and thus the whole operation is ended.

* N.B. Point *with a great* P, *signifies the Angle of the Bastion; but with a little* p, *it only means a Geometrical point.*

The

Plate 3. Fig. 9. The computed Tables are alſo very neceſſary to draw a Polygon upon the ground, which has been deſign'd or drawn upon Paper: for, as one cannot deſcribe a Circle upon the ground as one may upon Paper, and as a Polygon is uſually drawn about a Town to be fortified, the foregoing Table muſt be us'd in this manner.

To draw an Heptagon, for example, upon the ground, Plate 1. Fig. 5. having choſen a point as B, where you wou'd have a Baſtion, ſtick there your firſt Stake, to draw with any proper Inſtrument the Angle ABC of the Polygon, (which in the Heptagon is of 128 degrees and a half, as one may ſee in the foregoing Table) made by the two lines AB,BC, which ought to be of 120 Toiſes each; and having ſtuck two Stakes at the ends A,C, make at C another Angle of the Polygon BCD of 128 degrees and a half alſo, by the line CD, which is likewiſe of 120 Toiſes, and at the point A, a third Angle of the Polygon BAO ſtill 128.30', by the line AO of 120 Toiſes as the others. Laſtly, having ſtuck two Stakes at the ends N,D, make there two other Angles of the Polygon ONR, CDR, of 120 Degrees and a half each, by the two Lines NR,DR, which will be of 120 Toiſes each, and thus an Heptagon will be drawn about the Place to be Fortified, and nothing will be wanting but to deſcribe the Baſtions.

To deſcribe a Baſtion, for example, upon the Angle B, having mark'd the Demi-gorges BF, BS, of 27 Toiſes each, as they ought to be in the Heptagon, at the points F,S, make with the Curtains EF, ST, the Angles EFH, TSV, each of as many degrees as the Angle of the Flank ought to be of in the Heptagon, (*viz.*) of 105 degrees, by the lines or Flanks FH,SV, which muſt be made of 28 Toiſes, as the foregoing Table ſhews, for the Heptagon, marking the two points H,V, by two Stakes: and if the ſame thing be done at the other Angles of the Polygon, all the Flanks will be drawn, and only the Faces will be wanting, which may be deſcribed ſeveral ways.

At the points H,V, make the two Angles FHK, SVK, each of about 125. 20', as may be found in the foregoing Table for the Angle of the *Epaule* in an Heptagon, by the

the two lines or Faces HK, VK, which will determine the Point of the Baſtion at K, where conſequently you muſt ſtick a Stake, or elſe faſten a ſmall Cord or *Line* at the point E, and draw it towards K, ſtretching it over the point H, and likewiſe another *Line* at T, to ſtretch it towards K, drawing it likewiſe over the point V, and theſe two Lines thus ſtretch'd will croſs one another at K the Point of the Baſtion. Or elſe faſten at H,V, two *Lines* of equal length (*viz.*) of 42 Toiſes and two foot, ſuch as is the Face of the Baſtion of the Heptagon in the foregoing Table; and theſe two *Lines* being ſtretch'd, and their ends being joyn'd together, will give the Point of the Baſtion at the point K, which will be an example to mark the Points of the other Baſtions by.

If you have no computed Tables, the Plan of what you intend to draw upon the ground muſt be exactly deſcribed upon Paper, that the quantity or aperture of the Angles may be known by a Protractor, and the length of the lines taken off of the Scale of the Plan. Right-lines are mark'd upon the ground by ſtretching the *Line* which is faſten'd to two Stakes, and their length is meaſur'd with the Toiſe or *Six foot-Rule*, then the ground muſt be dug or furrow'd along the *Line* that when the Stakes and *Lines* are taken off there may be the mark of a right-line left.

Two kinds of Stakes are commonly us'd, ſome about a foot or two in length at moſt, which are us'd to ſtretch the *Lines* upon ſmooth ground; and others of 4 or 5 foot in length, which are us'd to draw lines, when the ground is uneven or full of Buſhes. The Strings are alſo of two kinds, Cords or thicker Strings to make lines, and ſmaller Strings or *Lines* to make Angles.

How to Fortify a Polygon with Caſemates and Orillons.

THE Two ſides AB, BC, belong to a Decagon Fortified according to our Method, which ſuppoſes the Demi-gorge AD of 30 Toiſes, and the Flank DM of 40 Toiſes, the inward ſide AB being always of *Plate* 3. *Fig.* 9.

120

120 Toises; and now the thing requir'd is to add Orillons and cover'd Flanks to this Part of a Decagon.

Take MN, the third part of the Flank DM, for the thickness of the Orillon, and from the opposite Flank'd-angle K, draw thro' the point N, the line NQ equal to MI or NI, the half of MN, for the depth of the Case-mate. Take again upon DK the great Line of Defence lengthned, the Part DR equal to NQ, to describe from the two points R,Q, with the distance RQ, two Arches, which here intersect one another at O, which must be the Center of the cover'd Flank R,Q, which we make round rather than flat, as Mr. *Vauban* and several others do, that it may be greater, its Merlons larger, and consequently stronger, and that it may last longer; that is, that it may longer withstand the effort of the Enemy's Cannon, which will happen by reason of its concavity, in which the Cannon-ball being, as it were clasp'd, and and consequently being resisted by a greater number of parts, won't have so much effect as against a flat Flank, which touching the Ball but in one point (as one may say) cannot make so great a resistance, as having fewer parts which bear the shock.

To describe the Orillon, which we shall also make round, that it may cover and resist the better, draw from I the middle point of MN, the right line IP, perpendicular to the said MN, and draw from M, the end of the Face MH, MP perpendicular to the said MH, which will intersect IP, at P, which must be the Center of the round Orillon, to the intent that it pass thro' the two points M,N, and that it touch the Face MH, at its end M, otherwise it wou'd bunch out and be an Eye-sore, as I have said elsewhere.

The cover'd Flank RQ being here of about 27 Toises, will be able to contain 9 Pieces of Cannon, three Toises distant from one another: and it may be made to hold more, if instead of taking the third part of the Flank DM, for the thickness of the Orillon MN, we let that proportion alone, and only make MN of 7 or 8 Toises, which is enough to resist all efforts.

The

The Use of Casemates or low Flanks, is chiefly for the defence of the Ditch, and with the Cannon to hinder the Enemy from making a **Traverse* in the Ditch to bring the Miner to the Face of the opposite Bastion, and to ruin all the Machines and Work of the Enemy in the Ditch, when he has taken it. *Plate 3. Fig. 9.*

The Orillons are a great help to the Casemates or *Places basses,* and in a manner necessary to cover and preserve them from the Enemy's Cannon: for an Orillon is made to cover the Artillery of the Flank in such manner, that one may fire against the Point of the opposite Bastion without being seen from the Counterscarp, and for that reason we have drawn the line DR from K the Angle of the opposite Bastion. Nevertheless some make the line or *Revers* QN, which we have call'd the *Platform* of the Casemate, not to answer to K the Point of the opposite Bastion, but to some point on the Face, 3 or 4 Toises distant from the Angle or the said Point, that the retir'd Flank may be better cover'd, which does not much hinder the Cannon that is hidden at Q from discovering the Ditch, which Cannon wou'd otherwise be seen from the *Counterscarp*, that is, from the outward Edge of the Ditch towards the Field, when the Flank'd-angle is very acute, which may happen in an irregular Place.

'Tis plain that if the Enemy shou'd attack the Curtain DE, and make a Breach in it, one might fire on that Breach upon the Rear, and expel the Enemy from his Lodgment there, from the *Revers* or Return QN, which being of Six or Seven Toises may contain Two Pieces of Cannon, which will be wholly cover'd. In such an Orillon the *false Gates* or *Posterns* are made for Sallies, and to be assistant to the Ditch and *Outworks;* that is, such Works as are separated from the Body of the Place.

When we have said that the cover'd Flank RQ is call'd *Low Flank*, or *Place Basse*, it was to distinguish it from

* *A little Trench with a Parapet one each side, which the Besiegers make thwart the Moat to be secure from Flank-shot.*

another cover'd Flank, which is yet retir'd farther inwards than the low Flank, and being rais'd above that Flank is call'd *Place haute*, or high Place, in which there is another Battery of Cannons, of which we shall give a Draught, when we have taught the way of drawing the Plan of the Rampart and Parapet.

Plate 3. *Fig.* 10. We have alſo ſaid that this low Flank or low Place RQ, is call'd *Caſemate*, becauſe Vaults as B are made in it to put the Cannons and Ammunition in when there is no farther occaſion for 'em. A, repreſents the *Place-Baſſe* or Caſemate, upon whoſe edge are the *Merlons* C next to the *Port-holes* D, thro' which the Cannon cover'd by the Merlon C is fir'd: The Port-holes are alſo call'd *Embraſures*, and when they are ſmall and only fit for a Muſket, *Meurtrieres* or *Murderers*.

That height of Earth which the Cannon is ſet upon to fire againſt the Enemy, is call'd *Battery* and *Platform*; but *Platform*, or *Platform of Batteries* more properly ſignifies ſtrong Joiſts or Planks which make a Square Floor for the Cannon of the Battery. Theſe Planks are call'd in French *Tablouins*, and ſuſtain the wheels of the Carriages, and hinder the Cannon from ſinking into the earth by its weight: and that the Cannon may not ſo eaſily roll back, the Platform is made to incline a little towards the Merlons, which are that part of the Parapet which lies betwixt two Embraſures of a Battery.

The Merlons C, are as you may ſee, cut obliquely as well as the Embraſures D, that the Cannon may alſo ſhoot obliquely, or on one ſide. The Merlons are commonly 9 foot wide within, and 6 without, and in the whole 12 foot, that thoſe which tend the Cannon may be cover'd when they bring it up again after its recoiling, which is uſually 10 or 12 Foot. It is plain that the Merlons being all that is left of the Parapet, when Port-holes, or Embraſures have been made in it, to put the mouth of the Cannon thro', their height and thickneſs muſt be the ſame with that of the Parapet. As for the Embraſures, their height above the Platform is 3 Foot towards the Cannon, and only a foot and a half towards the

the field, that by help of that ſlope or *Glacis* the Cannon may *dip*, that is ſhoot downwards. They are about 3 foot wide on the inſide towards the Town, Six or Seven on the outſide, and two foot in the narroweſt place. The firſt Embraſure begins upon the lengthened line of defence, and the laſt at the *Enfoncement* or depth of the Caſemate towards the Orillon.

Batteries are made not only for the defence of, but for attacking, a Place; whence there are ſeveral ſorts of Batteries, which we ſhall deſcribe occaſionally.

Therefore when the Platform of a Battery is lower than the *Level of the Field*, that is, than the Surface of the Ground, it is call'd a *Buried Battery*, or Battery *ruinante:* and *Battery de Revers* or *Murthering Battery*, is one that beats upon the back of any Place: *Battery en Echarpe*, is that which plays obliquely: and *Battery d' Enfilade* is that which ſcours, or ſweeps the whole length of a ſtraight line; for in Terms of Fortification to *Enfiler* or *ſcoure*, is to diſcover and ſhoot along or ſweep the whole length of a ſtraight line: and *Enfilade*, is the diſpoſition of a piece of ground ſo Situated that one may thence diſcover a *Poſt* where the Enemies retrench themſelves to fight.

E, repreſents that little or falſe Gate that we have call'd *Poſtern*, and the double line FGH repreſents the * *Cordon* of the Wall, which is only us'd for Ornament, above which is the Parapet IK, which we ſhall treat of more particularly in the deſcription of the *Profil* of the Wall.

* *A row of Stones made on the outſide.*

Of the Enceinte of a Place.

A *Place*, or *place of War*, generally ſignifies a Place or City regularly or irregularly Fortified, and when it is encompaſs'd with Walls it is call'd *Place revêtue:* and *Regular Place*, when the parts of its Enceinte are ſimilar, equal to one another, and equally fortified: and *Irregular Place*, when it wants the Properties of a regular one.

By the *Enceinte of a Place* we do not only underſtand, the Curtains, Flanks, and Faces of the Baſtions, with the

the Rampart and its Parapet; but alſo all the other Works made about it, as Ditches or Moats, Half-Moons, Horns, and Crown-Works.

To be more plain, we ſhall diſtinguiſh three ſorts of Enceintes, (*viz.*) the *firſt* or *ſimple Enceinte*, which contains a Rampart, a Ditch, and an *Eſplanade* or Glacis: the *Second Enceinte*, which beſides all this has a Wall, call'd *Chemiſe*, when it is not very thick, with a *Chemin des Rondes* cover'd with a little Parapet for the Watch or Rounds to go about at night: and the *third Enceinte*, which is call'd *Falſe-Braye*, and alſo *Baſſe Enceinte*, becauſe it is a kind of Parapet, which is carried all round on the Level between the Rampart and the Ditch, to defend and hinder the paſſing of the Ditch. Inſtead of this Work, moſt us'd by the Dutch, Mr. *Vauban* only uſes a Tenaille with Flanks, which he raiſes in the Ditch before the Curtain, as we ſhall further explain when we ſpeak of this manner of Fortifying in *Part* III.

We call *Fortreſs*, *Strong Place*, or City, a Fortified Place, which contains a conſiderable number of Houſes, and *Fort* a leſs Fortified Place, which is call'd *Fort-Royal*, when it has 120 Toiſes for its Line of Defence: which is a rule for all the other lines of the *firſt Draught*, according to our Method, where it is eaſy, by the Compaſſes of Proportion, or by a particular Scale, or rather by Trigonometry, to know the length of all thoſe lines, ſuppoſing the inward ſide of 120 Toiſes, becauſe the Line of Defence is pretty near its length, as you have ſeen. But as the Rampart, the Ditch, and the Glacis bear no proportion to the Line of Defence, the following Rules muſt direct the Undertaker of ſuch a Work.

The Rampart.

THE Rampart is a Maſs of Earth which is rais'd about a Town, to cover the Houſes from thoſe that are in the Field, and to raiſe thoſe that defend the Place as much as is needful, that they may ſee the Field

Field round about, diſcover the Enemy, and fire on him as far as their Pieces can carry.

The height of the Rampart is uſually 3 Toiſes, more or leſs according to the height of the Ground without the Place, which may command it: and its breadth at bottom is about 15 Toiſes, with the **Talu's* which make it narrower at top, more or leſs according as thoſe Slopes are, which can't be determin'd, as depending on the quality of the Soil; ſome Sort of Earth ſticking faſter together than ſome again: for when we have good Earth we give the *outward Talu*, that is, the Talu towards the Field, the leaſt Slope we can, that the Rampart may be the wider at top, and its aſcent more difficult for the Enemy. As for the *inward Talu*, that is, the Slope towards the Place, it may be as great as you pleaſe, that the Beſieged may aſcend it with the greateſt eaſe and bring up the Cannons and Carriages neceſſary for the defence of the Place.

* *Slope.*

The line AD repreſents the thickneſs of the Rampart, and EF its height, ED, the breadth of its inward Talu; and AB, that of its outward. Here the Earth of the Rampart fills up the whole Baſtion, and in ſuch a caſe it is call'd a *full Baſtion*: and when the Rampart follows the *Maſter-line* or Firſt Draught in a parallel line, as the Parapet does, the empty ſpace which is in ſuch a Baſtion gives it the name, of *Empty* or *Hollow Baſtion*; and in this empty ſpace *Magazines* may be made to keep Proviſions and Ammunition: and when inſtead of this empty ſpace the Baſtion is full, one may make *Terraſſes*, that is, Mounds of Earth more or leſs rais'd as there is occaſion, call'd *Cavaliers*, which are ſometimes Wall'd round; and always have, like other Works, a Parapet to cover the Cannon there ſet in Battery to defend the Face of the oppoſite Baſtion, and the Baſtion it ſelf, if the Enemy makes a Lodgment in it.

Plate 3. *Fig.* 12.

Cavaliers are often made upon the middle of the Curtain near the Parapet, to ſee the Field from the Place, to diſcover the Enemy in his Works, and to double the Fire which defend ſuch places of the Town as may be attack'd.

attack'd. Or else they are plac'd in such a Part of the Fortification as might be commanded or *enfiladed* by the Enemy, to cover it.

Fig. 11. As their use, so their figure differs: it is usually a long Square, or round, as A, and sometimes an Oval, which is rais'd in the Ditch of a Fenny Place, to cover a Gate, or to lodge in it a Guard against surprises, and then such a Cavalier is call'd *Horse-shoe* and *Pâté* when its Figure is very irregular.

Instead of Cavaliers, little Mounds of Earth are sometimes rais'd at the Points of the Bastions in the shape of Traverses, which are call'd Platforms, because Cannons are play'd on 'em to fire over the Parapet, and also *Barbettes*, whence Shooting along the Glacis of the Parapet, is a call'd *to Shoot en Barbe*, (*in the Beard*) because the fire of the Cannon raising the Glacis of the Parapet, and burning its Grass, does as it were *Shave* it.

Lastly, Upon the edge of the Rampart towards the Ditch, in all the Saliant Angles of the Place *Guerites* or *Echauguettes* are built, which are little round Towers of Stone, when the Rampart is wall'd, and of Wood, when it is only of Earth, in the Shape of a Lantern, or as we usually call'd them in English, *Centry Boxes*, to lodge in each of them a *Centry*, or Soldier to hearken, look about, and watch over the Ditch against the surprises of the Enemy, and also to shelter the Centries in foul weather.

As these little Towers stand out upon the edge of the Rampart towards the Ditch, the Parapet of the Rampart is cut, that one may go from the Terre-plain of the Rampart to each *Guerite* or Box. They are made about 7 or 8 foot high, three or four foot wide, and open on all sides, that the Centries may look every way about them. A Centinel on Horse-back who stands at any place to prevent surprises is call'd *Vedete*, and to stand Centry is call'd *to be upon Duty*. (in French, *être en faction.*)

The

*The * Parapet.* * *Breast-work.*

UPON the edge of the Rampart towards the Field, there must be rais'd to cover the Besieg'd, a Bank of Earth about 20 foot thick, and 6 foot high, with a little Step or Stair 3 foot thick, and 2 foot or 2 foot and half high. This Elevation of Earth is call'd *Parapet*, and the Foot-pace or Step *Banquette*, which must be towards the Place for the Defender to get upon to see the Field the better, and fire when there is occasion. The upper part of the Parapet must have a Slope call'd *Glacis*, that the Musketeers which are upon the *Banquette* may fire downwards into the Ditch, or at least upon the Counterscarp, that is, upon the edge of the Ditch towards the Field. Such a Parapet is call'd *Royal Parapet*, to distinguish it from the Parapets of other Works.

A little way or Passage is left upon the outward edge of the Rampart, 3 or 4 foot wide to receive the Earth which may roll from the Parapet, and keep it from falling into the Moat; and this Space is call'd *Berme*, as also *Orteil*, *Liziere*, *Retraite*, *Relais*, and *Pas de Souris*. What is left of the Rampart towards the Place is call'd *Terre-plain*, that is the Horizontal Surface, where the Defenders stand and go to and fro': it must be broad enough to draw and turn the Carriages of the Cannons upon.

The Line OF represents the Terre-plain of the Rampart, and the line BC, the Basis of the Parapet, whose height LM above the Rampart must be Six foot at least, as being only made to cover the Soldiers which defend the Place from the Enemy's Cannon; and for that reason its thickness ought to be no less than 3 Toises, that it may be proof against Cannon-shot, and be capable of a Banquette to raise the Soldier high enough to fire over it: and that one may the easier get up upon this Banquette, which is two or three foot high, it must have at least 2 or 3 steps: and that the Cannon may hurt the Enemy, the Parapet must have Embrasures, whose *Plate* 3. *Fig.* 12.

Merlons ought to be of good Earth, the better to reſiſt the Enemy's Cannon.

As the Embraſures do ſomewhat weaken the Parapet, and the Cannons ſhooting thro' an Embraſure can diſcover but ſuch a Space, according as the Embraſure is bigger or leſs, and that ſo there may be ſome place very little ſeen from the Town, and that in the Second Flank the Embraſures muſt be very oblique to be uſeful; the *Chevalier de Ville*, to have the Parapet every where of equal ſtrength, and to have no need of a Banquette, wou'd have the Parapet rais'd no higher than four foot, that the Cannon might fire *en Barbe* and on every ſide; and as in ſuch a caſe the Soldiers wou'd not be cover'd enough in ſiege time, he makes the Parapet higher with *Gabions* or Baskets full of Earth, five or ſix foot high, and four wide from top to bottom, between which the Musketeers may eaſily fire: and when a Cannon is to be fir'd, only one of theſe Gabions muſt be remov'd, and ſet up again after the ſhot; but leaſt the Cannon and thoſe that tend it ſhou'd be too much expos'd after the Piece is fir'd, he makes the Rampart go down with a Slope towards the Place to facilitate the recoil of the Cannon.

Inſtead of Gabions, which are alſo call'd *Manequins*, and *Corbeilles* when they are leſs, and wider at top than at bottom; one may alſo uſe *Barrels of Earth*, which the Souldier carries to make his Lodgment, or Sacks of Earth, which are Sacks of courſe Cloath, fill'd with Earth, about 1 $\frac{1}{2}$ Foot thick, and 1 $\frac{1}{2}$ Foot high, uſeful to cover one's ſelf in haſte, and make Retrenchments, the Spaces between ſerving as Embraſures to fire thro' upon the Enemy.

But as ſuch a low Parapet does not ſecure the Souldiers from Cannon-ſhot, unleſs they ſtand at ſome diſtance from it, and that whilſt a Gabion was remov'd the Enemy being upon the watch might ſo level his Cannon as to endamage that of the Defenders; to me it ſeems better to have a Parapet 6 Foot high, with its Banquette, and Embraſures in fit places: or if one wou'd have

have no Embraſures, a Terraſs muſt be rais'd with Turfs, and Boards over them to ſuſtain the Cannon, which may be levell'd any way that one ſhall think fit, without fearing the Enemy's diſmounting it, becauſe it hides it ſelf in its Recoil, if the Rampart be of a reaſonable height; and the Enemy ſeeing no Embraſures does not know what place to level his Cannon at, as not knowing whether the Cannon be ſtill in the ſame place that it fir'd from: ſuch a thing may be eaſily done, and the Enemy will that way be much fatigu'd, as being forc'd to alter his manner of ſhooting every now and then: for as the Defenders expoſe the Mouth of the Cannon they fire it, and the Cannon recoiling at the ſame time ſecures it ſelf from the Enemy, who has nothing left to do, unleſs it be by beating down the whole Parapet to lay the Beſiegers Cannon bare, which wou'd be a tedious piece of Work.

The Ditch.

THE *Ditch* or *Moat* is a Hollow or Channel made about the Place which we wou'd defend, to avoid Surpriſes. The Earth of the Rampart muſt be dug up juſt by, that the Rampart and Ditch may be made at the ſame time: whence it follows that they bear a Proportion to one another; for if the Rampart is made of ſuch a bigneſs, one muſt dig the Ditch till Earth enough be got to make the Rampart, Parapet, and Eſplanade, leaſt one ſhou'd be at unneceſſary Expences.

Between the Rampart and Ditch a Way or Space is left, call'd the *Berme*, of about 3 or 4 foot, as we have *Plate* 3.
already ſaid, to receive the Earth that may roll down *Fig.* 12.
from the Parapet, either of it ſelf or from the Shock of the Enemy's Cannon.

That Edge of the Ditch which is towards the Place as GIKH, is call'd *Scarp*, and that which is towards the Field, as PQ, is call'd *Counterſcarp*, which is commonly made round over againſt the point of a Baſtion, that one may have the more Room in the Cover'd-way, and the reſt is parallel to the great Line of Defence.

All

All the Heights or Depths must be made with a Slope.

The breadth and depth of the Ditch can't well be determin'd, as depending upon several Circumstances, and especially the Nature of the Soil: for in Fenny places, or where Water does not lie very deep, it ought to be made wider and shallower, that it may be more troublesome to the Enemy that wou'd pass it, obliging him to *Bleed the Ditch*, that is to drain it dry. This we may say nevertheless; that the Breadth of the Ditch ought to exceed the length of the tallest Trees, or else Bridges might be made to pass the Ditch tho' it were full of Water.

In Rocky, Stony and High Places, the Ditch must be made as deep as possible, and narrower, the easier to avoid Surprises, and to hinder a *Scalade*, that is to keep the Enemy from climbing up the Rampart with Ladders; besides the Stone dug up in such a place when a a deep Ditch is made, will be useful in the Mason's Work.

Lastly, In such places as the Soil is good, the breadth as well as the depth of the Ditch is moderate. In the following Plans and Profils we shall allow 20 Toises for the breadth of the Ditch over against the Flank'd-angle, and 15 foot for its depth. The Edge of the Ditch towards the Field over against the middle of the Curtain, where it is broader than about the Point of the Bastion, is terminated by a *re-entrant* Angle, usually call'd the *Angle of the Counterscarp*, that each part may be seen and Flank'd by the opposite Flank.

In a Ditch which is full of Water, about the middle, a Bank of Earth or Sand is left, or great Stakes are driven and stand about a Foot above Water, to hinder the Enemy from passing in Boats: and in a dry Ditch, about the middle of it is made another less Ditch or Trench call'd *Cunette* and *Cuvette*, Sunk till you come to water, to avoid surprises, hinder the Besiegers from mining, and that the Water may drain and leave the main Ditch drier: it shou'd be five or six Foot in Water if possible, to hinder the Enemy from wading thro' it and

and covering himſelf in it. When a Ditch is lin'd with a Wall, Stairs are in all its Angles for the Service of the Counterſcarp.

The Wall.

THO' Clay be very good to build a Rampart, and will not eaſily moulder; yet in *France* and *Italy* they build a good Wall, which has its Foundation below the bottom of the Ditch, and which is ſo high that one may from it diſcover the Field, without its hindring the proſpect of the Rampart, to keep the Rain from deſtroying the Works, and ſuſtain the Earth when it is not good, that the Rampart may laſt the longer, and not have ſo great a ſlope.

When a Rampart has a Wall, it is call'd *Rampart revêtu*, and the Wall is call'd *Revetêment*, whoſe height above the bottom of the Ditch may be 4 Toiſes. Its Foundation ought to be of Stone, and its Body is uſually of Stone; but it is better to build it of Brick if one can, becauſe it does not fly in pieces ſo as it does when of Stone. A *Cordon* is made from its top for Ornament only, with a little Parapet, or Breaſt-work, Six Foot high, and Two Foot thick, which we ſhall ſpeak of more particularly in the Second Profil. *Plate 3. Fig. 12.*

The Wall ought to have a conſiderable *Talu*, as for example, the Fifth or Sixth Part of its Height: And that it may be able to ſuſtain the weight of the Rampart, it's proper to ſupport it on the inſide with *Eperons*, * or *Counter-Forts*, which are Buttreſſes, or Walls, which go partly thro' the Rampart, and begin at the *Revetêment*. The Counter-Forts are generally Vaulted, and their intervals which are of about 18 Foot, are fill'd with Earth to make them firmer.

* *Spurs.*

The Chemin des Rondes.

THE *Chemin des Rondes*, or Way of the Rounds, is a ſpace left upon the Wall between its Parapet and the Rampart of a Place of War, to hear and ſee from it what is done in the Ditch. This ſpace does not ex-

exceed 9 Foot, the usual breadth of the top of the Wall, and the Way to it, is thro' *Posterns*, or little Gates, which are near the Gates of the Place, that are generally made about the middle of the Curtains, as being the freest and most convenient place.

We have already said that the Parapet of the *Chemin des Rondes*, which is rais'd upon the Cordon of the Wall, is but Two Foot thick, being built rather to keep the *Rounds* (*the Night Watch*) from falling into the Ditch, than for Strength, and as it is Six Foot high without any Banquette, it ought to have small Embrasures or *Meurtrieres* every Four Foot. This *Chemin des Rondes*, which is made of Brick, will easily be conceiv'd by observing the Second Profil, which we shall explain anon.

The False-Braye.

THE *Dutch*, who make their Works of Earth only, instead of a Wall, make upon the Level between the Rampart and Ditch, a kind of Parapet, call'd *Fausse*, or false *Braye*, Six Foot high, and Three Toises thick, with such a Banquette as the other Parapets have, leaving between the Rampart and this Parapet, a Way Three Toises wide, to receive the Ruins which the Cannon of the Besiegers may cause to fall in this Way, or Terre-plain of the False-braye.

The chief use of the False-braye is to Defend the Ditch; but I wou'd not have it continu'd along the Faces of the Bastions, because in those places it wou'd be *Enfiled* by the Enemy's Works: And tho' one may raise higher Parapets towards the Point of the Bastion to hinder the *Enfilade*, yet it makes it no better, because such a Parapet being once Demolish'd, which is soon done, the False-braye will always be *Enfiled;* besides, the Ruins of the Rampart obliges those that are in the False-braye to abandon it, which gives the Enemy an easy possession of it. Hence it is plain, that in wall'd Places False-brayes are altogether useless, because the pieces of the Wall which the Besiegers Cannon causes

causes to fall into the Terre-plain of this Work, kills all those that happen to be in it.

The Cover'd Way, and Esplanade.

A Way is left upon the Counterscarp 4 or 5 Toises broad, call'd the *Covert* or *Cover'd Way*, or *Coridor*, which is as another Terre-plain, because it is cover'd with a Parapet Six Foot high, with a Banquette like the rest. This Parapet has a great Glacis, which insensibly loses it self towards the Field for 15 or 20 Toises, and for this reason it is call'd *Esplanade*, or only *Glacis*.

The Cover'd Way, with its Parapet and Esplanade are made parallel to the Counterscarp of the Place, and that of the Out-works, when there are any, to Defend the Field, and hinder the Enemy from coming near and taking the Ditch: And for greater security, a little way off of the Parapet *Palissadoes* are set, which are great Stakes sharp-pointed, commonly set upright 3 Foot within the Earth, and 4 or 5 Foot above Ground, and stand so close together, as to admit only the Muzzle of a Musket, or at most a Pike. These Palissadoes are set in the form of a Lozange, making an Angle towards the Field, and another towards the Place, and so on.

To hinder the Enemy still more from an easy taking of the Counterscarp, or of the Cover'd Way, a Ditch is made full of Water, call'd *Avant-fossé*, or *Ditch of the Counterscarp*. Some would have the Cover'd Way sunk below the Level of the Field, about 2 Foot, that the Horse may there be cover'd, and that the low Flanks may better Command and Defend it.

To disturb the Enemies Works, beyond the Glacis or Esplanade, *Redoubts* are built within Musket-shot of the Out-works of the Place, that they may thence be Defended. These Redoubts are little Square Works, with a Ditch and Parapet, and are Retrench'd *Corps de Guard*.

How

How to describe the Profil of a Fort that has an Enceinte of the first kind.

Plate 3. Fig. 12. THE *Profil*, or *Porfil*, or *Orthography of a Work*, is the Section of that Work, when cut by a Plain, perpendicular to the Plain of the Horizon. This is easy to be comprehended by the Figure ADFOR, which is the Profil of a Rampart, and its Parapet; which Profil shews the Heights and Breadths of all the parts of that Work.

To describe the Profil of the Rampart, the Ditch and Esplanade, draw first the Level of the Field AB, and take upon it the Lines

Plate 4. Fig. 13.

AC, Basis of the Rampart, 15 *Toises.*
CE, Breadth of the Ditch, 20 *Toises.*
EF, Cover'd Way, 5 *Toises.*
FB, Breadth of the Esplanade, 20 *Toises.*
CD, † *Berme,* 4 *Foot.*

Draw from the points, A, C, D, E, F, as many Perpendiculars to AB, and take upon them the lines

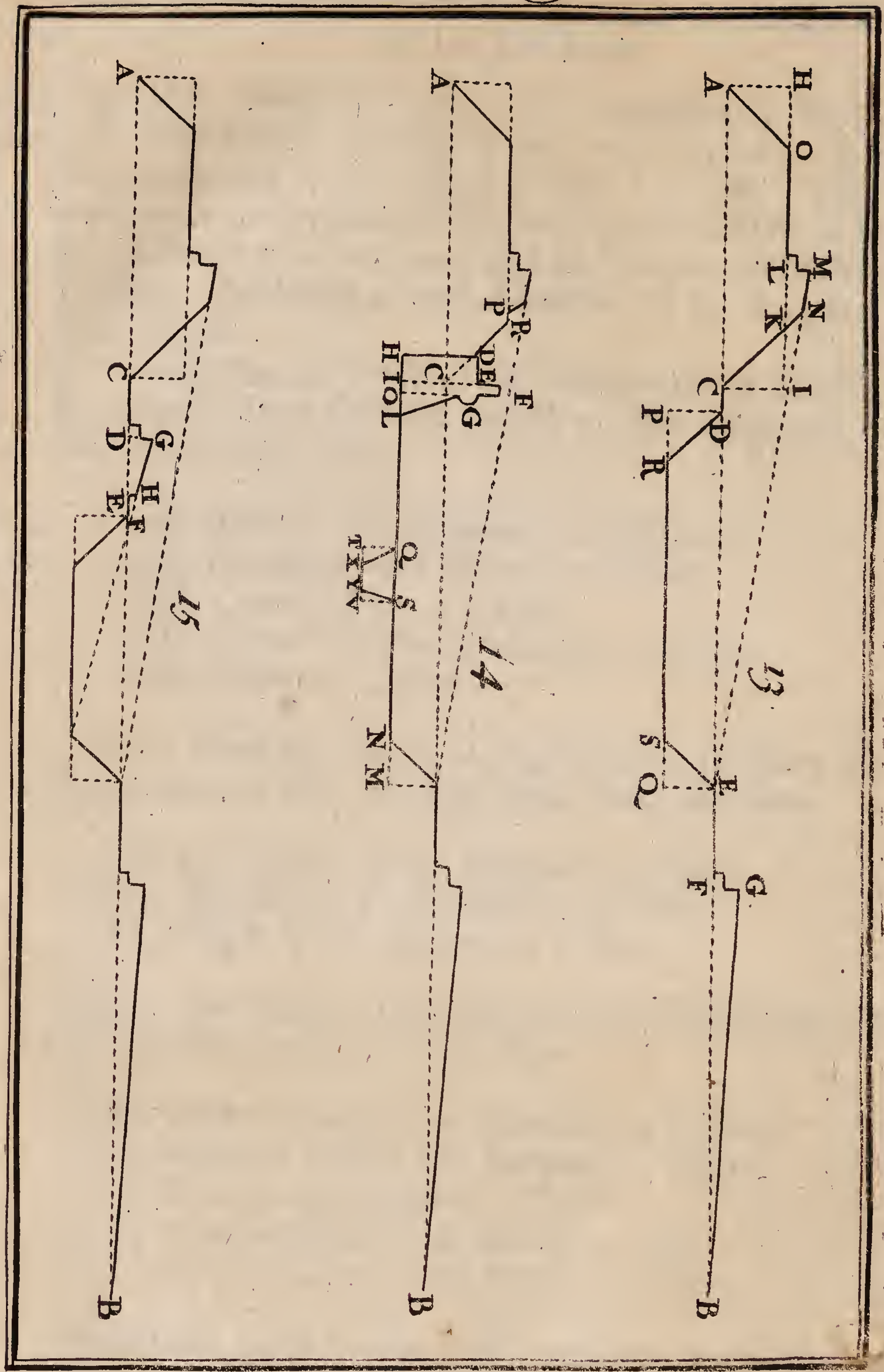
AH, CI, Height of the Rampart, 3 *Toises.*
DP, EQ, Depth of the Ditch, 15 *Foot.*
FG, Height of the Esplanade, 6 *Foot.*

Draw the Glacis GB, and join the Right-lines HI, PQ, to take upon them the lines

HO, Inward Talu of the Rampart, 3 *Toises.*
IK, Outward Talu of the Rampart, 3 *Toises.*
KL, Basis of the Parapet, 3 *Toises.*
PR, Inward Talu of the Ditch, 15 *Foot.*
QS, Outward Talu of the Ditch, 15 *Foot.*

From the point L, upon HI, raise the perpendicular LM, of 6 Foot for the Height of the Parapet, as that of the

† *The Berme is so call'd, whether it be made on the outside of the Parapet upon the Rampart, or between the Rampart and the Ditch.*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the Esplanade FG, and the Banquette added to each Parapet must be at least 2 Foot high, and 3 Foot broad. The Glacis MN of the Parapet is drawn by the point E of the Counterscarp, and ends where CK lengthned cuts ME.

How to describe the Profil of a Fortress whose Enceinte is of the second kind.

HAving, as before directed, drawn the Profil of the Rampart, Ditch, Esplanade, take the lines *Plate 4. Fig. 14.*

HL, Basis of the Wall, 9 Foot.
IO, Basis of the Parapet, 2 Foot.
OL, Talu of the Wall, 4 Foot.
CE, Height of the Wall, 9 Foot.
DE, Chemin des Rondes, 9 Foot.
EF, Height of the Parapet, 6 Foot.
PR, Berme, 2 Foot.

In the lower part of the Wall, over against the *Chemin des Rondes*, is added a *Cordon* G, which is a Rib, or Row of Stone-Work made round, and standing out. As such a Cordon is but for Ornament, I think it useless, and even prejudicial, as being a good aim for the Enemy's Cannon. It is plain that the lower the Walls are, the less they are expos'd to the Enemy's Cannon, and the less apt to fill the Ditch with Ruins; but they are easier to be Scal'd.

If you wou'd have a Cunette in the middle of the Ditch, which is seldom done, allow 15 Foot for its Breadth QS, or TV, and as much for its Depth QT, SV, and about 5 Foot to each of the Two *Talu's* TX, VY. When such a Trench is Palissado'd, 'tis always on that side towards the Place.

How to describe the Profil of a strong Place, whose Enceinte is of the third kind.

HAving, as before, drawn the Profil of the Rampart, Ditch, and Esplanade, take the lines *Plate 4. Fig. 15.*

CD,

Plate 4. Fig. 15. *CD, Space of the False-braye, 3 Toises.*
DE, Breadth of the False-braye, 3 Toises.
DG, Height of the False-braye, 6 Foot.
EF, Berme, 3 Foot.

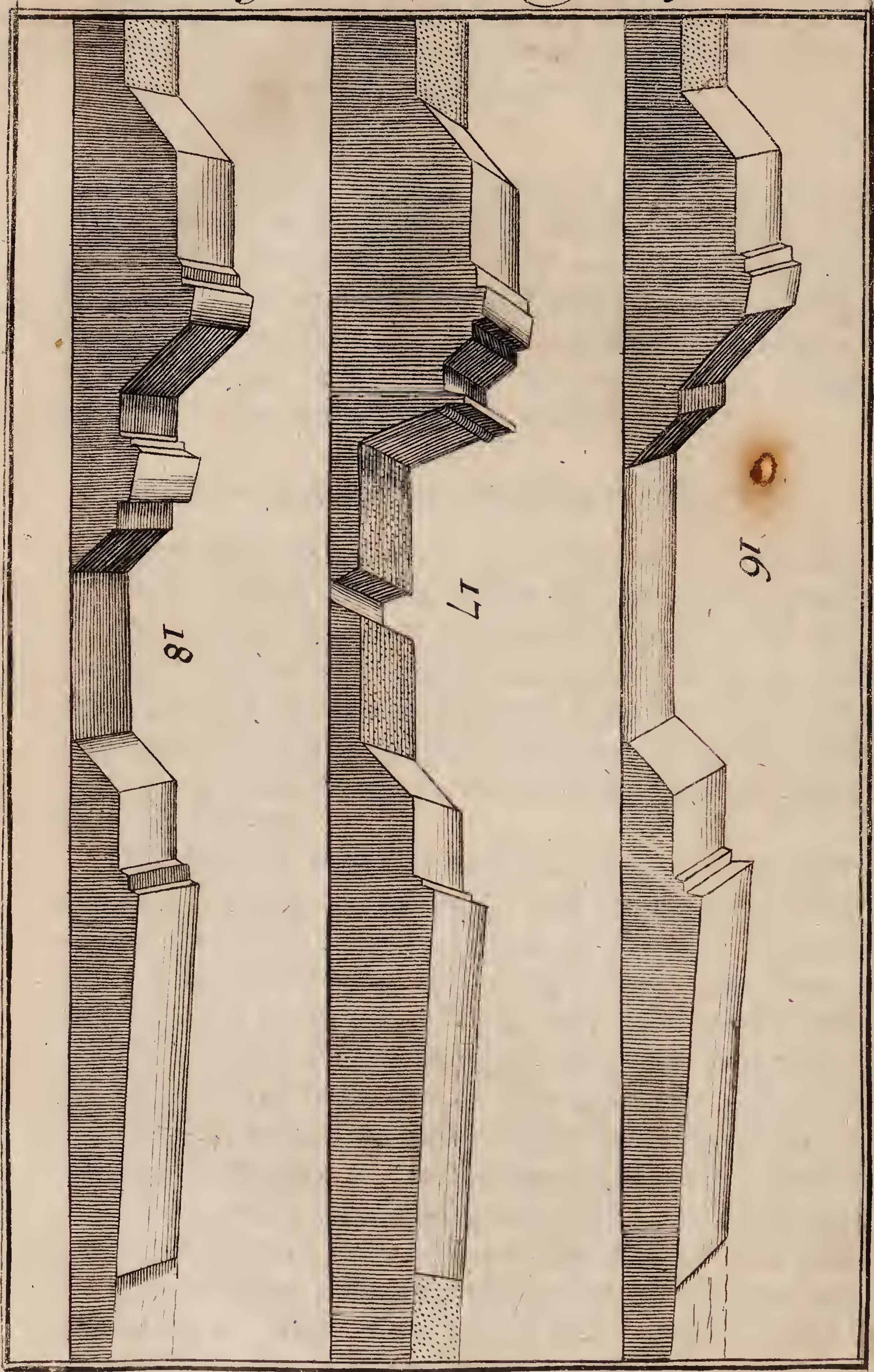
Tho' the False-braye be approv'd of but by few, and made several ways, and almost wholly neglected by the greatest Practitioners, at least if made after the *Dutch* Method, yet I thought my self oblig'd to give first the Profil, then the Plan of it, that you might the better understand what it is, before we explain Mr. *Vauban's* Method, who makes it more useful.

How to represent a Profil in Perspective.

PErspective in general, is the Art of representing in the *Table* or *Section*, visible Objects, as they appear in the Table, for that Reason suppos'd to be Transparent. Hence it is plain that an Object to be visible must be at a reasonable distance from the Eye, which also must be at a reasonable distance from the Table, which we must always conceive to be between the Eye and the Object.

This kind of Perspective is not often us'd in Fortification, as being too difficult, and altering the Proportion of the Parts to be represented, such as are most distant from the Table, appearing always less than the nearest; but a more simple and easy Perspective is us'd, which is call'd *Perspective Cavaliere*, and *Military Perspective*, which supposes the Eye infinitely distant from the Table, which in such a Supposition does not at all change the Figure of the Geometrical Plain: And tho' this is naturally impossible, the Sight not being able to reach an infinite Distance, yet the effect of it is Useful, because it shews distinctly the thing to be Represented; therefore we shall make use of it as well for the Plans as for the Profils, in the following manner.

Plate 5. Fig. 16, 17, 18. Having drawn the Profil according to the foregoing Rules, draw on which side of the Profil you please, from all the Angles, Straight Lines parallel to one another,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other, and of ſuch an equal Length as you think fit; then join the ends of all thoſe Lines by Right-lines, which will make a Second Profil, and ſhade the Parts which you wou'd repreſent to be from the light.

How to draw the Plan of a Fort, whoſe Enceinte is of the firſt kind.

WE call *Plan* or *Ichnography*, the Horizontal Section of a Military Work: That is, the Section of that Work when cut by a plain Parallel to the Horizon, where the length of Lines, the aperture of Angles, the thickneſs of the Rampart and Parapet, and the breadth of the Ditch, *&c.* are ſeen, in their true Proportion.

To deſcribe the Plan of the Rampart, Ditch and Eſplanade, draw the Polygon you want, with the *Maſter-line*, which we ſhall always make thicker than the other lines, for diſtinction ſake. Or if the Paper is not big enough, deſcribe the Triangle of your Polygon as ABC, with a Curtain and Two Demi-Baſtions. Within the Polygon draw the Line DE parallel to the Maſter-line, and 3 Toiſes diſtant from it, for the Baſis of the Parapet, which muſt every where follow the Maſter-line at 3 Toiſes diſtance. Draw again, within that, the Line FG, parallel to the Curtain, and 15 Toiſes diſtant from it, for the Baſis of the Rampart, as you find it in the foregoing Profils. *Plate 6. Fig. 19.*

Then deſcribe outwards, from the point of the Baſtion for your Center, the Arch HI, with the diſtance of 20 Toiſes for the Breadth of the Ditch, and the Counterſcarp IK, which touches in one Point, or raſes the Arch HI, and tends to the end of the Orillon of the oppoſite Flank, when there is one, as there is commonly, that the Ditch may be defended from all the retir'd Flank. Draw LMN for the breadth of the Cover'd Way, 5 Toiſes diſtant from the Counterſcarp and parallel to it. Laſtly, draw OPQ parallel to the foregoing Line LMN, and 20 Toiſes diſtant from it, for the Breadth of the Eſplanade.

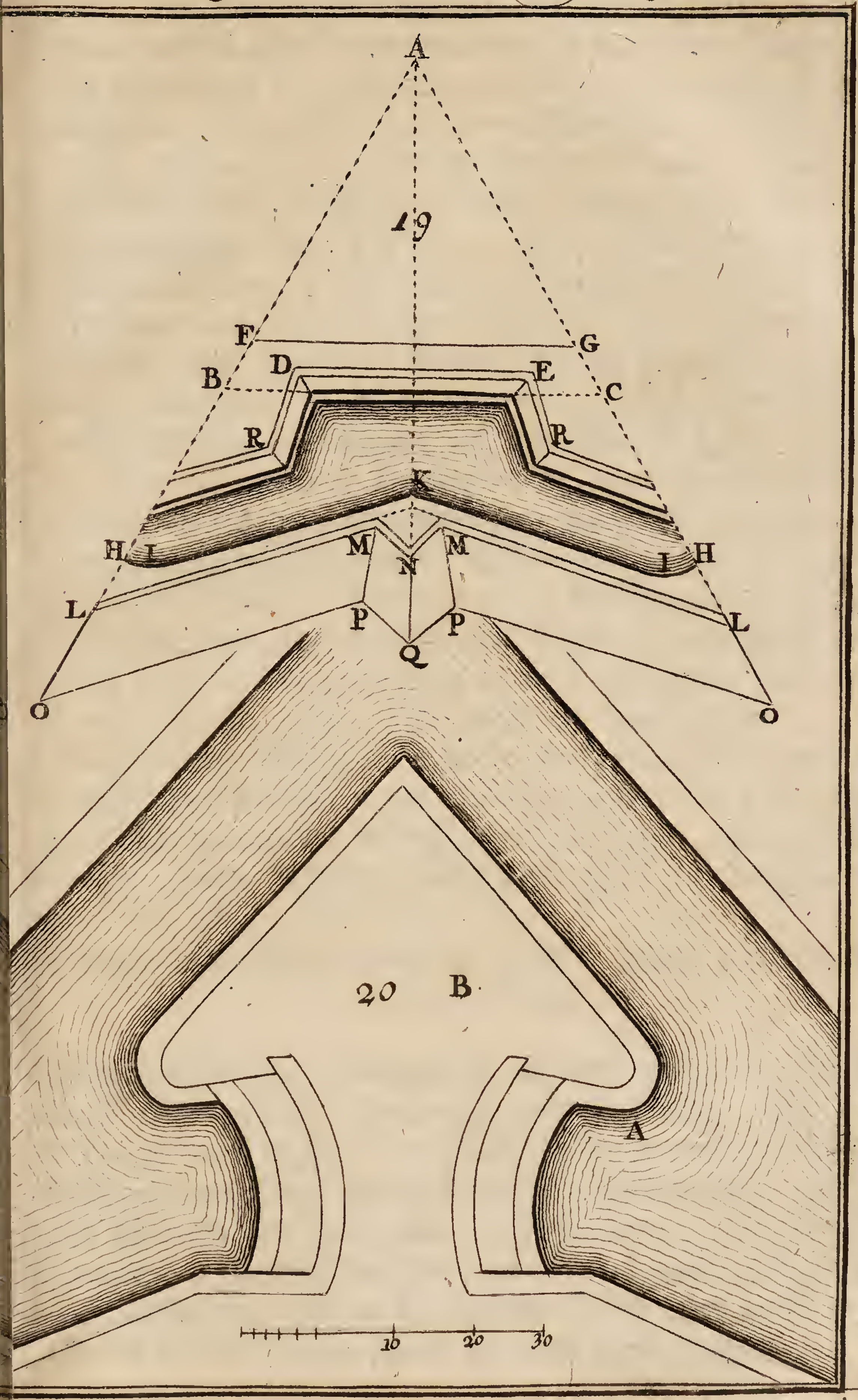
The point K is the Angle of the Counterſcarp, where we have added the place of Arms MNM, the better to flank the Field. We have made its Demi-gorges of 10 Toiſes each, and allow'd 12 Toiſes for each of its Sides or Faces MN.

The Coridor or Cover'd Way LM, is ſaid to be Enfiled from the oppoſite Angle of the Epaule R, becauſe from that Angle one may ſee all the Cover'd Way. The double Line which we have added upon the Terre-plain of the Rampart, and in the Cover'd Way, repreſents the Banquette; and that which we have added in the Ditch beyond the Maſter-line, repreſents the Berme. Laſtly, the tranverſal Lines drawn in the Parapet of the Rampart, and in the Eſplanade, repreſent the Glacis or Slopes.

How to draw the Plan of a Fort, whoſe Enceinte is of the ſecond kind.

HAving, as before, deſcrib'd the Maſter-line of the Baſtions and Curtains, draw parallel Lines according to the Breadths mark'd in the ſecond Profil, which will be taken, upon the Scale of the Plan, if there be one, or upon the Compaſſes of Proportion, having open'd them in ſuch manner that the diſtance of the points mark'd 120 on both ſides upon the Two Lines of equal Parts, be equal to the inward ſide, becauſe it is always ſuppos'd of 120 Toiſes.

As for the Rampart and its Parapet, they are mark'd within the Maſter-line, and the Wall beyond, which always is parallel to it in every place. The Rampart is ſo too ſometimes, and then the Baſtions are Hollow, and eaſy to *Countermine;* that is, to make *Countermines* and *Caſcans* in; which are holes in the Ground like Wells, from whence a paſſage under Ground is made to give Air to the Enemy's *Mine*, which is a Hollow that he makes under the Place which he has a mind to blow up with Powder. The firſt Paſſage or *Head* that is drove under Ground, which is but juſt wide and deep enough for a Man to work upon his Knees, is call'd *Gallery* or *Ca-*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Canal, in the end of which is made the *Chamber* of the Mine, call'd alſo *Fourneau*, which is a little Room that the Barrels of Powder are put into, to blow up the place deſign'd. This Chamber is a hollow Cube 6 Foot every way. Sometimes 'tis made in the Shape of a Croſs, to give the fire a paſſage upwards, when it is fear'd that the Beſieged ſhou'd give it Air, which may happen when the Enemy makes it in an empty Baſtion. When the floor and ſides of the Chamber are dry, inſtead of Barrels, only Sacks of Powder are us'd; when they are wet, the Mine is made as a hole, which the Barrels, call'd *Caiſſons*, of Powder are put into, and they are fir'd at once by ſeveral *Saucidges*, or Pipes made of tarr'd Cloth, ſew'd in the form of a Gut, which reach from the *Fourneau* to the place where the Engineer ſtands to ſpring the Mine. Laſtly, when the Earth is rocky, or ſuch, that one cannot make the Mine in the deſign'd Place, little Fourneaux are made, and an *Araignée*, which are ſeveral little Heads that end in ſmall Mines, call'd *Faugates*, or *Fougaſſes*, which are ſprung all at once, being fir'd by ſlower or quicker Saucidges, as every Mine or Fourneau is nearer to, or farther from, the main Saucidge, which begins at the entrance of the *Araignée*.

How to deſcribe the Plan of a Fort, whoſe Enceinte is of the third kind.

DRAW the Polygon as before, with the Maſter-line of the Curtains and Baſtions. Make the Rampart and Parapet as uſual, in ſuch manner that the Parapet be always parallel to the Maſter-line. Beyond the Maſter-line deſcribe the Falſe-braye every where parallel to the Maſter-line, by Two Lines, 3 Toiſes diſtant from it, and as far diſtant from one another for the Way or Terre-plain of the Falſe-braye, and the ſpace of it.

We have already ſaid that ſuch a Falſe-braye is not much approv'd of, and that Mr. *Vauban* makes it after another manner, as we ſhall explain in its proper place.

We have not mark'd the *Talu's* in *Fig.* 19. to avoid confusion. They are only represented in large Plans.

How to draw the Plan of a Fort with High and Low Flanks.

LOW Flanks are made in a Fortress to hinder the Passage of the Ditch, by ruining with Cannon all the Enemy's Works and Machines; especially the Gallery, which he usually makes in the Ditch, when he has pierc'd thro' the Counterscarp, which is call'd *Sapping*, to bring the Miner to the Face of the Bastion, and after to come to an Assault, when the Mine has made a considerable Breach.

Plate 6. *Fig.* 20. Since then Low-flanks are made to discover and flank the Ditch, and especially the Face of the opposite Bastion, they should be cover'd by an Orillon, whose make has been taught in *Fig.* 9. *Plate* 3. where it appears, that the Cover'd or Low-flanks answer to the Face of the opposite Bastion, it being superfluous that they should discover more, because they wou'd be more expos'd, which wou'd be a great fault.

As in these Low-flanks pieces of Cannon are put for the defence of the Ditch, they ought to have a Parapet upon their outward edge, 6 Foot high, and 18 or 20 Foot thick, with Embrasures, whose Merlons must be of good sound Earth, and a Platform of above 4 or 5 Toises for the Cannon to be in. This Parapet covers the Cannons, and the Orillons preserve the Low-flanks, that is, hinder them from being broken by the Enemy's Cannon.

The High-flanks are absolutely necessary in a fortified Place, because they supply the want of the Low-flanks, when they are out of order and broken: Besides, they oblige the Enemy to make his Trenches, Batteries and other Works the higher. They have a Parapet level with, and like that of the Place; that part of the Rampart which remains in the Demi-gorge, serving as a Battery for the Cannons, which are plac'd behind that Parapet every 3 Toises, when there are Embrasures, as in the Low-flanks. This

This Figure, and one's own Reaſon ſhews, that the High-flank ought to be more retir'd towards the Demi-gorge than the Low-flank, whoſe Battery may be level with the Field, that is, 15 Foot above the Bottom of the Ditch, which we have made of that depth; tho' this Battery may be made a little lower, to be more out of the reach of the Enemy's Cannon. Whence it is plain, that when the Demi-gorge is very large, as by our Method it happens in the great Polygons, one may have 3 Cover'd Flanks, a Low one, a Middle one, and an High one, after Count *Pagan*'s Way, as we ſhall ſay more fully, when we explain his Method. But as our Flanks are great, methinks Two Cover'd Flanks ought to ſuffice, Three ſeeming more prejudicial than uſeful; becauſe when ruin'd by the Enemy's Cannon, they would be as ſo many Steps for him to Mount as he gives *Aſſault*; that is, to Mount the Breach that is made by the Cannon in the Body of the Place; and when a conſtant fire is made upon ſuch a Breach to defend it, as it may happen, if the Breach is in the Flank, becauſe it may be ſhot at from the other Flank, we call it *Voir en Breche* (to ſee in the Breach). *Plate 6. Fig. 20.*

In each Cover'd-flank there ought to be a little Gate or *Poſtern*, thro' which one may go out at the bottom of the Ditch without being ſeen by the Enemy. Such a Poſtern is of uſe, not only to let out a Guard to go to the Out-works, and to *make Sallies*, that is, to go out of the City in order, openly to attack the Beſiegers, and oppoſe his Deſigns, but alſo to go to the *Coffers*, which are made before the Low-flank in caſe of an Attack, without which Poſterns, one cou'd not go into the Ditch, nor conſequently defend thoſe *Coffers*, which are only a little Ditch made in the great one, when it is dry, over againſt the Low-flanks, 15 or 20 Foot wide, and 6 or 7 deep, cover'd with Planks and Earth, rais'd about 2 Foot higher than the bottom of the Ditch, after the manner of a Parapet, to have ſeveral Port-holes in, for ſmall pieces of Artillery, which are plac'd in ſuch a Coffer to defend the Face of the op-

 poſite

posite Bastion, and to hinder the passing of the Ditch. Such Coffers are only made in a dry Ditch, and when there is no False-Braye, which wou'd cover them, and render them useless. But in instead of Coffers we often make in the Ditch, before the middle of the Tenailles, *Caponieres*, which are Lodgments 4 or 5 foot deep, having on each side a palissado'd Parapet, about 3 foot high, as a double cover'd Way, to cover the Musketeers lodg'd in it, who fire thro' the *Meurtieres*, and pass thro' such a Caponiere to get to the Out-works. These are often made upon the Glacis of the Esplanade, to repel the Enemy when he endeavours to take the cover'd Way. They are useful to scoure the Ditch, and hinder its passage.

These Posterns are made at A, the Bottom of the Orillon, as we have said elsewhere; and below the *Place haute*, or high Flank, a Vault is made to bring the Cannon into the Casemate, or low Flank, thro' the Passage B, in the upper part of the Orillon, by which one descends to go to the bottom of the same Orillon. From the Postern A one may go into the Coffers, which are over against the low Flank thro' a cover'd Alley or Trench, which is made in the Ditch near the Orillon. The length of the Coffer takes up all the breadth of the Ditch, in this it differs from the Caponiere, which takes up but part of it. The Besieger *Epaules*, that is, covers himself against Coffers, by casting up Earth on that side that the fire of the Musketeers comes from the Coffer.

Near the high Flanks, upon the Rampart at the Angle of the Flank, Cavaliers are added to fire a great way, disturb the Enemy in his Trenches, break his Batteries, and dismount his Cannon. Cavaliers are also erected upon the Curtains for the same end, especially when there are, within Cannon-shot, rising Grounds which command the Place, and which one has not time to level; otherways it is better to level them, or leave them when they are out of Cannon-shot, which is about 1000 Toises point blank, because then only Steeples or Chimneys can be hit.

When rising Grounds beat *de revers*, (that is, in the Rear, or upon the Back of any place,) in the Baſtions, which is the worſt command, or upon ſuch places as ought to defend others, then *Traverſes* or Parapets are built along the Capitals, about Six Foot high towards their Banquette, and five on the other ſide; and when ſuch Hills *enfile* the Faces, and diſcover Flanks, Traverſes are made in the Ditch. But one may hinder a Baſtion from being commanded, by raiſing its Point; or keep a Battery from playing *de revers* in a Baſtion, by raiſing the Flank and Face on the ſide towards that Hill.

Laſtly, when riſing Grounds are near the Place, they muſt be encloſ'd in ſome Out-work, as in a Horn-work, or in a Crown'd-work whoſe conſtruction will be taught in the Second Part: or elſe when they are very near the Place, as within Musket-ſhot, a Citadel is built about them, as will alſo be taught in the Second Part.

How to deſcribe a Fort with the Place of Arms, and the chief Streets.

THE *Place of Arms of a Fortified Town*, is a great Space free and void, of the ſame ſhape with the Fortified Polygon, uſually left in the middle of a Town, to diſcover equally from all ſides, and where the chief Streets end, which ought to anſwer not only towards the middle of the Curtains to the Gates, for the conveniency of the Inhabitants, but alſo to the Gorges of the Baſtions; that from O, the Center of the Place, the Governour or Major may ſee what's done in all the Attacks, and ſend a ſpeedy relief where it is neceſſary, without being oblig'd to go to inform himſelf where 'tis needful, and that thus ſeeing all the Baſtions, the middle of the Curtains, and the Gates, the Town may be kept in Subjection. In the Place of Arms the Souldiers are call'd together to receive Orders, and to Exerciſe. *Plate* 7. *Fig.* 21.

Its bigneſs ought to be proportion'd to the Fortified Polygon; which, methinks, can be done no way bet-

ter, than by allowing for its Radius OG, or OH, when the Place is regular, the Length of a Demi-gorge, which always increases as the Polygon has a greater number of Bastions, according to our Method, at least as far as the Decagon.

As for the Streets, there is always one made along the Rampart, a little wider than the rest, as of 8 Toises, that there may be room to retrench in case of necessity, and to set the Souldiers in Battle-array, and put 'em in Order of defence upon a sudden Alarm. This Space serves also to hinder the Houses from having any Communication with the Rampart, the better to secure the Rounds, the Guards, and the whole Place.

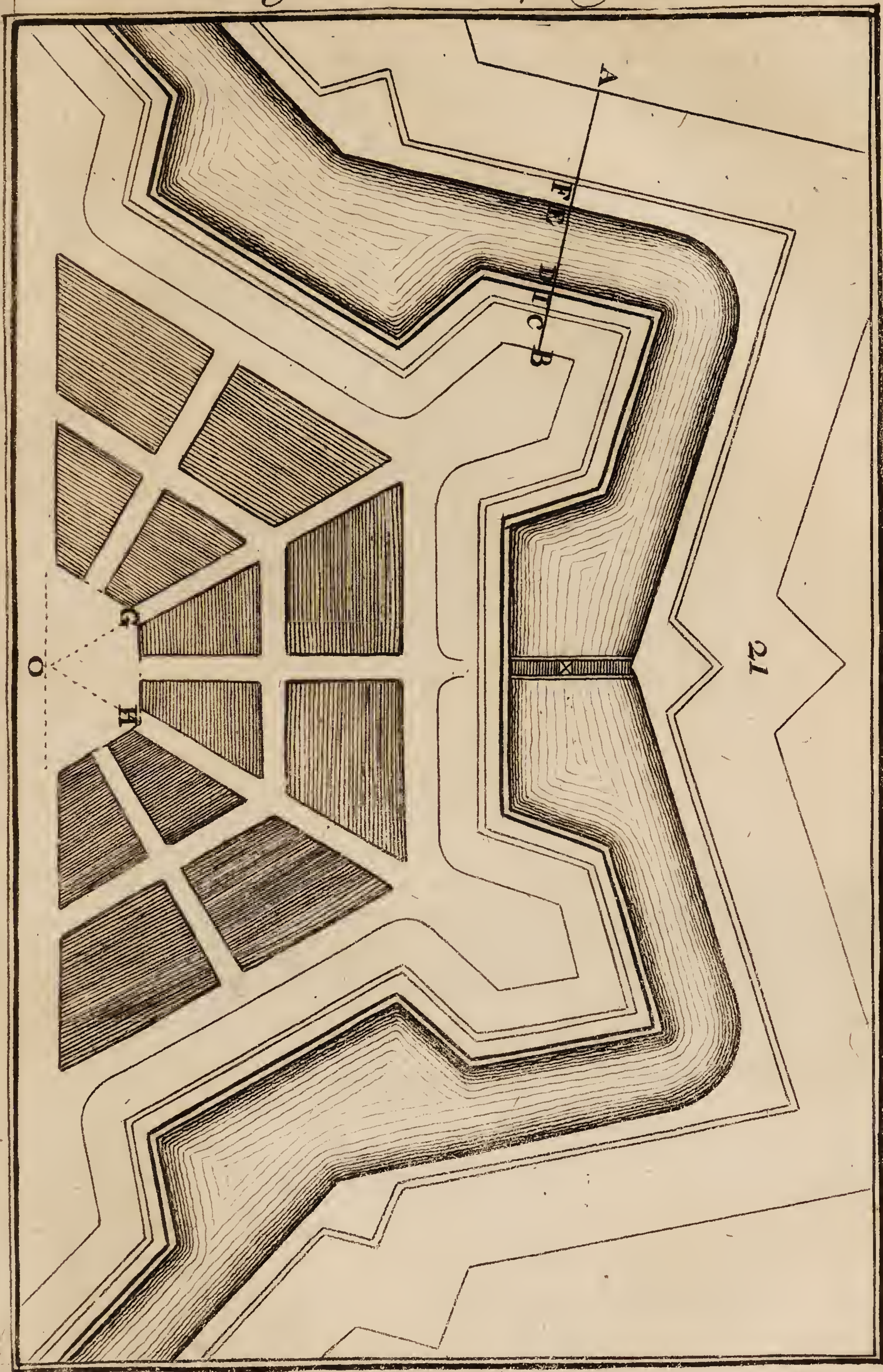
The other Streets are made narrower, as of Six Toises at most in great Places only, for their breadth is less in little Places, as of 3 or 4 Toises. They have a Communication one with another, and the chief are drawn along the Radii of the Polygon, and the lines drawn from the Center of the Place thro' the middle of the Curtains, to answer to the Center of the Bastions and the Gates.

The Gates of the City are generally made at the end of such Streets as answer to the middle of the Curtains, which is the strongest place, as being defended by both the Flanks, which ought to be so, because the Gates are in more danger of a Surprise than any other Part. They ought to be 10 or 12 foot wide, and 15 or 16 high.

The fewest Gates that may be, are made in a Place of War, because then there won't be occasion for so many Corps de Guard, which are necessary to prevent Surprises. The Way to them is thro' Vaults made under the Rampart, in the middle of which there

Plate 8 *Fig.* 22. are *Orgues*, which are several long and thick Pieces of Timber, arm'd with Iron at bottom as A, half a foot distant from one another, and suspended by a Rope C, from the Roller B, which is above, and is let loose when we wou'd have these *Orgues* fall all at once upon the Passage in case of any Enterprise, to barr up the

Way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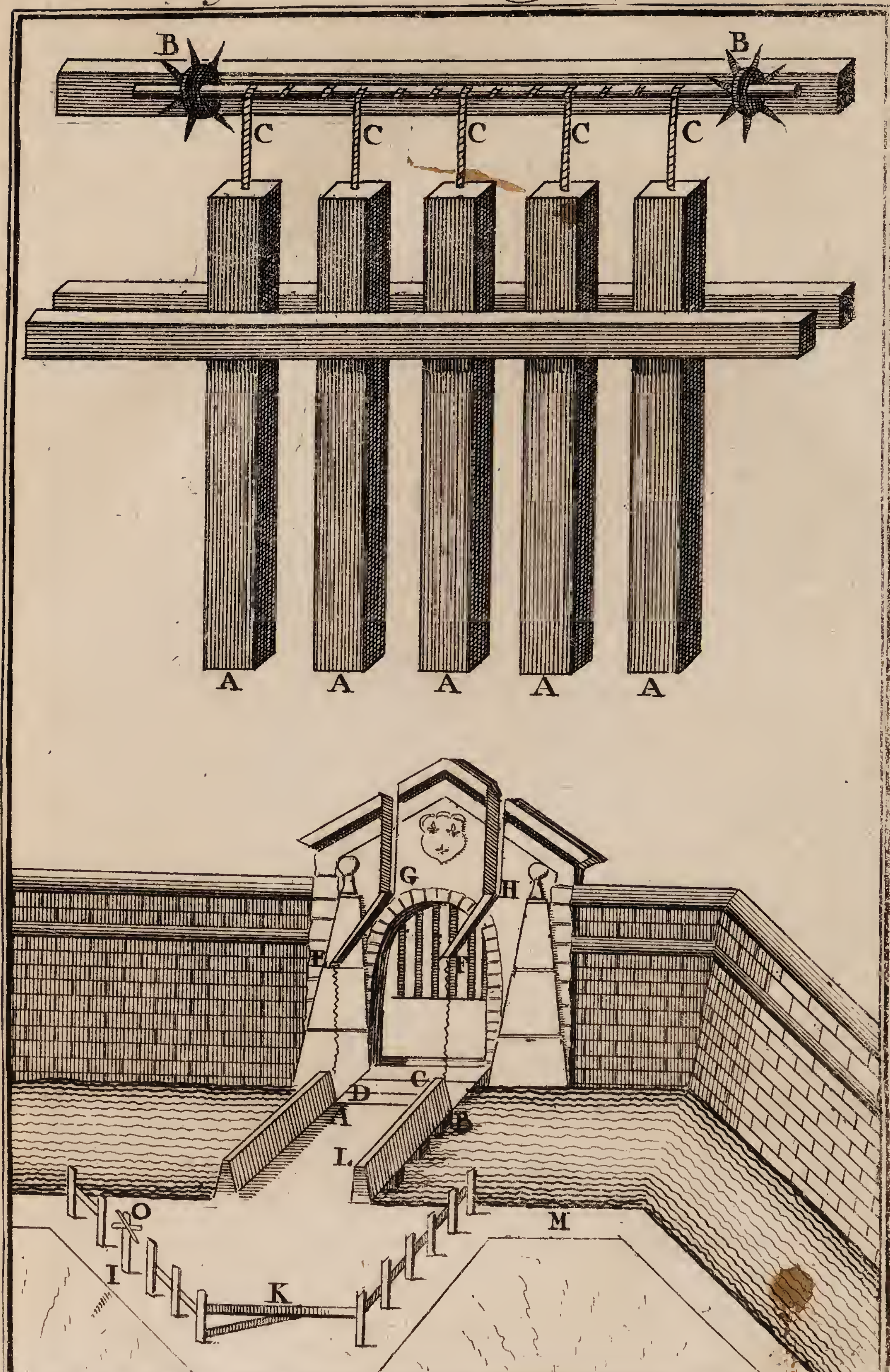
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Way as a Door wou'd, and tho' broken by a * Petard or Cannon they fall inſtantly and by their length ſtill Stop up the Way.

Orgues are better than *Herſes, Lattices* or *Portcullices*, which are a kind of Door made in the form of a Harrow of ſeveral Pieces of Timber, arm'd with Iron Spikes as AB, let down by means of a *Moulinet* or Roller, where it is ſuſpended by a Rope as the *Orgues*, when the City-Gate has been broken; becauſe a Petard may break the *Herſes* without any ſpeedy and eaſy Remedy, and they may be kept from coming down by ſetting a Piece of Wood upright in the Grooves made in the Door-caſe for them to ſlide in: or elſe by putting under them * *Chevalets*, which are a kind of four-footed Tables, as AB, or CD; or an over-thrown Waggon. *Plate* 9. *Fig.* 25. * *Treſſels.*

The Vault mention'd, has two other Vaults, one on either ſide, call'd *Corps de Guard* or Guard-Houſes, becauſe a Guard of Souldiers is there plac'd to prevent Surpriſes. Several Gates are uſually made in the ſame Vault or Entrance, with ſeveral Guard-Houſes to defend the Town from ſurpriſes, and hinder the effect of the Petard, but they ought to be made in a winding Paſſage, becauſe the Petard or Cannon wou'd break ſeveral of them at once, if they ſtood in a right line. *Fig.* 28.

Each Gate is cover'd by a Half-Moon or Ravelin, the Way to it being over the great wooden-Bridge L, which is made in the Ditch of the Place, and is unmoveable, being ſupported by Pillars of Wood, and ſometimes of Stone as B, and for that reaſon it is call'd a *Dormant* (Sleeping) *Bridge*, being diſtinct from another Bridge near the Gate of the Place, call'd *Pont levis*, or Draw-Bridge, becauſe it is drawn up, or let down at pleaſure. *Plate* 8. *Fig.* 23.

Theſe Draw-Bridges are of two kinds, of which the moſt common are call'd the *Ponts à Fléches*, which are drawn up or let down all at once as ABCD, whoſe motion is at CD towards the Gate, and which is ſuſpended at the other end AB by two iron Chains AE, BF, which

* *An Engine of Metal fill'd with Powder to break Gates, Portcullices, &c. deſcrib'd at the end of the Sixth Part, which ſee.*

are

are fix'd to the *Flèches* or ſmall Beams EG, FH, whoſe motion raiſes or ſinks them.

Such a Bridge is none of the beſt; becauſe, as the *Chevalier de Ville* ſays, it is diſcover'd when drawn up, and if the *Flèches* break, it can neither be rais'd nor let down. The beſt are the *Baſcule-Bridges*, which are ſuch as riſe by the means of a Counterpoiſe within. Bridges ought to be built ſo low as not to be commanded from the Field, and ſo wide, that two Waggons a-breaſt may go over them.

A Draw-Bridge is often made only with *Traps*, that is, Planks which are taken off and put on every day. There are generally two Draw-Bridges joyn'd to the main Bridge, one in the middle, and one next to the Gate, and a Centinel is plac'd at each Draw-Bridge: and then the Bridge ſhou'd wind a little, as the Entrance of the Place does.

Draw-bridges are made in every Work, that the high-way goes thro', and when there is no Ravelin before the Gate of the Place, a Guard-Houſe is built in the middle of the Dormant-bridge; but otherwiſe 'tis built in the Ravelin: tho' when there is no Ravelin, there ought at leaſt to be a cover'd Way M, and before the great Bridge or *Pont dormant*, a good Paliſade I, with *Turn-Stiles* O, and a *Rail* K, to open and ſhut for the paſſing of Carts.

Laſtly, in the City along the Rampart are built *Caſerns* or little Houſes for the Souldiers of the Garriſon, that they may be leſs burthenſome to the Inhabitants. Each Caſern has generally two Beds, to lodge Six Souldiers, three and three. Half of theſe mount the Guard, and the reſt ſtay to ſecure that Quarter. The Horſe uſually lodge three or four in a Caſern, and the Foot Six. Magazines alſo are made in the neareſt Streets to the Rampart, that the Ammunition may be carried the eaſier. The Magazines of Powder ought to be the fartheſt from one another, and in the cloſeſt places that may be.

How

How to draw the Plan of a Fortification upon the Ground.

THE Way to deſcribe a Fortification upon Paper differs much from that of drawing it upon the Ground; for here you cannot uſe a Scale and Compaſſes for meaſuring of Lines, nor a Protractor for meaſuring Angles: but inſtead of a Protractor, you muſt have a **Graphometer*, to draw upon the Ground Angles of as many degrees as you pleaſe, as we have taught in our *Introduction to Mathematicks;* and inſtead of a Scale and Compaſſes, you muſt have Stakes to faſten *Lines* to, and a pretty long Chain divided into Toiſes and feet to meaſure lines.

* *An Inſtrument mention'd by the* French *Author.*

We have already taught how to deſcribe upon the Gound the *Maſter-line* of the Curtains and Baſtions, when we ſhew'd how to Fortify by computed Tables: what remains is to teach how to draw about ſuch a Place the Plan of the Rampart, Berme, Ditch, Cover'd Way, and Eſplanade, which muſt be done before the Earth be dug up.

Plate 7. *Fig.* 21.

Having drawn thro' the middle of the Face of the Baſtion the line AB, as near as one can, perpendicular to it; take on the inſide of the Face from I to B, 15 Toiſes for the Baſis of the Rampart; and on the outſide of it from I to D, 4 Foot for the Berme, from I to E, 20 Toiſes for the breadth of the Ditch, from E to F, 5 Toiſes for the Coridor, and from F to A, 20 Toiſes for the Eſplanade.

We have not mention'd IC the breadth of the Parapet, becauſe that breadth muſt be taken upon the Terre-plain of the Rampart, which is not yet rais'd. Having prepar'd things thus, with a Spade or a Plough make on the inſide thro' the point B, a furrow parallel to the Maſter-line for the beginning of the Rampart, if you wou'd have hollow Baſtions, and on the outſide thro' the point I, another furrow parallel to the Maſter-line alſo for the Berme.

The Ditch is eaſily drawn, if thro' the point E, the Counterſcarp be deſcrib'd parallel to the Face of the Baſtion, when all the cover'd Flank can defend the Ditch

Ditch, otherwise it must be drawn to the revers or back part of the Orillon, if there be one. For the rounding of the Ditch, take a *Line* 20 Toises long, or as long as the breadth of the Ditch, and fastening one end of it to the Point of the Bastion, the other will describe the Arch requir'd.

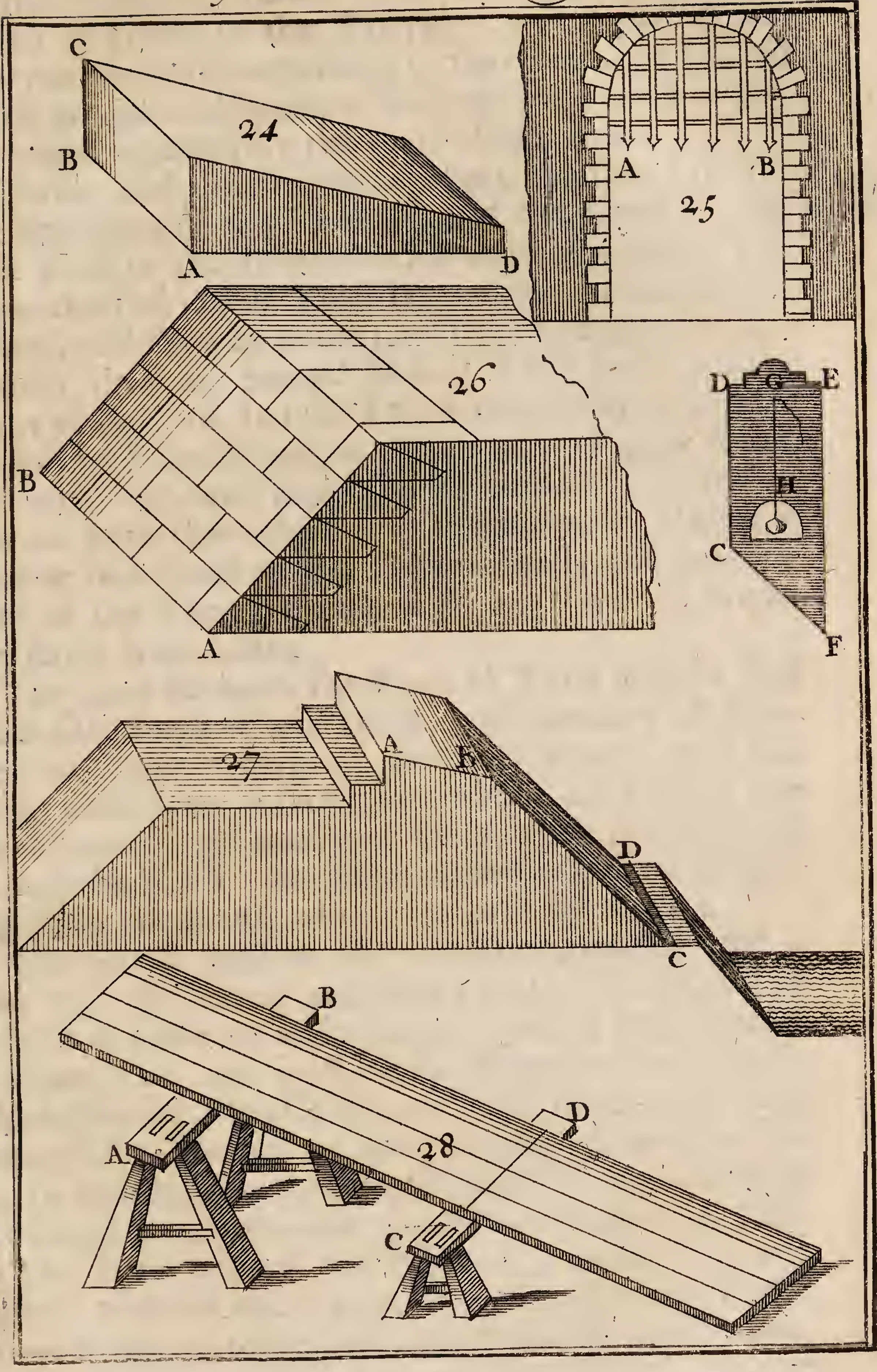
Lastly, Thro' the point F, draw a line or furrow parallel to the Counterscarp for the cover'd way. You may do as much thro' the point A for the Esplanade, but it is not so necessary, nor so easy by reason of the unevenness of the Ground, 'twill suffice to make the Glacis of the Esplanade lose it self gradually towards the Field, about 15 or 20 Toises off from the Cover'd Way.

How to raise the Rampart with its Parapet, and the Esplanade with the Earth dug out of the Ditch.

HAving describ'd upon the Ground the Plan of the Rampart, Ditch, and Esplanade as before taught, we must begin with the Foundation of the Rampart, which is dug down five or six foot, when the Earth is not firm enough, and deeper, as 7 or 8 foot, when the Soil is Sandy, and in such a case it is pav'd with Planks, sometimes *Piles* are stuck in it, which are great Oaken Stakes, sharp at one end and arm'd with Iron at each end. These Piles are driven into the Earth as far as the *Ram* * can drive them, then their heads are fix'd with Stones that are forc'd between them; or else when there is Water or moving Sands, the Piles are fasten'd together with cross-pieces of Wood, like a close Lattice pinn'd with Iron, to fix the heads of all the Piles, upon which a Floor is made for a Foundation; and if the Earth be moving, the Piles are driven a little slanting to resist the push of the Sands.

* or Beetle.

Plate 9. *Fig.* 24. That the outward part of the Rampart may bind the better, 'tis cover'd with Turfs made in the Shape of a Wedge to cleave Wood as ABCD, and whose length AD is 15 inches, breadth AB 6 inches, and thickness at BC, the bigger end which stands outwards, 5 inches. As for its thickness at the other end D, we shall leave it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it undetermin'd, becauſe it depends upon the ſlope which is given to the Rampart; and the Workman muſt cut the Turfs according to that ſlope. The beſt Turfs are got in Meadows, becauſe their earth is better bound together by the roots of the Graſs.

Having laid the firſt Turfs along the line AB, for the beginning of the Foundation of the Rampart, peg each Turf to the Ground with a good wooden Pin. Upon this firſt Bed of Turfs lay another, and upon that a third, and ſo on up to the top of the Rampart, in ſuch manner that the Second Bed of Turfs cover all the joints of the Firſt, and the Third likewiſe may cover all the joints of the Second, and ſo on as the Figure ſhews. *Plate 9. Fig. 26.*

Every Turf muſt have a ſtrong wooden Pin thro' it, and to bind them the better, Branches of Willow about an inch thick are ſtuck Horizontally into the fore-part of the Rampart, each thro' two Turfs to hinder the Earth from falling.

The ſpace between the Rows of Turfs muſt be fill'd with the Earth of the Ditch, well beaten with Rammers or *Pilons*, which are pieces of Wood 5 or 6 foot long, with Iron at each end; which are held by two Handles about the middle of them, when they are us'd to beat the Earth. Pavers commonly uſe them in beating down the Stones when they pave the Streets.

The Earth muſt be wet when it is ramm'd, that it may joyn the better, and thus a foot of Earth will be reduc'd to ſeven or eight inches; and to keep true to the ſame Talu, one muſt uſe the *Hanging-Level* CDEF, whoſe *Bevel* or Slant CF ſuits with the ſlope of the Talu, fitting CF to a board laid along the beginning of the Talu, in ſuch manner that the Thread GH may fall upon the line, which ought to be parallel to the ſide CD, or EF.

The Terre-plain of the Rampart is made to incline a little towards the Town, as well to let the Water run that way, as to facilitate the Recoil of the Cannon, which thus goes out of Sight of the Enemy recoiling after every ſhot: and 'tis levell'd about a Foot higher than it ſhou'd be, becauſe of its ſinking as it recoils.

Trees

Trees are planted here, and eſpecially Elms, which are the beſt: their Roots are uſeful in binding the Earth: of their Timber are made the *Carriages* for the Cannons, and of the Boughs *Faſcines*, which are Faggots of ſmall Branches.

Upon the outward part of the Rampart, at the foot of its Parapet, that is, level with the Terre-plain, are ſtuck *Fraiſes*, which are Square Oaken Stakes, ſtuck Horizontally, within Six inches of one another, ſtanding 10 or 12 foot out of the Ground, to prevent Scalades and the Deſertion of Souldiers.

Plate 4. *Fig.* 27. When the Rampart is finiſh'd, and the Earth well ſettled, the Parapet muſt be rais'd upon it after the ſame manner, Six foot high on the fore ſide A, and 4 foot high at the back ſide B, covering it with Turfs on both Sides, and alſo on the top, tho' 'tis better to Sow it with Hay-Seed, or ſome Herb which does not grow very high.

You muſt leave, as we have already ſaid, a Way CD between the Rampart and Ditch, call'd *Berme*, to Strengthen the Rampart, and ſtop the Earth of the Parapet, which might fall of it ſelf, or by the Shock of the Enemy's Cannon, and without that Berme wou'd fall into the Ditch and Spoil it.

In making the Talu's of the Ditch, which muſt be the laſt things that are made, becauſe they might be broken during the other work; their Earth may ſerve for the Eſplanade, which ought to be Six Foot high, and 15 or 20 Toiſes broad, as we have ſaid elſewhere, and which, as much as may be, ought to be of Stones or Pebbles cover'd with Turf.

Works of Earth ought not to be made in Winter, by reaſon of the Froſt, but in Summer, when the Earth is dry; the beſt kind of it being Clay or Marle, which binds beſt, and does not require ſo much Talu, beſides it produces a great deal of Graſs, whoſe Roots make it firm and ſupport it.

Sandy Earth is not good, and gravelly worſe, being hardly kept up without a Wall, whoſe Foundations muſt

muſt be laid as thoſe of the Rampart, or elſe by digging down to firm Ground, and filling up the Hole with Mortar made of hot Lime, mix'd with Gravel and Pebbles, which bind ſo well together, that Stone it ſelf is not harder.

The depth of the Foundations is reckon'd from the firm Ground to the Level of the Ditch. They ought to contain the Talu's of the Wall, which beſide its Talu ought to widen a Foot and a half at bottom on each ſide to make it the more Solid: and this is call'd *Retraite.* This Talu is commonly equal to the ninth part of the Wall's height, which is about 4 Toiſes above the bottom of the Ditch. Whence it follows, that to lay the Foundations of the Wall right, you muſt know what height 'tis to be of.

Tho' in the Profils we have made the thickneſs of the Wall 9 Foot, nevertheleſs that ought not to be taken for a general Rule, becauſe that thickneſs ought to alter, according to the Nature of the Soil that it is to ſupport, firm Clay not wanting ſo thick a Wall as a lean, or ſandy Soil. It is generally made four Foot and a half thick about the Cordon, that is the height of the Rampart; and three may do, if the Maſon's Work be good.

We have ſaid elſewhere that *Eperons* or Counter-forts are added to the Wall, to make it Stronger, and the better able to ſuſtain the Earth which is always preſſing outward: and here you muſt note, that when there is a Wall, there is no need of a Berme or Fraiſes, becauſe a Wall is ſteep and can't be ſo eaſily Scal'd: but one may have Paliſado's at the Foot of the Wall, or round ſtraight pieces of Wood, about a Foot thick and 12 Foot long, ſunk half way into the Earth, as near to one another and the Wall as poſſible, to hinder it from being under-mined.

The Rampart muſt be made before the Wall is built, otherwiſe it wou'd hinder one from carrying the Earth out of the Ditch, and alſo to give the Earth of the Rampart to or three Years time to ſink and ſettle; other-

otherwise as it pushes mightily, and especially after Rain, the Wall wou'd have much a-do to sustain it; and be in danger of being overthrown. The Wall may be built of Stone; but Brick is better, because a Cannon-ball only makes a hole in it, without beating it to pieces.

When the Ditch is digging, Pillars are left untouch'd of the same Earth which is taken away, call'd *Temoins*, (Witnesses) by which one may find out how many Cubick Feet or Toises of Earth have been taken away. Several Sorts of Instruments are us'd to dig and carry up and down the Earth and Stones, the most common of which shall be here explain'd in their Order. But before, we must tell you that when the Ditch is dug so deep that the Earth cannot be thrown out without going up, a Bridge must be made for the Workmen about 7 or 8 Foot high, that such as bring up the Earth, and such as go down empty-handed, may pass at the same time without disturbing one another. This Bridge is *Plate* 9. *Fig.* 28. made of several strong Planks fasten'd to one another and set upon Tressels of different height, that the Bridge may rise gradually, and Barrows of Earth may be easier wheel'd up.

An Explanation of some Instruments us'd in Fortification to move the Earth, and break Ground.

THE Instruments us'd to *break Ground*, that is, to dig and open the Earth in order to carry it elsewhere, and make Ramparts and Parapets of it, are first the *Pick-Ax*, so call'd because it pecks and opens the Ground. Its Iron is sharp pointed and a Foot long, with a wooden Handle about 3 Foot long.

The *Mattock*, of about the same bigness as the foregoing Instrument, but different in this, that its point is 3 inches broad, and that 'tis very useful in hard and stony Ground, where the Pick-Ax makes only an hole.

The *Wooden-shovel*, of about the same length as the other two foregoing, to throw the Earth into the Barrows, or into the Tumbrels, to carry it to another place.

This

This Shovel is usually all of Wood, and sometimes it has Iron at the end, or all round the Bit, that it may last the longer and enter better into Clay or hard Earth. Turfs are usually cut with it. *Plate 6. Fig. 20.*

The *Spade*, whose lower part is all of Iron and Square at top to set one's Foot upon, when earth is dug without Pick-Ax or Mattock.

The *Crow*, which is a Bar of Iron about three or four Foot long, to heave up Stones by putting one end of it under them, to separate and lift them from the Earth, and hoist them upon the Hand-Barrows.

That the Crow may take the better hold, it is generally bent and turn'd up at the end, and when it is so bent, the French call it *Pied de Chevre* (Goat's foot:) and the better to raise Stones, under the Crow, which is instead of a Leaver, a little Stone or a piece of Wood is laid to serve for a fix'd Point, or Center of Motion, which the Masons call a *Beat*.

An Explanation of some Instruments us'd in Fortification, to carry Earth from one Place to another.

THE Instruments us'd to carry Earth, are first the *Wheel-Barrow*, which consists of two Arms, a Wheel and a Box or Trough in the middle, which is fill'd with the Earth of the Ditch with a Shovel, and serves to carry it to the Rampart, Parapet, or cover'd Way, *&c.*

Its figure shews, that by lifting up its Arms or Handles one Man may run it along. These Handles ought to be about a Foot and a half long besides the Trough, which is 7 inches deep, 16 long and 15 broad to hold about a Cubick Foot of Earth.

The *Tumbrel*, which is a Cart with two Wheels, rail'd at bottom, with Planks on the sides held by four large Pieces of Timber like Joists, call'd *Blades*; this is us'd to carry Earth and such little Stones as are dug up; for the great and heavy Stones, which the Ornaments and Angles of the Bastions are made with, are carried in Waggons which are are made stronger and go upon four Wheels

The *Doſſer*, or *Panier*, made of Ozier twigs pretty deep, carried generally upon the back with Braces or Straps of Leather, that Men thruſt their Arms thro' to carry the Panier, which is only to take the Earth a little way. It holds about half a ſolid Foot of Earth.

The *Hand Barrow*, which is an Inſtrument of Wood, uſually of Beech, with four Handles about 2 Foot long a piece, with a Grate or Bars in the middle, to lay the Stones upon, that are found in the Ground, that they may be carried by two Men to the place where they are wanted.

Maſons uſe a Stronger Hand-barrow to carry heavy Stones by the help of ſeveral Men. They alſo uſe one of another kind, made like a Ladder, to carry carv'd Stones to build with, and other Materials in Troughs, which hang from this Barrow by four Cords and an Iron Hook.

How to raiſe the Plan of a Fortification in Perſpective.

THE Art of repreſenting in Perſpective an Object rais'd upon its Geometrical Plan, is call'd *Scenography*, whether this repreſentation be drawn in true, or in Military Perſpective, which we ſhall uſe here, as being fitteſt for the Matter we treat of, becauſe as we have ſaid elſewhere, it does not alter the Geometrical Plan, and ſhews what is rais'd, and diſtinguiſhes every Part in particular as well as true Perſpective, which requires a deal of time, and is very hard to ſuch as wou'd draw according to all its rules, and changes the Proportion of the Baſtions, which tho' regular, appear too irregular, and as it were, lame and defenceleſs.

Having drawn in White (that is, with the points of your Compaſſes upon Paper) the Ichnographical Plan
Plate 3. of what you wou'd raiſe in Perſpective, draw from all
Fig. 12. the Angles of that Plan, except thoſe of the Talu's, White lines parallel to one another, and Perpendicular to one ſame right line, drawn at pleaſure, as VX. Then begin to raiſe the higher Part, (*viz.*) the Parapet, becauſe the higheſt Parts hide from the Eye thoſe that are lower

lower behind; the higheſt Parts therefore you may draw in Black without fear of a miſtake. Taking then all the Perpendiculars, which go from the Angles of the Plan of the Parapet, of one length, according to the height that you wou'd give the Parapet, which ſhou'd be greater than it is in reality, that the Plan may look the better; joyn the ends of all thoſe lines with Black lines, and the Parapet will be rais'd, if you draw lines downwards from the outward ends to the Talu's which anſwer to the right lines, as AR, *&c.*

Then draw the Perpendiculars, CO,EF ,and all the others which go from the Angles of the Plan of the Rampart, of the ſame length as you deſign for the height of the Rampart, and joyn the ends of thoſe Perpendiculars by the right lines OF, OL, *&c.* that is, ſuch as are not hid by the Parapet, whoſe height LM above the Rampart muſt only be mark'd on the inſide, and the Rampart will be rais'd, if likewiſe you draw its Talu's, as FD, *&c.*

Laſtly, take downwards the Perpendiculars PS,HT and all the others, which ſhall go from the Angles of the Plan of the Ditch, equal to the depth that you wou'd give the Ditch, and joyn likewiſe their ends by right lines, at leaſt ſuch as are not hid from the Eye, and you will have the Ditch ſunk without any Talu, which it will not be difficult to add, or to end the reſt without any further directions.

The Second Part.

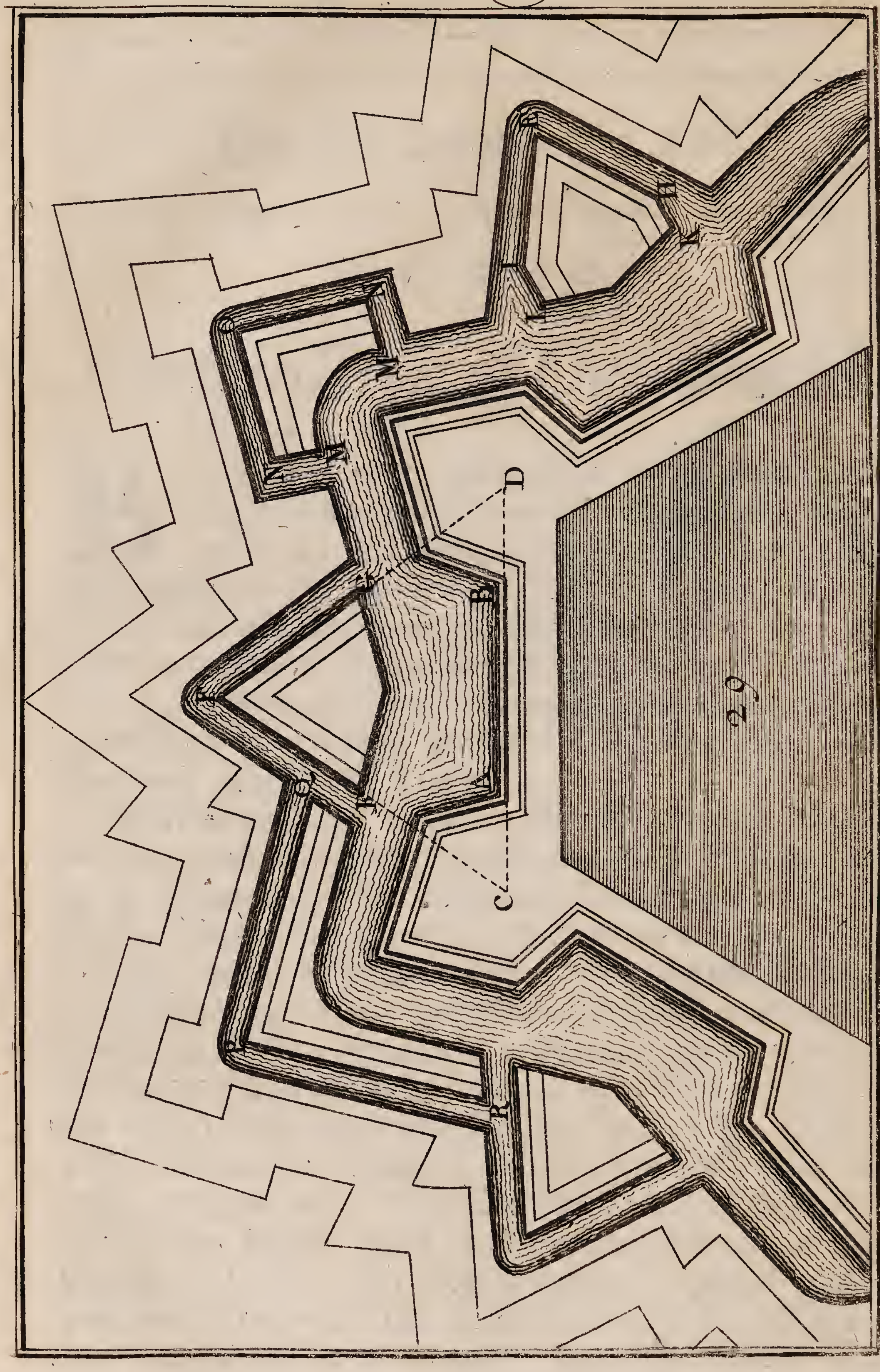
OF THE

Construction of Out-Works.

BY *Out-Works* we underſtand Works made and rais'd beyond the Ditch of a Fortified Place, to cover it, and hinder the Enemy from taking Advantage of the Hollows, or of the Riſing Grounds, which are uſually without Counterſcarps; for ſuch Hills or Hollows may make Lodgments and Curtains, that is, Coverings for the Beſiegers, and ſtand them in good ſtead in carrying on their Trenches, and raiſing Batteries againſt the Town.

This ſhews how neceſſary Out-Works are, which are the moſt important Pieces of a Place of War, and as the Armour of a Fortification: So that Places which have no Out-Works, can't be ſaid to be well Fortified; becauſe, as *De Ville* ſays, however ſtrong the Rampart of the Place be, if it is not Arm'd, that is, cover'd with good Out-Works, it cannot reſiſt long, being continually batter'd by the Enemy who is very near it; whereas the Out-Works ſtop the Enemy, retard his Deſigns, while Succour is expected, if they are well made and kept, and eſpecially if the Ditches be full of Water.

Tho' one might think that for the defence of Out-Works, a ſtronger Garriſon is requir'd, becauſe otherwiſe the Troops of the Beſieg'd wou'd be oblig'd to diſ-unite, which wou'd render them leſs able to withſtand a ſurprize: Yet if it be conſidered that the Town being beſieg'd, the Enemy is forc'd to attack the Out-Works, which may be defended by the ſame number of Soul-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Souldiers as wou'd defend the same Place if it had no Out-Works, whilst the Body of the Place is safe, there being no need for Souldiers to defend the Bastions, which are not attack'd; it will appear that there will scarce be occasion for any more Souldiers, because a few Centries in the Bastions, and a few Souldiers in the Guard-Houses to prevent surprises, will suffice.

We shall endeavour to make the Out-Works as large as possible, because little ones cannot be long kept; for if their Parapets are once broken and open, they must be defended by a great number of Souldiers, which they are too small to contain, and one cannot make Retrenchments in them to defend Breaches with safety; besides there is no room to get out of the Way if the Enemy shou'd throw in any Bombs.

We shall here only describe the Plan or Ichnography of these Out-works, without mentioning their Profil or Orthography, because their Heights differ according to the situation of the Place where they are made, and according as the Place is higher or lower. If therefore you always remember this Maxim, (*viz.*) That the Works, which are farthest from the Place, ought to be seen and commanded by those that are nearest, there will be no other Rule, but Judgment and Experience, to determine the Heights that must be given to the different Kinds of Out-works, the Construction of which follows.

How to describe the Plan of a Fort with Ravelins and Half-Moons.

THE Name of *Half-Moon* is generally confounded with that of *Ravelin*; but here by a *Half-Moon* we Understand a Kind of a detach'd Bastion, as MNONM, made just beyond the Ditch, over against the Point of the Bastion to cover it, so call'd because its Gorge is an Arch in the shape of a Crescent: And by a *Ravelin* a little Out-Work plac'd upon the Angle of the Counterscarp, over against the Curtains of the Place, and made at least of two Faces, and two Demi- *Plate* 10. *Fig.* 29.

Plate 10. *Fig.* 29. gorges, as FEG, to cover not only the Flanks, but alſo the Bridges and Gates, and to defend the Half-Moons, which as we have ſaid, are before the Angle of the Baſtion, when it is too acute.

The Conſtruction of Ravelins is different, according to the Manner of Fortifying: But it ſignifies little which way they are made, if their Faces do not make too acute an Angle, and be neither too long nor too ſhort; 40 or 50 Toiſes being ſufficient for their length, and about 60 Degrees at leaſt for the Aperture of their Angle. To follow this Maxim, we ſhall thus deſcribe the Ravelin.

Deſcribe from the Two ends A,B, of the Curtain AB, with a diſtance equal to the lengthned Curtain AD, or BC, Two Arches, which here interſect at E, from which point you muſt draw to C and D the ends of the inward ſide CD, the Faces EF, EG, which will be terminated by the Counterſcarp at F and G, for I ſuppoſe the Ditch already deſcrib'd; and you will have the Maſter-line of the Ravelin, to which add on the outſide a Ditch about 10 Toiſes broad, and on the inſide a Rampart of the ſame breadth, and a Parapet 3 Toiſes broad, that it may be Cannon-proof, and all this by lines parallel to the Faces, and to one another.

We have drawn the Faces EF, EG, to the Centers C,D, of the Baſtions, that this may be defended by the Faces of the ſaid Baſtions. Flanks are ſometimes added to ſuch a Ravelin, as HK, IL, which we have made perpendicular to the Curtain of the Town, after we have made the Faces EH, EI, of Forty Toiſes each: Which is ſeldome done, becauſe ſuch Flanks are of little Advantage, encreaſe the Expence of the Ditch, and hinder the Flank of the Baſtion from being ſo well cover'd, becauſe they ſerve as Parapets to Beſiegers when they have taken the Ravelin.

All theſe Ravelins are uſually made of Earth, and ſometimes they are Wall'd, eſpecially when they cover Gates and Bridges, that they may laſt the longer, as it is neceſſary in ſuch Places: And when they are not

Wall'd

Wall'd, and consequently must have a greater *Talu*, they ought to have Fraises, at least, below the Parapet, lest the Enemy shou'd run up easily into the Ravelin along the Slope, which must be great, supposing the Work to be of Earth.

To describe a Half-Moon, lengthen beyond the Ditch of the Place, that Part of the Parapet which is near the Banquette of the Face of the Bastion, in such manner that the Two lines MN be each equal to half a Demi-gorge of the Bastion, and draw thro' the Two ends N of those Flanks MN the Faces NO parallel to the Counterscarp: And you will have the Master-line of the Half-Moon, to which add on the outside a Ditch 10 Toises broad, and parallel to the Master-line, and on the inside a Rampart of the same breadth, and a Parapet 3 Toises broad, with Two lines parallel to the Faces NO.

As this Work is only made to cover the point of the Bastion, which is seldom attack'd, unless its Angle be too acute, which can only happen in an irregular Place, it is not often made: And as the Face of the Bastion is sooner attack'd than its Angle, which is defended by Two Flanks, that all the Face may be cover'd, it will be better to lengthen the Faces of the Half-Moon quite to the Counterscarp of the Ravelin, as PQ, PR, and then that Work RPQ changes its Name, and instead of being call'd *Half-Moon*, 'tis call'd *Counter-Guard*, or *Enveloppe*, tho' by that Word *Enveloppe*, call'd also *Sillon*, be usually meant an Elevation of Earth made in the Moat to Fortify it, when it is too broad.

By *Counter-Guard* is also meant a Tenaill'd Horn-Work, whose Front is open'd on each side from the Scarp to the Counterscarp of the Ditch of the Ravelin, so that One side of the Tenaille have no Communication with the other. Tho' such a Work, whose Construction we shall teach in the Third Part, when we shall explain Mr. *Bombelle*'s Method, be of excellent use to cover a Ravelin, which then is call'd a *Tenaill'd Half-Moon*, or *Horn'd-Half-Moon*; yet considerng the

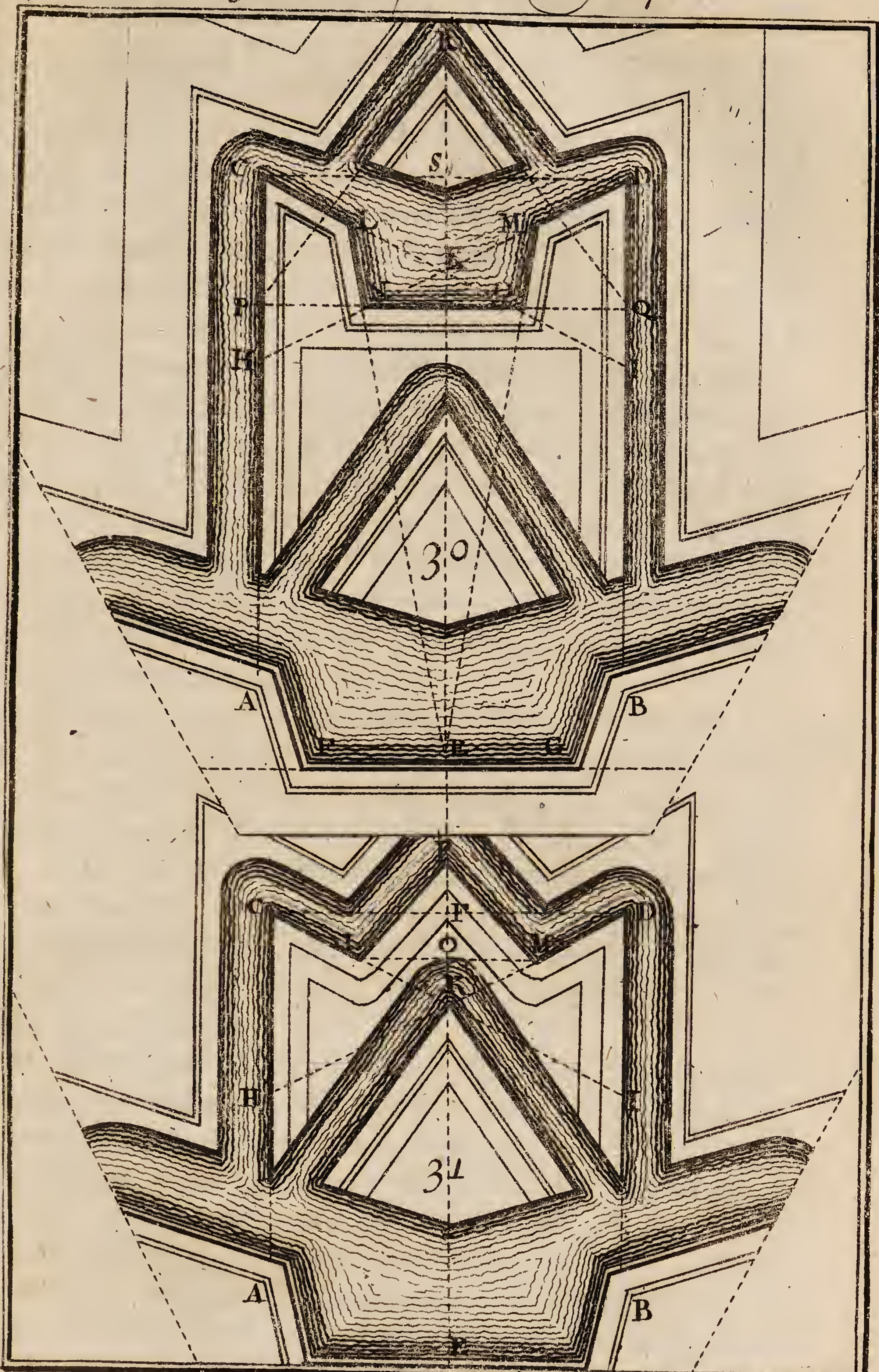
Ground it takes up, 'tis better to have a Horn-Work with Demi-Bastions, whose Construction follows.

How to describe a Horn-Work.

HORN-*Works*, usually call'd *Horns*, are made several Ways, according to the quality of the Soil, and the Place where they are to be erected, which is generally beyond the Ravelin, to cover it and the Curtains of the Place. Some of them have their long sides, which are call'd *Wings*, parallel to one another, and perpendicular to the Curtains of the Place, and these are the best: Others grow narrower towards the Place, and others on the contrary, widen towards the Place. Some are made with a single, and others with a double Tenaille, but the best of the Horn-Works is that which is made with Two Demi-Bastions, which we shall begin with.

Plate 11. *Fig.* 30. Draw from the Two Angles A, B, of the Epaule of the Parapet, the right lines AC, BD, parallel to one another, and to the line ES, which divides the Curtain FG into Two equal parts, at right Angles, and each of them equal to the Great Line of Defence, or 120 Toises, or a little longer, if you wou'd have some Space left betwixt the Rampart of the Horn and the Ditch of the Ravelin, and have the Horn defended from the Body of the Place, and the Wings AC, BC, from the Face of each Bastion. Draw the line CD, which will be divided into Two equal parts at S, by the interfection of the line ES. On the Two parallels AC, BC, take the lines CH, DI, each of them equal to CS, or DS the half of the line CD, and draw the lines CI, DH, which make the single Tenaille CKD, which wou'd be enough if you wou'd have a Horn-Work with a single Tenaille, but if you wou'd have a *Reinforc'd Tenaille*, that is, a Tenaille fortified by a Curtain between Two Demi-Bastions, divide each of the Tenailles CK, DK into Two equal parts at L, M, and the lines CL, DM, will be the Faces of the Demi-Bastions: And to have their Flanks and Curtain, draw from the Two points

L, M,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L,M, to the point E the Middle of the Curtain of the Place, the Right-lines EL, EM, which will be cut at N,O, by the Two lines DH,CI. Laftly, draw NO for the Curtain, and the Two Flanks will be LN, MO. Thus you will have the Mafter-line of the Horn, to which you muft on the infide add a Rampart with its Parapet, and on the outfide a Ditch, all of the fame breadth with thofe of the Ravelin; as for the Height, 'tis evident that it ought to be lefs than that of the Ravelin.

It will be proper to add to the Angle of the Counterfcarp of this Horn, a Ravelin to cover its Flanks and Curtain: This Ravelin will be defcrib'd the fame way as that of the Place, (*viz.*) by lengthning the Curtain NO on either fide as far as the Mafter-line at P, Q, and defcribing from N, O, the ends of the Curtain NO, with the diftance NQ, or OP, Two Arches, which here interfect at R, *&c.*

To make a Horn-Work with a double Tenaille having, as before, divided each of the Tenailles CK, DK, into Two equal parts at L, M, draw LM, which will be divided by EF at right Angles, and into Two equal parts at O, from which point take upon the lines EF, OP equal to CL, or DM, to join the Right-lines PL, PM, which will end the Mafter-line of the Tenaille, to which you muft add a Ditch and a Rampart with its Parapet, as before. *Plate* 12. *Fig.* 31.

The Situation of the Ground does not always give you leave to make the Wings of a Horn-Work parallel to one another, and in fuch a cafe you may give it another Shape: And when it widens towards the Field, it is call'd a *Queuë D'Irondélle*, or *Swallow's-Tail*, and when its Wings come nearer to one another towards the Field, it is call'd a *Contrequeuë D'Ironde-Work*, or Counter-Swallow's-Tail, whofe Wings, if poffible, muft be drawn Perpendicular to the Faces of the Baftion, that from them, they may be the better defended. When a Swallow's-Tail has Two Tenailles it is call'd *Bonnet à Prêtre*, or *Prieft's-Cap*, the Conftruction of which follows.

How

How to describe a Bonnet à Prêtre.

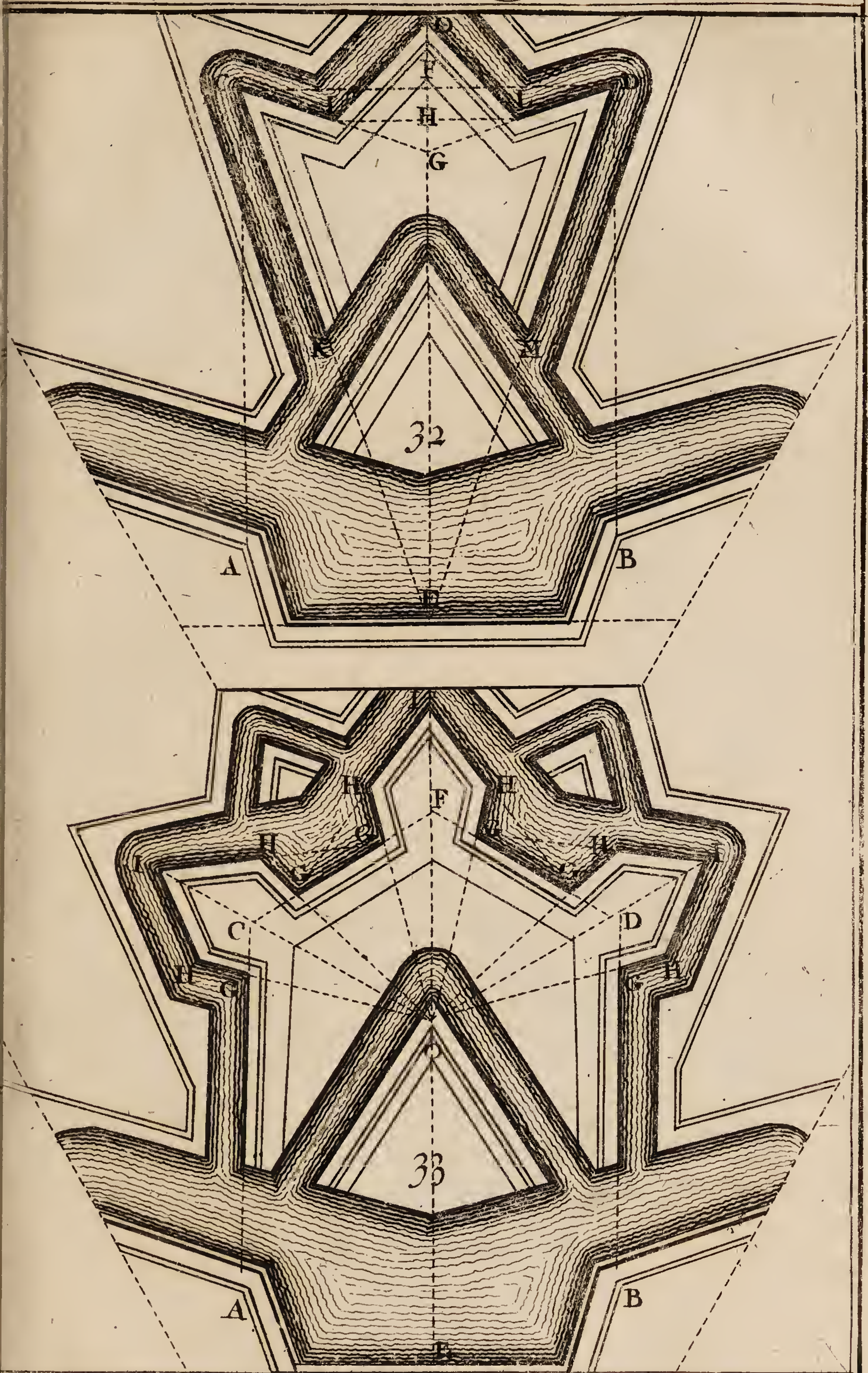
Plate 12. Fig. 32. HAving drawn, as before, the Two lines AC, BD, parallel to the line EF, and of what length you pleafe, fo that it does not exceed the reach of a Musket, take upon that line EF, FG, the third part of CF or DF, and make the fingle Tenaille CGD, each Face of which muft be divided into Two equal parts at I, L, to make there a double Tenaille as before, by joining the Right-line IL, and making the line HO equal to CI, or DL, then draw OI, OL, and the Two Wings CK, DM, to E the middle point of the Curtain of the Place, which muft be terminated at the Counterfcarp of the Ravelin, when there is one; or when there is no Ravelin, at the Counterfcarp of the great Ditch.

This Work has been call'd *Bonnet à Prêtre*, becaufe being made with Three Saliant-angles, which in Terms of Fortification are call'd *Angles Vifs*, and of Two Reentrant Angles, which are call'd *Angles Morts*, and being wider at the Head than at the Gorge, it is like a Prieft's Cap. It was chiefly invented, to take in Springs and rifing Grounds, whofe lofs may be hurtful to the Inhabitants, and hinder them from defending the Place well.

How to describe a Crown.

Plate 12. Fig. 33. THE *Crown* call'd alfo a *Crown'd-Work* and *Crown-Work*, is a kind of Horn-Work made up of Three Baftions, or Two Demi-Baftions and a whole Baftion in the Middle: this laft is ufually made like a *Queuë D'Irondelle*, and the firft has its fides parallel, as you will fee.

Firft then, to defcribe a Crown with Three Baftions, draw, as before, the Two lines AC, BD, parallel to one another, and alfo to the line EF, and fomething fhorter than the reach of Musket-fhot, that the middle Baftion, which projects towards the Field, may be defended from the Body of the Place: And at C,D, with the lines AC, BD, make the Angles ACF, BDF of 120 Degrees each, which will be eafily done by defcribing from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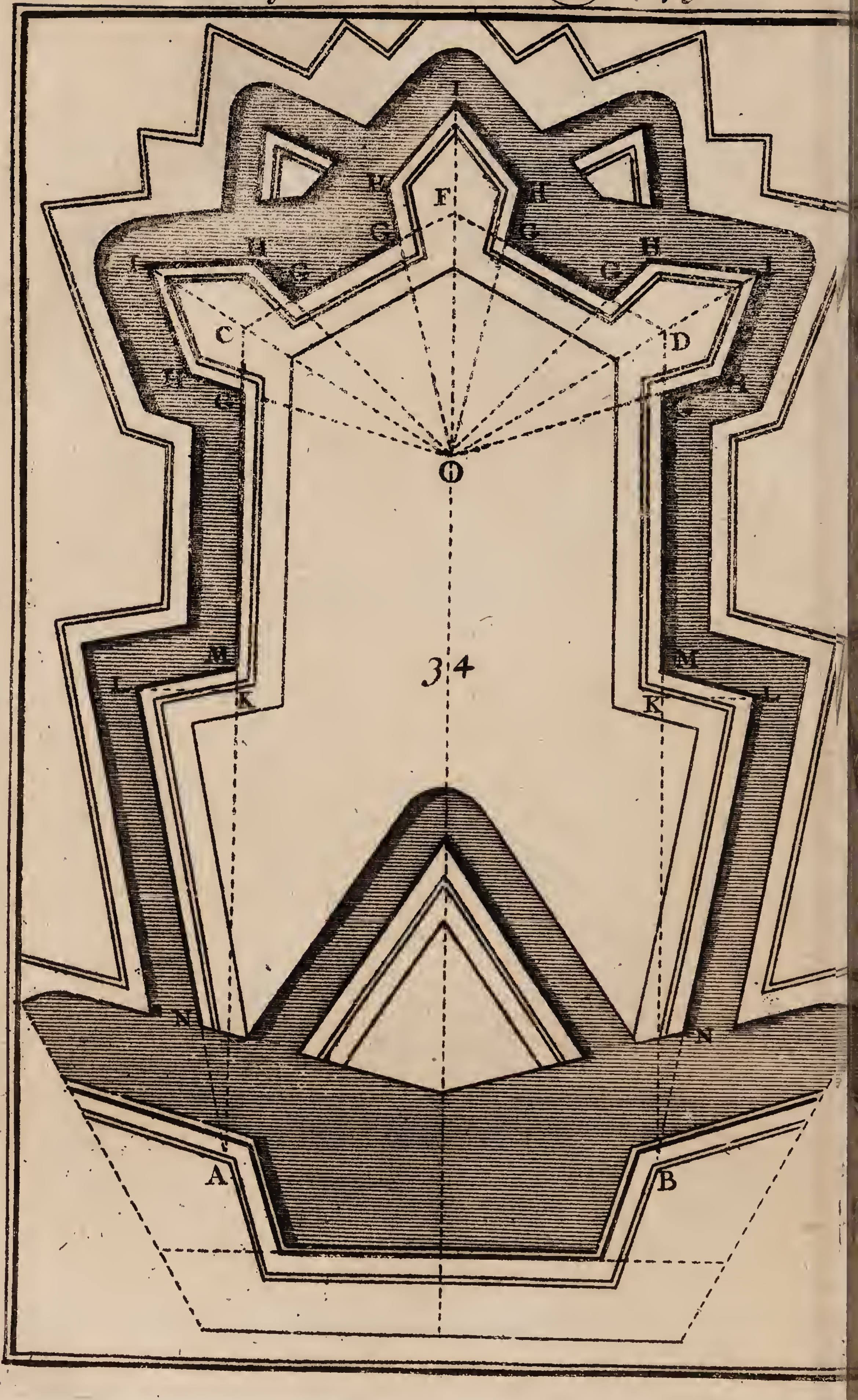
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from the points C,D, Two Arches to set off upon each of them the same distance with your Compasses, *&c.* *Plate* 12. *Fig.* 33. Carry the length of the line CF, or DF to O upon the line EF, which will be the Center of an Hexagon, of which CF and DF will be Two sides, and C,F,D, Three Angles, where you must make as many Bastions, allowing the fourth part of the inward side CF or DF, for their Demi-gorges CG, FG, DG, and for their Flanks GH.

This Work must have a Ditch and a Rampart with its Parapet, like those of the foregoing Works, observing that the Parapet ought always to be 3 Toises broad, at least, that it may resist the Enemy's Cannon: Yet we have given it but half that breadth along those Flanks GH, which are almost parallel to the Face of the Bastion of the Place, that from that Face it may be ruin'd with Cannon, in case that the Enemy shou'd take it, to hinder him from covering himself with it.

When, to take in some Commanding Ground, Passage or *Cavin*, that is, Hollow Place where the Enemy might intrench or cover himself, or for any other reason, you are oblig'd to make the Wings of a Crown, or of a Horn-Work out of Musket-shot, Returns or Shouldrings must be made about the middle of such Wings in the following manner.

Having taken off the lines AK, BK, of about 100 Toises, more or less, according to the length of the lines AC, BD, draw from the Two points K, the lines KL perpendicular to AC, BD, and 20 Toises long each, and draw from the points L towards the Shoulders A,B, of the Parapet of the Place, the Two sides LN, which will be terminated at the Counterscarp of the great Ditch, to which at the same points L, draw perpendicular Flanks or Shoulderings LM, which will serve to defend the Crown that will be made at CD, as before taught. *Plate* 13. *Fig.* 34.

We have in this Work, as well as in the foregoing Horns, drawn the sides LN, to the Angles A, B, of the Epaule of the Parapet of each Bastion of the Place, that they might be defended by the Faces of those Bastions;

ſtions: and we have made the Flanks L M perpendicular to the Sides NL, that the Angles L being right ones might be ſtronger and larger, and that the Flanks LM might by their obliquity better defend the Field.

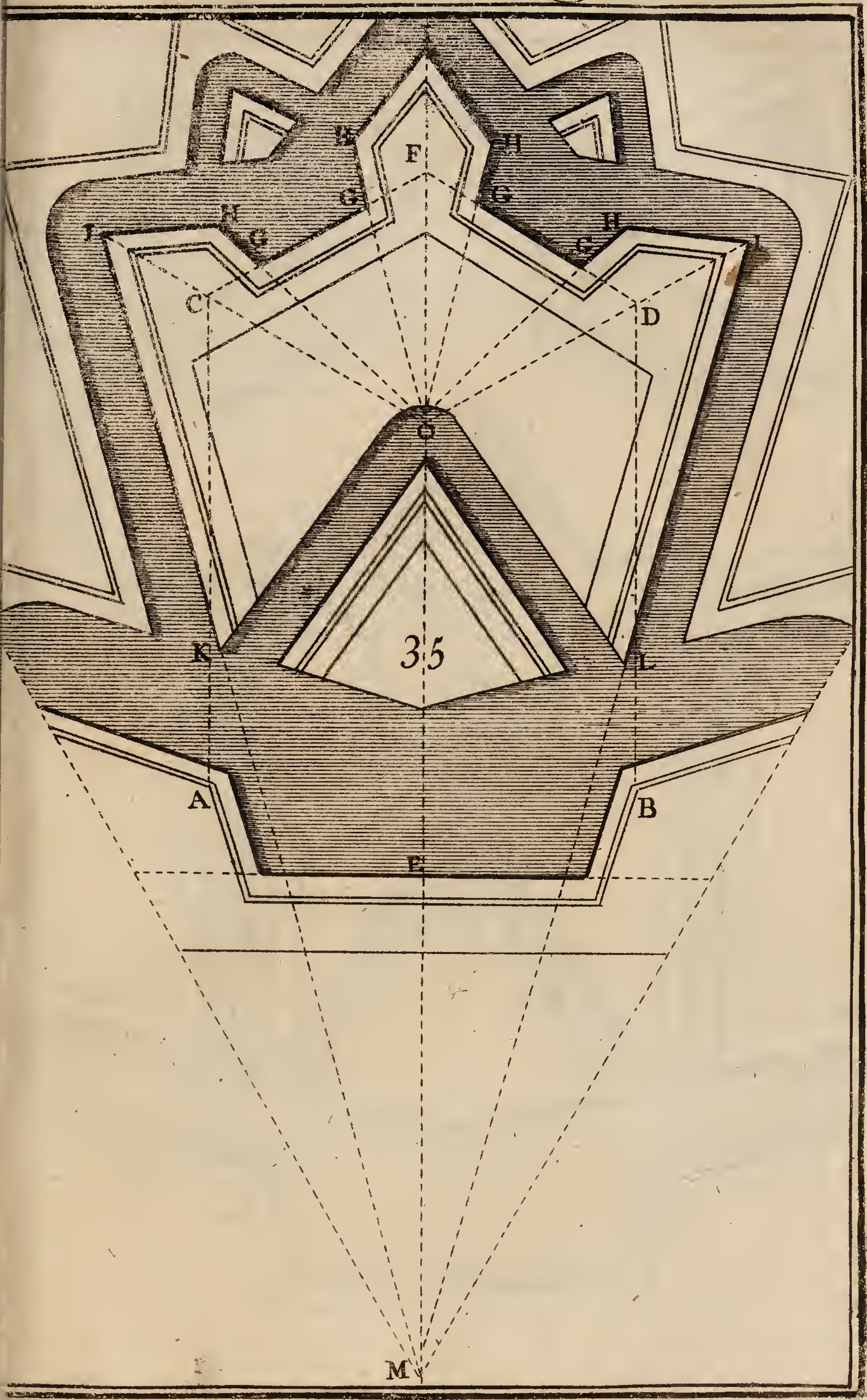
Plate 14. Fig. 35. To deſcribe a Crown in the Figure of a Swallow's Tail, draw as before, AC,BD, parallel to EF, and about the length of the inward Side of the Place, and as before make a Baſtion at the Angle F, and only two Demi-Baſtions at the Angles CD, to draw from their Points I to M the Center of the Place, or to any other point of the line MEF, the Wings IK, IL, which the Counterſcarp will terminate, *&c.*

This Work, by reaſon of its largeneſs, is to be preferr'd to other Horn-works, eſpecially to take in a Palace, or any other Building of importance. When a way into the Town goes thro' a Crown, it muſt be carried on thro' the middle of one of the Curtains, and Cover'd with a ſmall Ravelin, as we have done in this Work, and the two foregoing.

How to deſcribe a Couronnement.

Plate 15. Fig. 36. THO' a Crown be alſo call'd Couronnement, yet by *Couronnement* we ſhall here underſtand a Work of Earth, ſometimes rais'd about the Head of thoſe Horns which have two Demi-Baſtions to cover them, to take in Ground and keep the Enemy at a greater diſtance. We ſhall make it larger and more capacious than uſual, the more conveniently to range in Battle-array ſuch as are neceſſary for its defence.

The Horn being deſcrib'd as before, lengthen beyond its Ditch that part of the Parapet which is parallel to the Face of each Demi-Baſtion, from C to D, ſo that each line CD be of 30 Toiſes, which ſhall ſerve as a Flank towards the Place, and which you muſt divide into two equal parts at the points G, thro' which you muſt draw from the Epaules A,B, the right lines GH, GI, each of them equal to CG or DG, to joyn the line HI, which will be divided at right Angles and into two equal parts by EF, which alſo divides into two equal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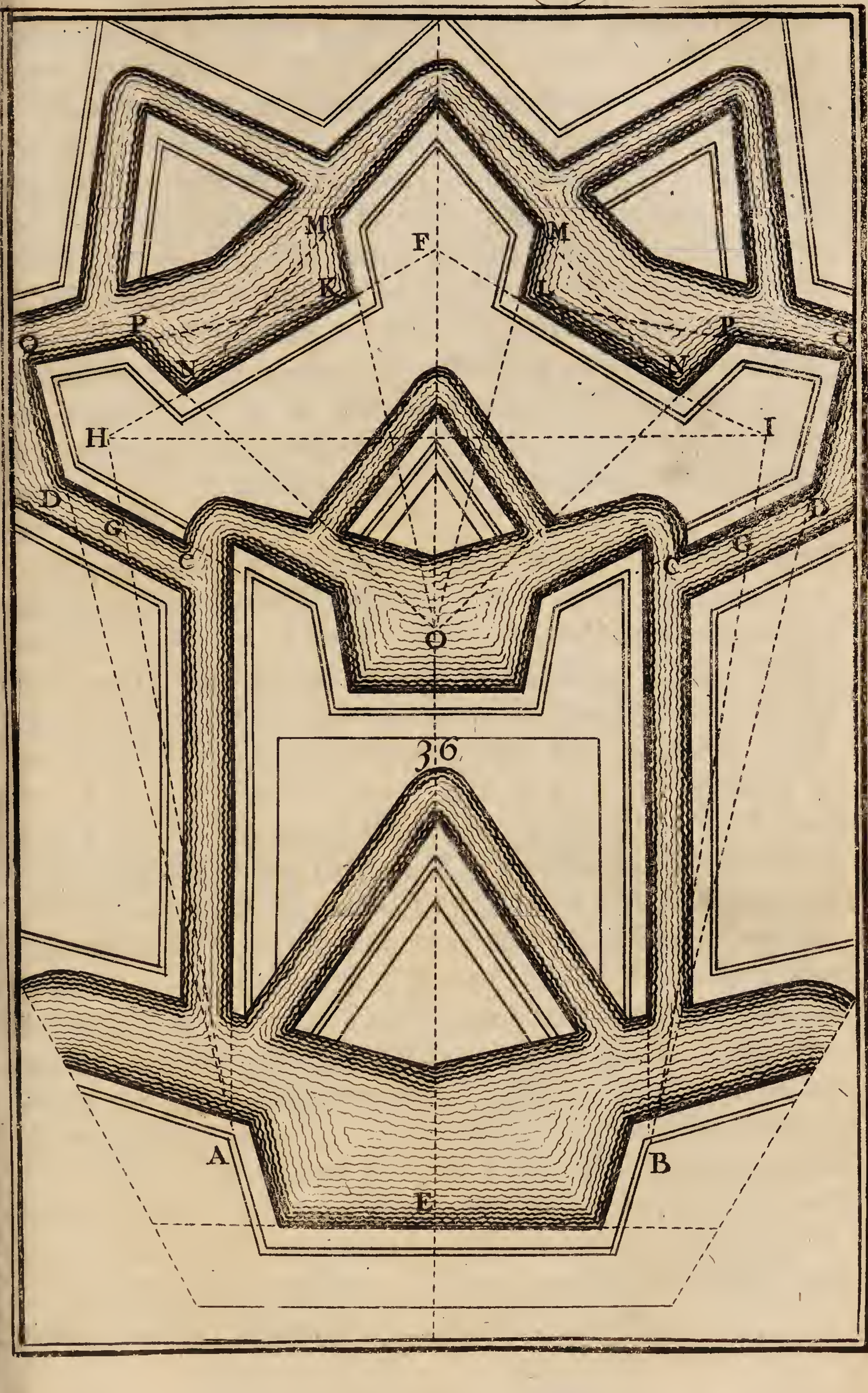
Fortification Plate 15, Page 76.



equal parts at right Angles, both the Curtain of the Place, and that of the Horn. *Plate 15. Fig. 36.*

Then at the two points H,I, with the line HI make the Angle HIF, IHF, of 30 degrees each, by the lines HF, IF, which will be equal and interſect at F upon the line EF, where a Baſtion muſt be made, whoſe Demi-gorges FK, FL, muſt be the fourth part of FH, or FI, and the Flanks KM, LM, the fifth part of the ſame inward ſide FH, or FI. Theſe Flanks muſt be drawn from the point O, which is found by making the line FO equal to the inward ſide FH or FI, as we have already done before, where that point O has been conſider'd as the Center of an Hexagon, whoſe ſides are FH, FI.

Again, make a Baſtion at each Angle H, I, making the Demi-gorges MN,IN, and the Flanks NP as before, then Q the Point of each Baſtion will be terminated by the interſection of the Lines of Defence KPQ, LPQ, ADQ, BDQ; and you will have the Maſter-line of the Couronnement, to which you muſt add a Ditch without, 8 or 10 Toiſes broad, and a Parapet within, at leaſt 3 Toiſes broad, without any Rampart, the Level of the Field ſerving inſtead of a Rampart, becauſe ſuch a Work being at a great diſtance from the Place ought to be very low to be commanded by the Horn, which in ſuch a caſe is call'd a *Crown'd-Horn*, which is alſo ſaid of a Horn made of two Demi-Baſtions and a whole Baſtion. The cover'd Way, which goes along the Ditch of the Couronnement, ought to be digg'd down below the Level of the Field, becauſe this Work being very low, as we have ſaid, and the Terre-plain of its Rampart juſt Level with the Ground, the natural fall of the Ground will be the Glacis of its Eſplanade. All the Ditches ought to have a communication one with another, when they are full of Water.

The Conſtruction of Traverſes.

TRaverſes are Works of Earth in the form of Parapets, made facing, or ſlanting toward the Enemy,

to

to keep him from passing a narrow place, and shut him out of the Ways thro' Marshes and Fenns, or between Rocks, or any other passage of importance: as also to cover one's self from being *enfil'd*, and then such a Traverse is call'd *Rideau* (Curtain) tho' in Terms of Fortification, *Rideau* signifies also a small rising Ground long ways upon a Plain, and which is sometimes almost parallel to the Front of a Place. This Traverse is us'd before Bridges and Gates to intrench one's self, to resist the longer, and to retard the Enemy's designs in expectation of Succour, or else when any unexpected Accident happens, to make a better and more advantageous Capitulation.

Traverses may be made how you please, and as many different ways you think fit. They are usually made with one or several Ravelins, or with a *flat Bastion*, so call'd because it is made upon a right line, that is, in the middle of the breadth of the Way which is to be shut up from the Enemy. We shall here mention four sorts, which are the most common, and I think the most useful.

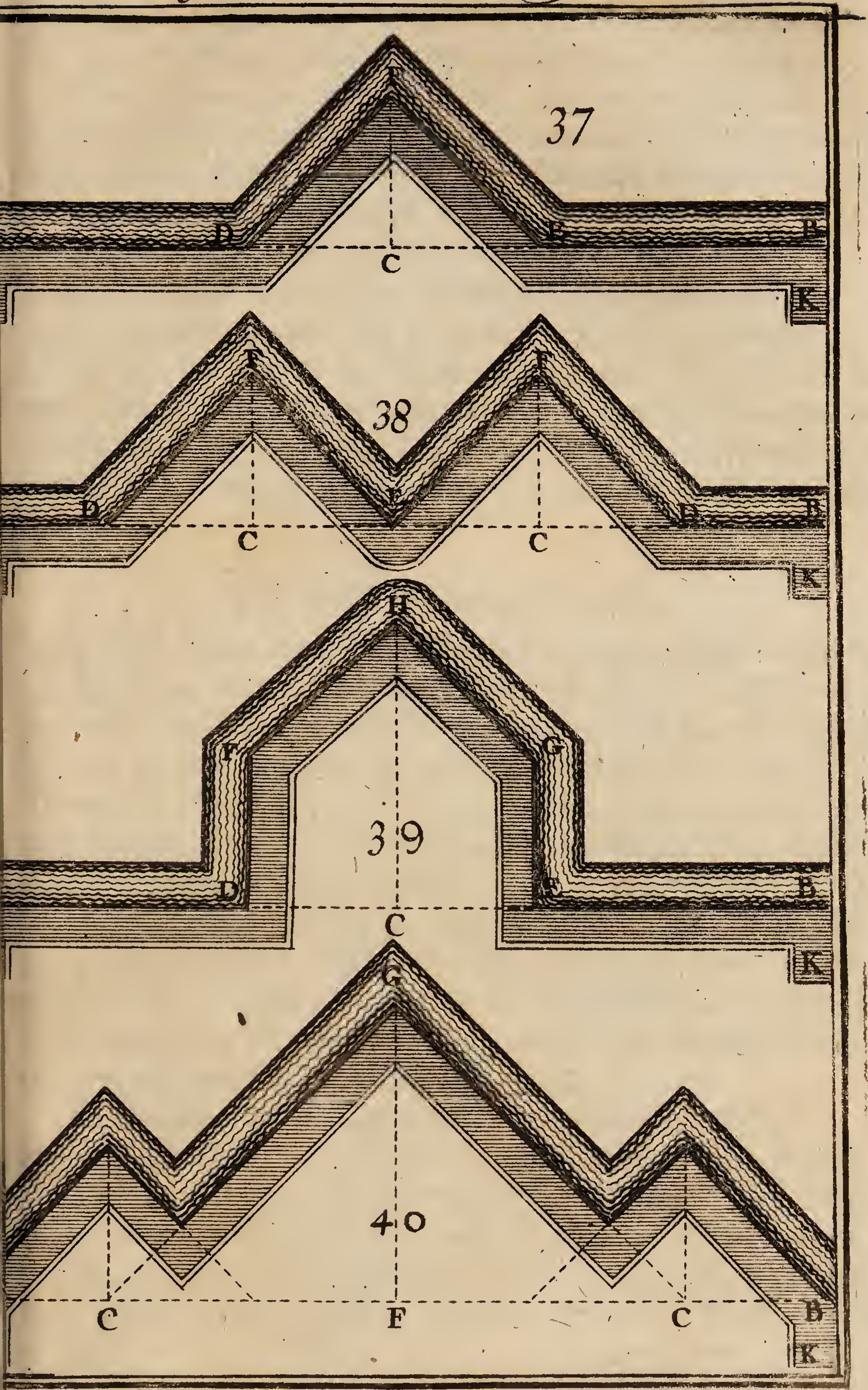
Plate 16. *Fig.* 37. The first kind of Traverses is made of a Ravelin having each Demi-gorge CD, CE, and the Capital F, equal to the sixth part of the breadth AB.

Fig. 38. The second Traverse is made of two Ravelins like the first, which make a double Tenaille.

Fig. 39. The third Traverse is made with a flat Bastion in the middle of the breadth AB, whose Demi-gorges CD, CE, and perpendicular Flanks DF, EG, are equal to the sixth part of the breadth AB, and the Capital CH equal to the Gorge DE.

Fig. 40. The fourth Traverse has two Ravelins, equal to and like the foregoing at each end of the breadth AB, with a third and greater Ravelin, whose Capital FG, and Demi-gorges FC are twice as long as those of the others.

You must not forget to make a Ditch 4 or 5 Toises broad, more or less upon occasion, on the out-side of these Traverses, and a Parapet, as usual, on the inside to cover those that are in it. You must add also at the


37
D
E
B
C
K
38
F
F
D
D
B
C
C
K
H
F
G
39
D
E
B
C
K
G
40
C
F
C
B
K

the Ends A,B, the Flanks AK, BK, to ſhoulder, that is, to cover the Souldiers that defend the Traverſe.

Traverſes are very uſeful, if they are plac'd one before another where a Town can be attack'd but on one ſide, tho' one may uſe other Out-Works one before another after the manner of Traverſes to fatigue the Enemy, and oblige him to raiſe the Siege.

Experience ſhews that ſeveral Traverſes rais'd one before another, are alſo very convenient to avoid the effect of the Bomb or Granado, becauſe if it falls in one Traverſe, it ſtays there and is conſum'd without execution, becauſe the Souldiers may hide themſelves behind the other Traverſes till the Granado or the Bomb is burſt.

The Construction of Citadels.

A *Caſtle* or *Citadel* is a ſmall Fortreſs, ſometimes of four, uſually of five, and ſeldom of ſix Baſtions, becauſe the Square is too imperfect, and the Hexagon is too great, which is uſually erected in conquer'd Towns by the command of the Prince, when he miſtruſts the Loyalty of the Inhabitants, to be ſecur'd againſt their rebelling, or to defend them when they are dutiful, or to puniſh them if they ſhou'd revolt. To this end, the Citadel ought to command all the Town, wherefore it muſt be built in the higheſt place, eſpecially when the Town is encompaſs'd with ſeveral Hills, to ſave the charge of levelling them.

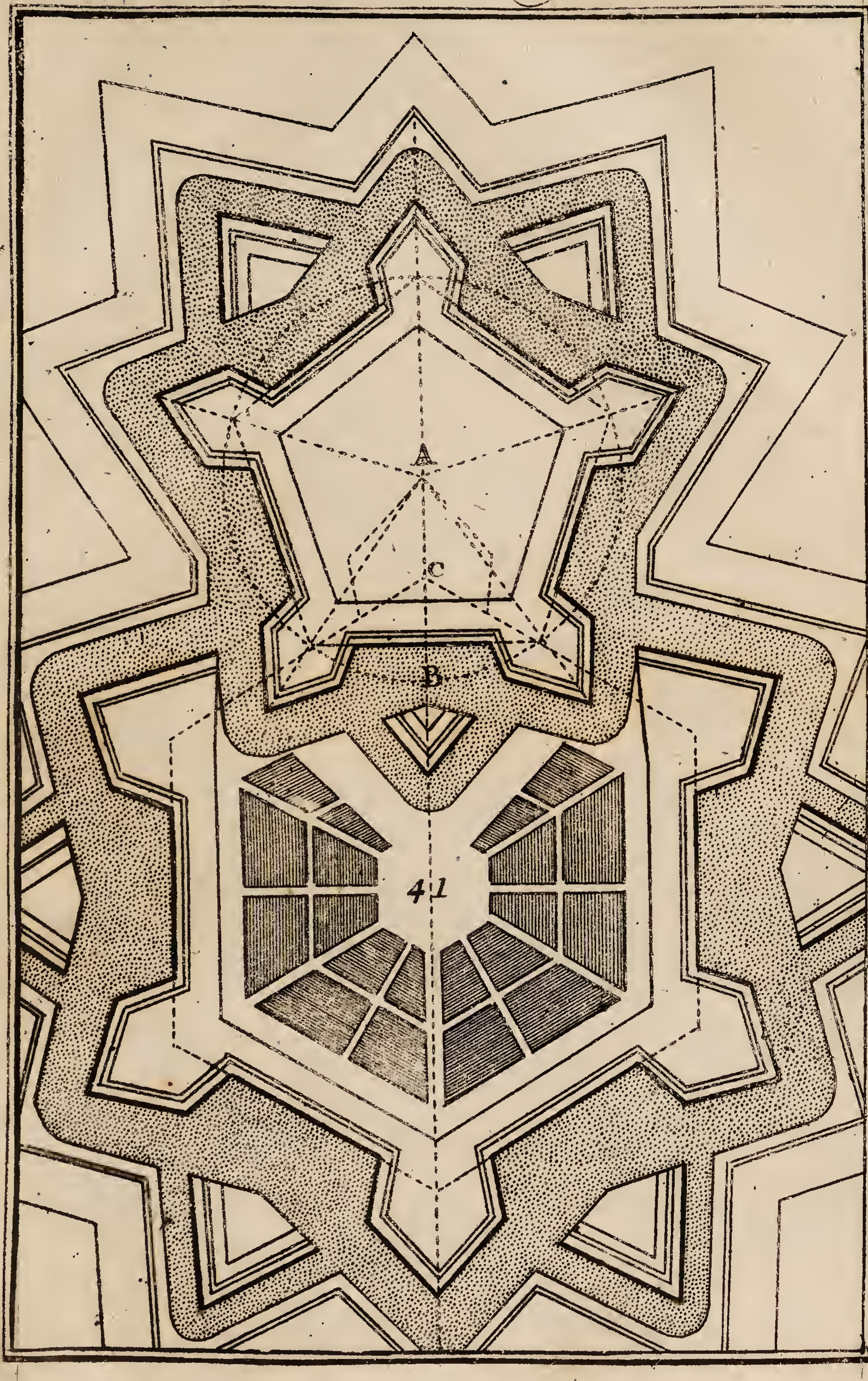
To deſcribe a Pentagon fit for a Citadel, its Center A muſt be the Point of one of the Baſtions of the Town, its Radius AB muſt be of 70 or 80 Toiſes. This Pentagon muſt be Fortified as before taught, in ſuch manner that two Baſtions advance into the Town, and that the whole Curtain may directly face the Town, and keep it in Subjection: which to do the more effectually, a Cavalier muſt be rais'd upon each of theſe two Baſtions to batter down the Houſes and Structures of the Place, in caſe that the Enemy ſhou'd have taken it. The Citadel muſt have a Rampart of 10 Toiſes with its *Plate 17. Fig. 41.*

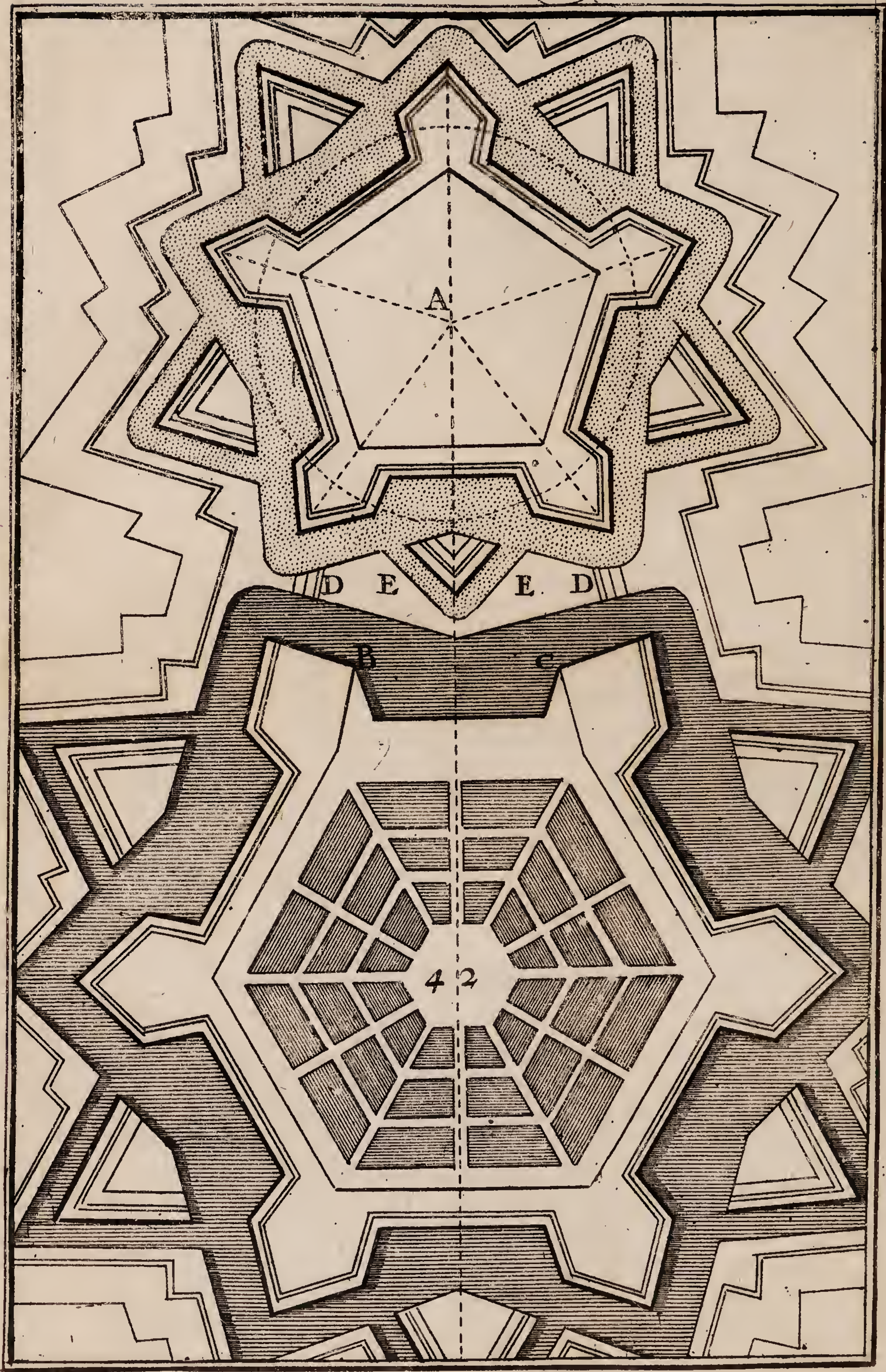
its Parapet of three, a Ditch of 12, a Coridor of 5, and an Esplanade of twenty.

The defences of the Place towards the Citadel must be broken, by continuing the Faces of each Bastion as far as the Ditch of the Citadel, least those defences might serve the Inhabitants in case of a Revolt, or the Souldiers to take the Citadel: for the Town ought not to be Fortified against the Citadel, but the Citadel against the Town. That therefore the Citadel may the better command the Town, it will be proper to have a Ravelin before that Curtain which enters into the Place, leaving a great space between the Ditch of the Citadel and the Houses of the Town, to hinder Plots which the Inhabitants might carry on against the Citadel; who in such a case can only approach it openly and with Trenches. Such a Space is call'd *Explanation*.

Citadels have usually but two Gates, one towards the Field, to take in Provisions and Ammunition, and Succour in time of need, and (hence it is call'd the Succour-Gate,) which shou'd never be open'd but to receive Succour. The Citadel is built part within and part without the Town, that the Governour of the Citadel may be Master of its Entrance towards the Field, as well as of its Entrance towards the City.

When the Citadels are detach'd from the Body of the Place, they ought to be rais'd between the Town and the place where the Enemy might encamp, to hinder his approaches on that Side. These are not so advantageous as when part of them enters into the Town because they cannot keep the Inhabitants in awe, nor easily hinder them from corresponding with the Enemies of the State. Citadels therefore ought not to be made thus, except in case of extreme necessity; These are built, as we have already said, upon the highest Hill, when the Place has several Hills about it, to save the charge of levelling them: And also when there are Castles already built, which by their advantageous Situation may easily command the Town: and especially in Sea-Port Towns, in such places as may command both the Town and the Harbour. The


A
C
B
41


A
D E
E D
B
C
42

The Center A of such a Place ought not to be distant from the two Epaules A,B, of each Bastion, more than Musket-shot if possible, that in case of necessity it may be partly defended by those two Bastions, whose Curtain and Flanks towards the Citadel ought to have no Rampart nor Parapet, but a bare Wall, that the Citadel may the better command the Town. *Plate 18. Fig. 42.*

From the Ditch of the Citadel to the Ditch of the Town must be carried on, on either side over against the Point of each Bastion, a little Ditch, whose Earth will serve to raise a small Rampart with its Parapet D, on each side, to cover the Place of Arms E, which is left between the Town and the Citadel.

Before we end this Second Part, we shall here teach how to make and use a very convenient Instrument to describe easily and readily upon Paper, the Master-line, of a regular Polygon Fortified after our Method, or any other, which allows no Second Flanks, from the Square to the Decagon.

Having drawn upon Brass, or any other Solid Matter, the line BC of 1000 parts taken upon a Scale exactly divided, describe from its middle Point A, thro' its ends B,C, a Semi-circle, to mark upon its Circumference from C the Radii of all the regular Polygons, according to the number of their Parts, which we have added to each of them, and mark the Figures IV,V,VI, *&c.* which are the marks of the Square, the Pentagon, Hexagon, *&c.* Draw from the points IV, V, VI, *&c.* to the Center A, right lines, to mark upon them from the Center A, the length of the Demi-gorges agreeable to each Polygon, such as you see it mark'd in the Table of the Lines and Angles, which we have given in the beginning of the First Part, for an inward side of 120 Toises, which is here the same with the Radius of the Hexagon; and join all the points mark'd upon those Semi-diameters by a crooked line, which shall be call'd the *Line of Demi gorges.* Do the same for the Capitals, setting off their lengths from the Center A, or as here, to avoid confusion, from the Line of *Plate 19. Fig. 43.*

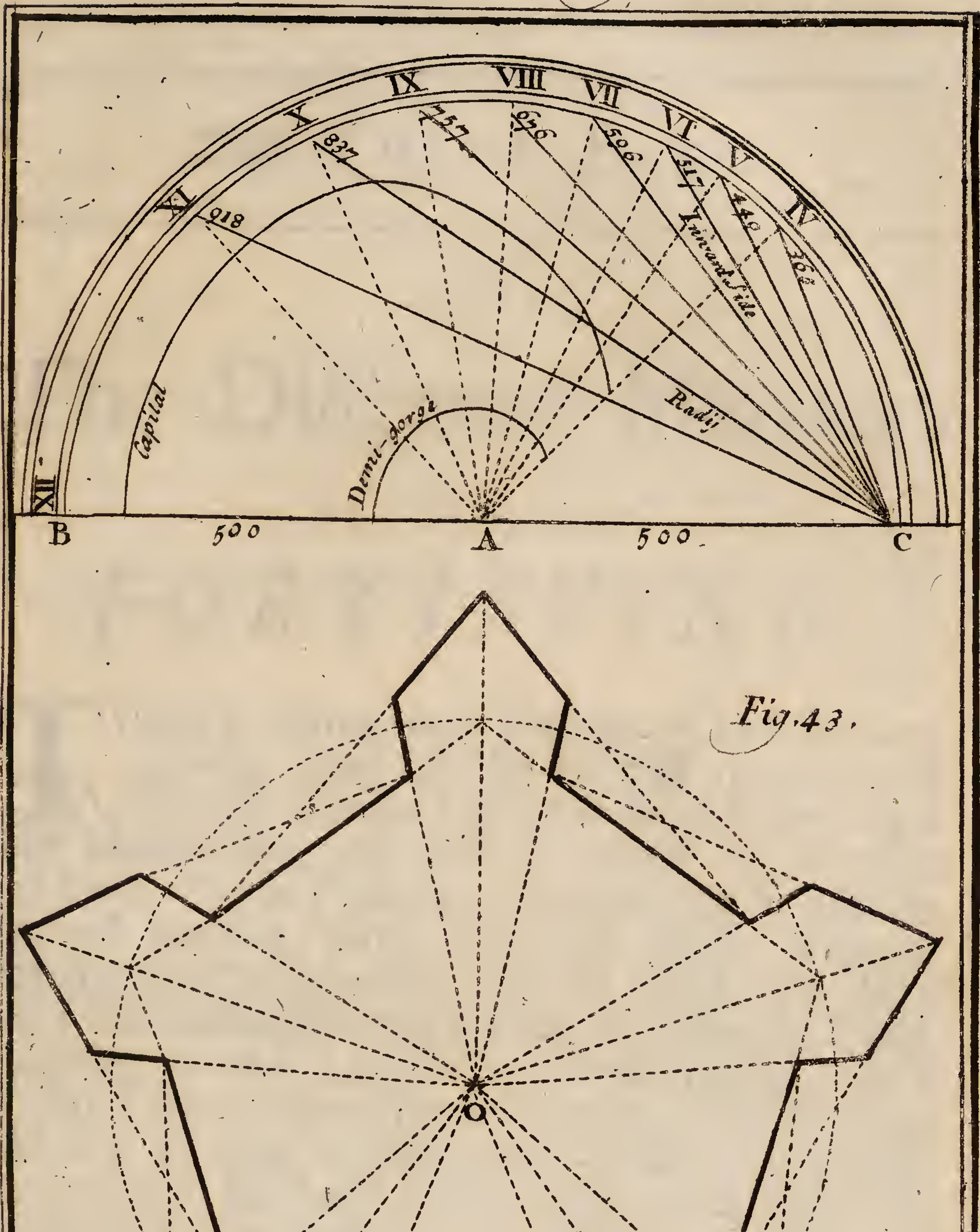
Plate 19. *Fig.* 43.

Demi-gorges upon the same Semi-diameters, to have a Second Curve, or crooked line, which we shall call the *Line of Capitals*; and the Instrument will be ended, whose use follows.

To Fortify a Pentagon (for example): Take upon the Instrument the length C V, and with that distance, from the Center O describe a Circle, which you must divide into five equal parts with the distance C VI, of the Radius of the Hexagon, which is the inward side common to all the regular Polygons, and drawing right lines from all the points of the division to one another, and from the Center O to those points you will have a Pentagon, which may be Fortified by means of the Instrument, thus:

Take upon the Semi-diameter A V, which belongs to the Pentagon, from the Center A, the length of the Demi-gorge, and set it off upon the inward side DE, and upon all the others, from D to F, and from E to G, *&c.* and the length of the Capital upon the same Semi-diameter A V of the Pentagon, from the Line of Demi-gorges to the Line of Capitals, and set it off upon all the lengthen'd Radii OD, OE, *&c.* from the same points D, E, to H, I, *&c.* draw the Rasant lines GH, FI, which will terminate the Flanks FL, GM, *&c.* which will be drawn from the Center O, thro' the ends F, G, of the Demi-gorges, *&c.*

The


XII
XI
X
IX
VIII
VII
VI
V
IV
918
837
757
656
606
517
440
362
Inward Side
Radii
capital
Demi-gorge
B
500
A
500
C
Fig.43.
O
D
F
G
E
H
L
M
I

The Third Part.

Of
The Different Manners
of
FORTIFYING.

THESE Three chief Maxims of Fortification, namely, That all the Parts of the Place be Flank'd, that the Line of Defence be not longer than Musket-shot, and that the whole Fortification and especially the Flanks be Strong enough to resist the Enemy's Cannon, have occasion'd several Ways of Fortifying to be invented, some of which have been call'd the *Italian*, some the *French*, and some the *Dutch*, according as their Authors have been *Italian*, *French*, or *Dutch Men*. But without following this distinction, which is of little consequence, we shall only explain the principal Methods of Fortifying, which have been found out by the most Famous Authors, and first our Method; not that I wou'd rank my self with those Famous Men, which I shall hereafter speak of; but to explain the better, and correct that Method of Fortifying, which I have hitherto us'd, and to give you something that is New.

Therefore to render our Method of Fortifying more perfect,(which makes the Flanks very long, tho', as I've already said, they may be made less,) it may be corrected by still allowing the length appointed for the Demi-gorge, and giving the Flank as many Toises as twice the

number of the Baſtions and 10 beſides, (*viz.*) making the Flank, which is always drawn from the Center, of 18 Toiſes in the Square, 20 in the Pentagon, 22 in the Hexagon; and ſo on as far as the Decagon, where the Flank being of 30 Toiſes as well as the Demi-gorge, muſt remain of that length, the inward ſide being always ſuppos'd of 120 Toiſes.

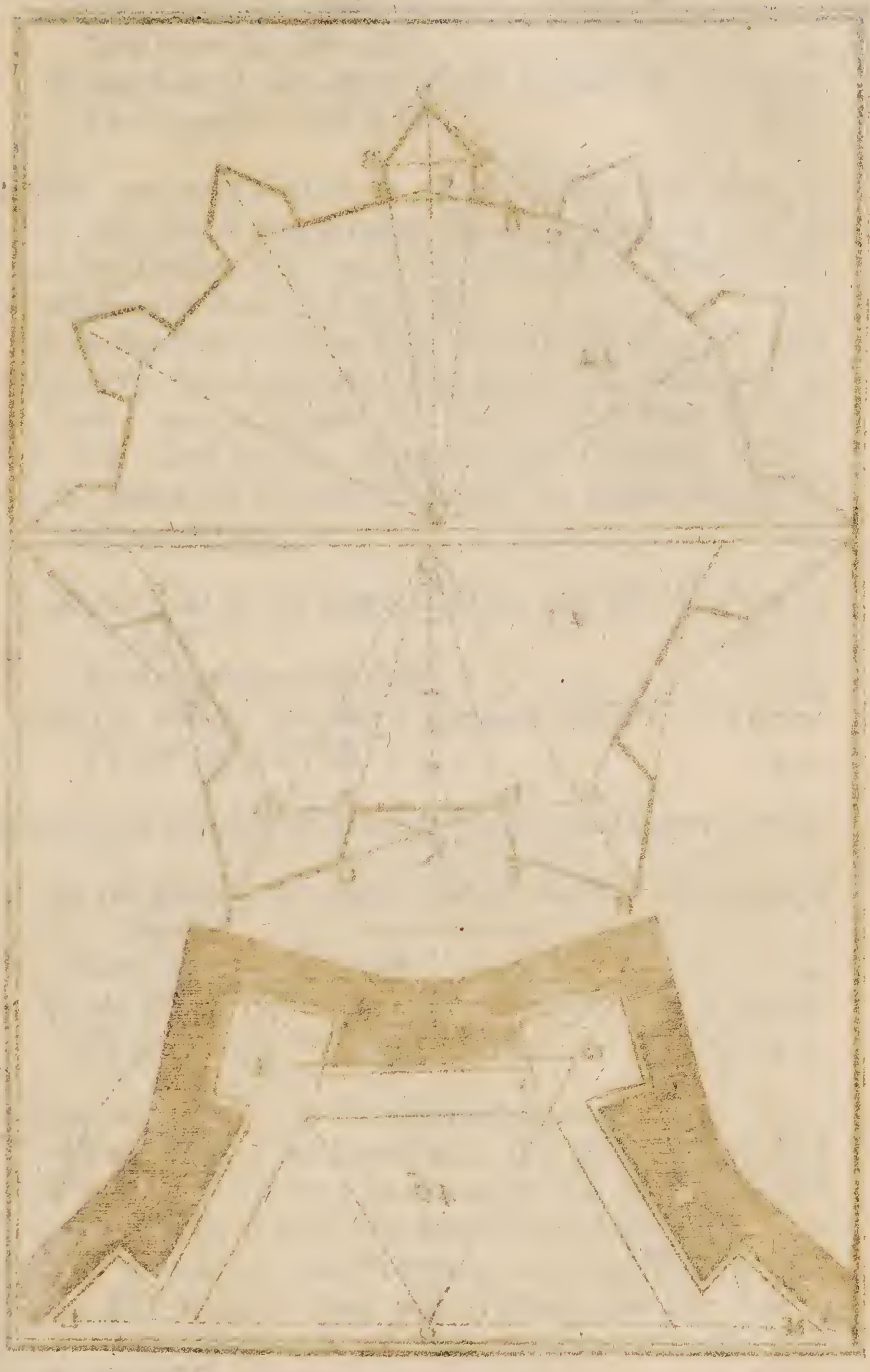
Plate 20. *Fig.* 44. As it happens by this manner of Fortifying, that in the Octogon and all the following Polygons the Flank'd Angle becomes obtuſe, and conſequently defective, it may be made right by means of the Semi-circle ADB, deſcrib'd upon the right line AB, which joyns the two Epaules A,B, of the Baſtion: and then a Second Flank GH will be had upon the Curtain, and two Lines of Defence; the one Raſant, as DH, and the other Fichant, as DG.

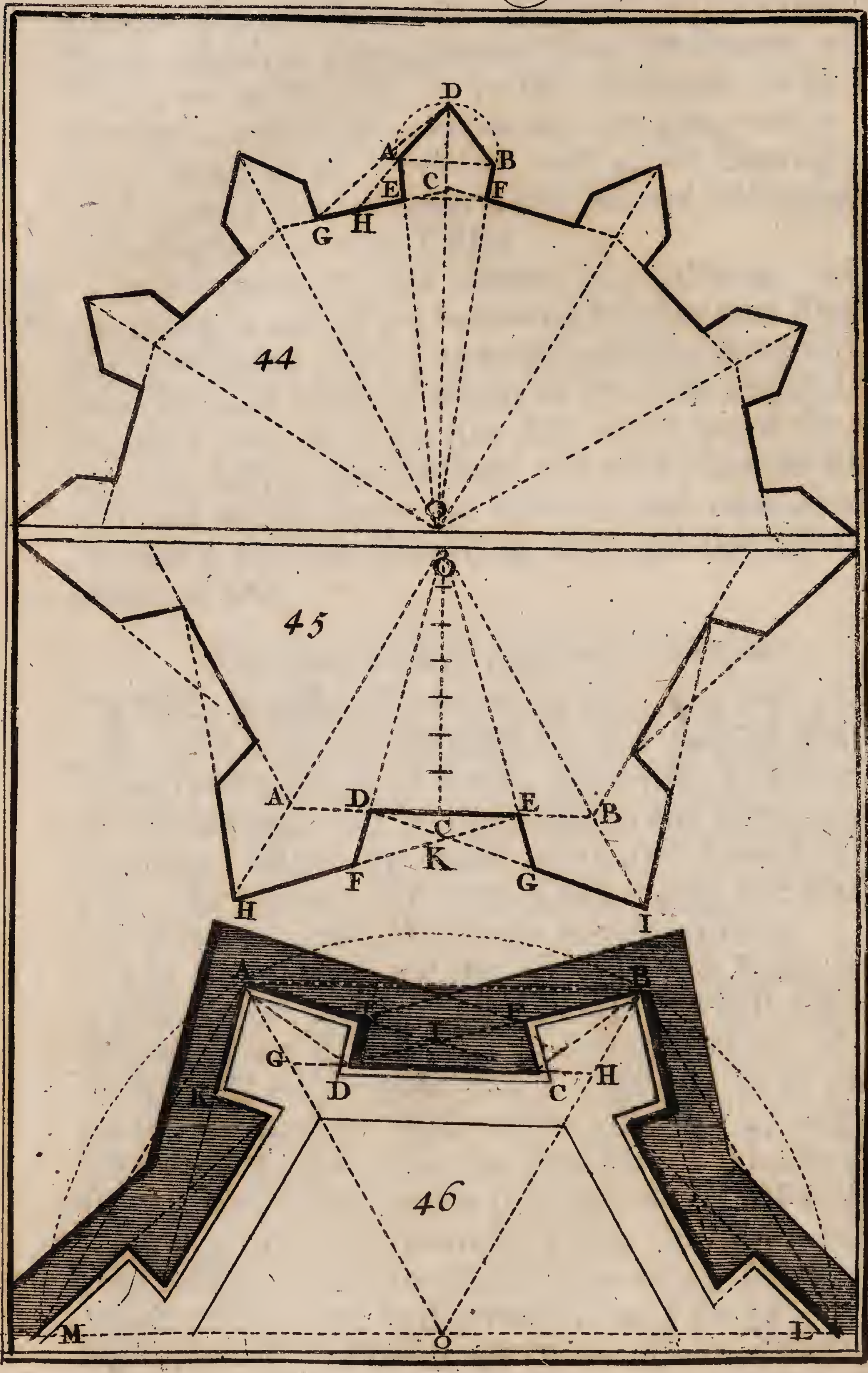
The Calculation of this Second Method.

THE Manner of computing the Lines and Angles of a Polygon Fortified by this Second Method, is the ſame as has been taught in the firſt, at leaſt as far as the Octogon; for in the Octogon and other greater Polygons, where we wou'd have the Flank'd Angle right, the Manner of Fortifying changing, the Manner of computing muſt alſo change as you will ſee.

The Radius CO, and the Angle of the Flank AEG, will be found, as has been taught in the firſt Method, wherefore we ſhall not ſpeak any more of it. As for the Flank'd Angle ADB, it is a right one, or of 90 degrees, and its half ADC conſequently of 45 degrees, which being taken from the Angle HCO, which in the Dodecagon, whoſe half is repreſented in the Figure, is of 75 degrees, the remainder will be 30 degrees for the Angle *diminuë* CHD, to which if you add the Angle of the Flank AEG, which in this Polygon is of 97. 38'. you will have 127. 38'. for the Angle of the Epaule DAE.

To find the Second Flank GH, you muſt firſt find the *complement* EH, in the obliquangled Triangle AEH, in which beſides the Angles, the Flank AE of 30 Toiſes is known, by this Analogy. *As*




D
A
B
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C
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G
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44
O
45
A
D
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K
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I
A
B
F
I
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G
D
C
H
K
46
M
O
L

As the Sine of the Angle diminué AHE	50000
To its opposite side AE	30
So the Sine of the Angle of the Epaule A	79193
To its opposite Side EH	47.3.

Plate 20. Fig. 44.

Which you will find to be of 47 Toises and about 3 Foot, and which being taken from the Curtain GE, or from 60 Toises, the remainder will be 12 Toises and 3 Foot for the Second Flank GH.

If to the Complement EH, found to be of 47 Toises and 3 Foot, be added the Demi-gorge CE, which is of 30 Toises, you will have 77 Toises and 3 Foot for the line CH: and in the obliquangled Triangle HCD, may be found the Capital CD, and the rasant-line HD, by these two Analogies:

As the Sine of the Angle CDH, half of the Flank'd Angle	70711
To its opposite Side CH	77.3
So the Sine of the Angle diminuë CHD	50000
To the Capital CD	54.4.

which you will find to be of 54 Toises, and about 4 Foot.

As the Sine of the Angle CDH, half of the Flank'd Angle	70711
To its opposite Side CH	77.3.
So the Sine of the Angle DCH	96592
To the rasant line DH	105.5.

Which will be of 105 Toises and about 5 Foot.

The Face AD will be found as in the first Method, and the fichant-line GD may be found in the obliquangled Triangle GCD, where the lengthen'd Curtain C G is known, which in this Polygon is of 90 Toises, the Capital CD of 54 Toises and 4 Foot, and the Angle GCD, which they make of 105 degrees; finding first the Angle CDG, thus:

Plate 20. Fig. 44.

As the Sum of the Sides CG, CD	144.4
To their difference	35.2
So the Tangent of a quarter of the Angle of the Polygon	76733
To another Tangent	18741

To which 10 degrees and about 37 minutes anſwer in the Tables, which being added to the quarter of the Angle of the Polygon, that is, to 37.30′. you will have 48.7′. for the Angle CDG, by means of which will be found the fichant-line GD, making in the Triangle GCD, this Analogy:

As the Sine of the Angle CDG	74450
To its oppoſite Side CG	90.
So the Sine of the Angle GCD	96592
To the fichant-line GD	116.5

Which will be of 116 Toiſes, and about 5 Foot.

A Third Method of Fortifying.

IF you wou'd have a greater Second Flank upon the Curtain, even in all the Polygons, having determin'd the length of the Flanks and of the Demi-gorges, as in the ſecond Method foregoing, inſtead of making the Angle of the Baſtion a right one, by means of the Semi-circle ADB, which does not ſignifie much, you may make it acute in all the Polygons, without fearing it ſhou'd become too acute, (*viz*) in making the Capital CD equal to the Gorge-line EF.

To know by Calculation the Aperture of the Flank'd Angle D, according to this third Conſtruction; firſt find out the Capital CD, or the Gorge-line EF, in the Iſoſceles Triangle ECF, where the Angle ECF is known to be of 150 degrees, and the two Angles at the Baſe, of 15 degrees, each, with the Demi-gorges CE, CF, which are of 30 Toiſes each in this Figure, which repreſents half of a Dodecagon. Therefore to find by computation the Gorge-line EF, make this Analogy:

As

As the Sine of the Angle CEF	25881	*Plate 20 Fig. 44*
To its opposite Side CF	30	
So the Sine of the Angle ECF	50000	
To the Gorge-line EF	59	

Thus the Gorge-line EF, or the Capital CD will appear to be of about 59 Toises, which may be more easily found by the following Analogy, which begins with the whole Sine.

As the whole Sine	100000
To double the Demi-gorge CE, or CF	60
So the Sine of half the Angle of the Polygon	96592
To the Capital CD	59

which will be of 59 Toises as before, and which being added to the little Radius OC, which will appear to be of 231 Toises and 5 Foot, you will have 290 Toises and five Foot for the great Radius OD.

As for the Angle of the Flank AEG, or CEO, it will be found of 97. 38′. by means of which you may, by computation, find in the obliquangled Triangle CEO, the line OE, by this Analogy:

As the Sine of the Angle of the Flank CEO	99113
To its opposite Side CO	231.5
So the Sine of ECO, half the Angle of the Polygon	96592
To its opposite Side OE	225.5

which will be of 225 Toises and about 5 Foot; to which adding 30 Toises for the Flank AE, you will have 255 Toises and 5 Foot for the line AO. Thus in the obliquangled Triangle ADO, the two Sides AO, DO, will be known, and the Angle which they make, or the Flank-forming Angle AOD, which in the first Method will be of 7. 22′. as it appears in the Table, which we have added at the beginning of the First Part. Therefore the Flank'd Angle may be known by finding out its half ADO, in this manner:

Take from 180 degrees the Flank-forming Angle AOD, or 7.22′. and half of the remainder 172.38′. will be 86.19′. whose Tangent is 1553398, which will be the third Term of the following Analogy:

As the Sum of the Sides AO,DO	546.4
To their difference	35.0.
So the foregoing Tangent	1553398
To another Tangent	99455

to which 44 degrees and about 51 minutes anſwer in the Table, which being taken from 86.19′. the foregoing half, there will remain 41.28′. for ADO, half of the Flank'd Angle, wherefore the Flank'd Angle ADB will be of 82.56′. which being thus known, the reſt may be eaſily known as before.

A Fourth Method of Fortifying.

WE will make an end by this fourth Method, for thoſe that wou'd not have Second Flanks; it is more general than the foregoing ones, becauſe it does not limit the Flanks, which are always drawn from the Center of the Place, nor the Demi-gorges, for tho' they always increaſe, yet they grow but very inſenſibly in great Polygons, ſo that as we have ſaid elſewhere, in a Polygon of 30 Sides, a Demi-gorge does not exceed 36 Toiſes upon an inward ſide of 120 Toiſes.

Plate 20. Fig. 45. Having drawn from O, the Center of the Polygon to be Fortified, thro' the point C the middle of the inward ſide AB, the right line OC perpendicular to AB, divide the ſaid OC into as many equal parts as the Polygon to be Fortified has ſides, and one more, as here for an Hexagon into 7 equal parts, and allow two of them for the Demi-gorges AD,BE, and three for the Capitals AH,BI: after which nothing will be wanting but to draw the raſant-lines EH,DI, which will terminate the Flanks DF,EG, drawn from the Center O.

In the Practice you need actually draw the perpendicular OC, and divide it into 7 equal parts; for if you apply the length of it, which you may have by deſcribing from the Center O, an Arch which raſes the inward ſide AB, to a number on each ſide of the Line of equal parts on the Compaſſes of Proportion, which is di-

divisible by 7, as from 140 to 140, whose seventh part is 20, and upon the said Line of equal parts you take the distance from 40 to 40, you will have the length of the Demi-gorge, and if you take the distance from 60 to 60, you will have the length of the Capital. *Plate 20. Fig. 45.*

The Calculation of this Fourth Method.

TO compute the Lines and Angles of a Polygon thus Fortified, as of an Hexagon, whose half this Figure represents, you must begin by the perpendicular OC, which may be known by computation, making in the rectangular Triangle ACO, this Analogy:

As the whole Sine	100000
To the Tangent of half the Angle of the Polygon OAC	173205
So AC half of the inward Side	60
To the perpendicular OC	104

which you will find of about 104 Toises.

If from 208 double, and 312 triple this perpendicular, you take the Seventh parts, you will have 29 Toises and about 4 Foot for the Demi-gorge AD, and 44 Toises and about 3 Foot for the Capital AH.

If from the inward Side AB, which is of 120 Toises, the Demi-gorge AD, or BE be taken, which we have found to be of 29 Toises and 4 Foot, there will remain 90 Toises for the lengthen'd Curtain AE: And in the obliquangled Triangle HAE, one may find the Angle *diminué* AEH, and half the Flank'd Angle AHE, by means of the two known Sides AE, AH, and the Angle HAE, which they make, which is what half the Angle of the Polygon wants of 180 degrees, in this manner:

As the Sum of the Sides AE, AH	134.5
To their difference	45.5
So the Tangent of a quarter of the Angle of the Polygon	57735
To another Tangent	19625

to which 11 degrees and about 6 minutes answer in the Tables,

Plate 20. Fig. 45. Tables, which being taken from a quarter of the Angle of the Polygon, that is, from 30 degrees, the remainder 18. 54'. will be the Angle *diminué* AEH; but if they are added the ſum will be 41. 6'. for half the Flank'd-angle AHE, wherefore the Flank'd-angle will be of 82. 12'. to which adding the Angle of the Center AOB, ſuppos'd here of 60 degrees, you will have 142. 12'. for the Flanking-angle HKI.

The Radius AO will be found of 120 Toiſes, and the Angle of the Flank ADO, or EDF, of 106. 18', as has been taught in the firſt Method, to which if DEF the Angle *diminué* 18. 54'. be added, you will have 125. 12'. for the Angle of the Epaule DFH.

If from the lengthen'd Curtain AE, which has been found to be of 90 Toiſes and two Foot, the Demi-gorge AD, which we have found to be of 29 Toiſes and 4 Foot be taken, the remainder will be 60 Toiſes and 4 Foot for the Curtain DE, by means of which one may find the Flank DF in the Triangle DEF, by this Analogy:

As the Sine of the Angle DFE	81714
To its oppoſite Side DE	60.4.
So the Sine of the Angle DEF	32392
To its oppoſite Side DF	24.

which will be about 24 Toiſes. The reſt may be known as before.

The Flank'd-angle begins to be obtuſe in the Enneagon; where it is of 92.6'. and it becomes ſtill greater in the following Polygons, but this exceſs is inconſiderable, being but of about one degree in each Polygon, ſince the Flank'd-angle is but of 96.26'. in the Dodecagon, whereas by Count *Pagan*'s, or Mr. *Vauban*'s Method, which are the ſame in reſpect of this Angle, it is of 103.8'. in the Enneagon, and of 113. 8'. in the Dodecagon, becauſe the Angle *diminué* is every where of 18. 26'.

Errard's *Manner of* Fortifying.

E*Rrard* Fortifies inwards, and makes the Flank perpendicular to the Face of the Baſtion, or to the Line of Defence, which is always Raſant in all the Polygons: As for the Flank, he makes it perpendicular to the Face, but from the Square to the Octogon; for in the other Polygons, he makes it perpendicular to the Curtain. The Flank'd Angle is of 60 degrees in the Square, of 80 in the Pentagon, and of 90 in the other Polygons.

But to come to the Practice: Let AB be the ſide of an Hexagon, whoſe Center is O. At the ends A,B, of the outward ſide AB, with the Radii AO, BO, make the Angles OAC, OBD, of 45 degrees each, and divide one of thoſe Angles, as OAC, into two equal parts by the line AD, which will terminate the line of Defence BD, which you muſt carry on to AC, to joyn the Curtain CD, and the Flanks DE,CF, each perpendicular to its oppoſite Raſant-line AC, BD, and thus each Angle of the Epaule E, F, will be a right one. If the ſame thing be done every where, the Hexagon will be Fortified, to which, on the outſide, muſt be added a Ditch, whoſe Counterſcarp is parallel to the Raſant-line, and is drawn from the Angle of the Epaule; and on the inſide a Rampart, whoſe breadth muſt be equal to the length of the Flank. *Plate* 20. *Fig.* 46.

Remarks upon Errard's *Fortification.*

THIS Manner of Fortifying is defective; firſt, becauſe the Flank (tho' being perpendicular to the Face of the Baſtion, it is better ſecur'd againſt the Enemy's Batteries, and contributes better to the defence of the Gates and Curtains, which are the Parts beſt defended by their nearneſs to the Flanks; yet it) diſcovers much leſs the Enemies Batteries, and cannot hinder them from advancing their Works as far as the Counterſcarp, which it can ſcarce defend, which is a great advantage to the Beſiegers.

Be-

Plate 20. *Fig.* 46. Besides, if one wou'd make Orillons in ſuch Flanks, the cover'd Flank will be ſo hid, that it will hardly diſcover all the breadth of the Ditch; and it can hold but few Cannons, the Angles of the Merlons alſo towards the Field will be ſo acute, that ſuch Merlons may be eaſily ruin'd by the Enemy's Cannon, which renders them uſeleſs.

Moreover, there is but little advantage reap'd by covering the Flanks from the Enemy's Batteries, becauſe when he is lodg'd upon the Counterſcarp he will always diſcover them: beſides, if the Flank be diſcover'd, it diſcovers alſo, with that advantage that the Parapets of the Place being of Earth that is well ſettled, will be more difficultly ruin'd then the Batteries of the Enemy, who is only cover'd with new rais'd Earth or Gabions.

Laſtly, as the Polygons increaſe, that is, as they have more ſides, the Faces increaſe likewiſe, and the Curtains decreaſe, which is a conſiderable fault, becauſe the weakeſt parts, (*viz.* the Faces, which are defended but on one ſide) increaſe, and the ſtrongeſt decreaſe, namely, the Curtains, which are flank'd and defended on each ſide by the Flank.

This Manner of Fortifying has this one Conveniency, that the breadth of the Ditch is ſeen and flank'd by the whole Flank, which does not happen in Places that have a great ſecond Flank, except the Ditch be made very wide over-againſt the Curtain, and alſo in Places that have a great Flank-raſant, unleſs you have a very large Ditch, which increaſes the expence.

The Calculation of the Angles and Lines, according to Errard's *Deſign.*

As this Figure repreſents a Demi-Hexagon, the Angle AOB of the Center will be of 60 degrees, and ABL the Angle of the Polygon of 120. KAE the Flank'd-angle having been made a right one, or of 90 degrees, as well as the Angle of the Epaule AED, the Angle *diminué* ACG will be of 15 degrees, the Angle of

of the Flank EDC of 75, and the flanking Angle EIF of 150. And since CAG half the flank'd Angle 45 degrees has been divided into two equal parts by the line AD, each of the Angles DAE, DAG will be of 22. 30′. And the Angle ADE its complement will consequently be of 67. 30′. The Angle ADG 37. 40′. The Angle ADC 142. 30′. The Angle AGC of 120 degrees, and the Angle DAB of 37. 30′. *Plate 20. Fig. 46.*

Thus all the Angles are known, but no Line is known, which yet is necessary, that one Line being known, may serve us as a Foundation to know the others by help of the known Angles: Therefore we must suppose one of a length agreeable to the Maxims of a good Fortification, as the Line of Defence AC, or BD, which we will suppose of 120 Toises, to compute in those parts the other Lines, in the following Manner.

First, find out the inward side AB by this Analogy in the obliquangled Triangle ADB,

As the Sine of the Angle DAB	60876
To its opposite Side BD	120
So the Sine of the Angle ADB	79335
To its opposite Side AB	156.2.

which will be of 156 Toises and about 2 Foot.

Because the Line AD is of use in finding out the Face AE, and the Flank DE, in the rectangular Triangle AED, we must find its length in the same obliqueangled Triangle ADB, by this Analogy:

As the Sine of the Angle DAB	60876
To its opposite side BD	120
So the Sine of the Angle ABD	25882
To its opposite Side AD	51

which we find to be of about 51 Toises, and which will serve to find out the Face AE, and the Flank DE, in the Triangle ADE rectangular at E, by these two Analogies:

As

Plate 20. Fig. 46.

As the whole Sine	100000
To the Hypotenuse AD	51
So the Sine of the Angle ADE	92788
To the Face AE	47. 1

which will be of 47 Toises, and about One Foot.

As the whole Sine	100000
To the Hypotenuse AD	51
So the Sine of the Angle DAE	38268
To the Flank ED	19. 3.

which will be of 19 Toises, and about 3 Foot.

By means of the Flank thus known, one may find out the Curtain CD, by the following Analogy in the Triangle CDE, rectangular at E,

As the whole Sine	100000
To the Secant of the Angle CDE	386370
So the Flank ED	19. 3.
To the Curtain CD	75. 2.

which will be of 75 Toises, and about 2 Foot.

The Capital AG will be found in the obliquangled Triangle, whose Angles are all known as well as the side AD, by this Analogy,

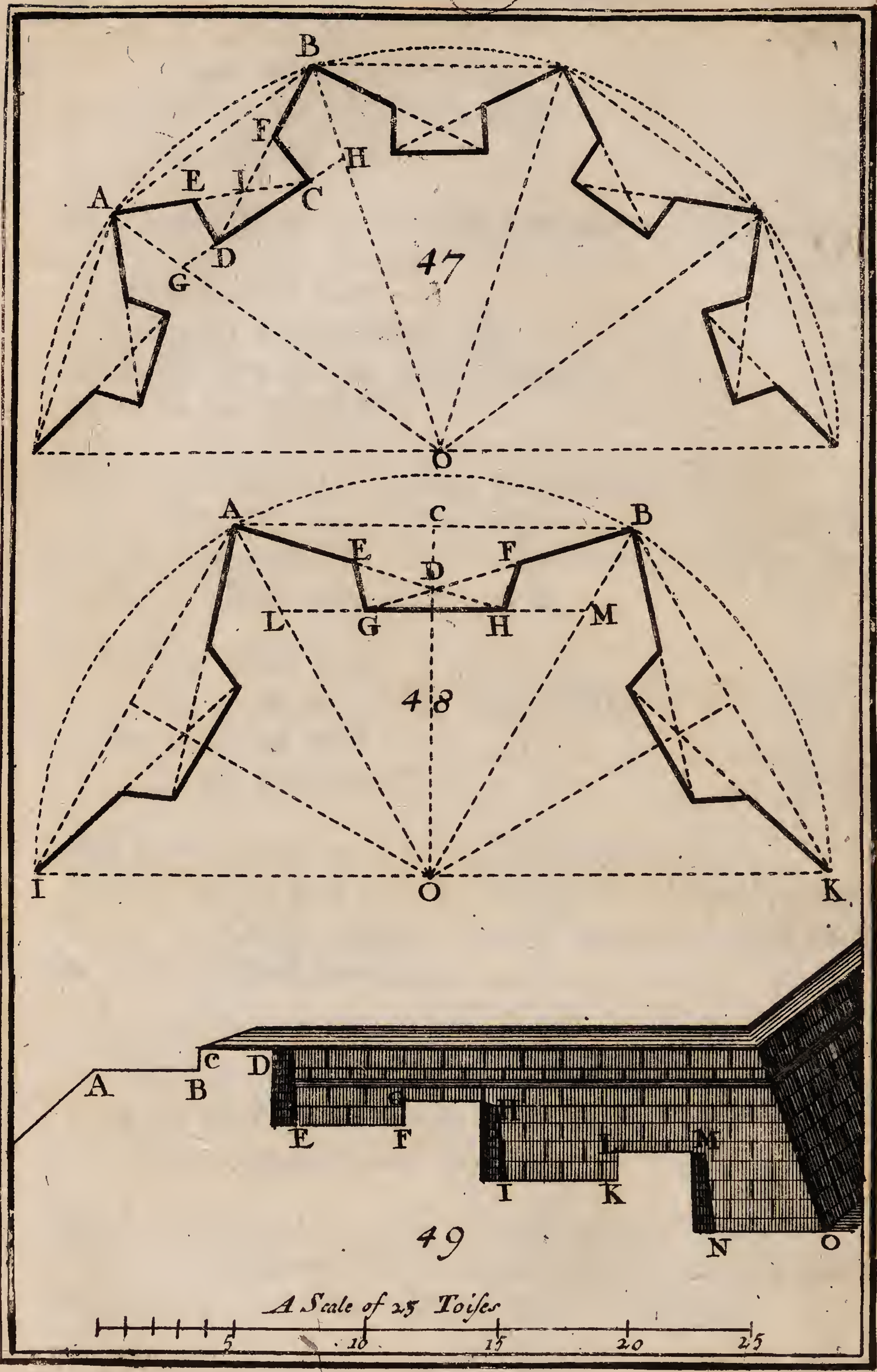
As the Sine of the Angle AGD	86602
To its opposite side AD	51
So the Sine of the Angle ADG	60876
To its opposite side AG	35. 5

which will be of 35 Toises, and about 5 Foot.

The Demi-gorge DG will be found by the following Analogy, in the same obliquangled Triangle.

As the Sine of the Angle AGD	86602
To its opposite side AD	51
So the Sine of the Angle DAG	38268
To its opposite side GD	22. 3.

which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L
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I
K
49
N
O
A Scale of 25 Toises
5
10
15
20
25

which will be found to be of 22 Toises, and about 3 Foot, whose double, or 45 Toises being added to the Curtain FG, which is of 75 Toises and 2 Foot, you will have 120 Toises and 2 Foot for the inward side GH.

The Computation will be no otherwise, when the Flank is perpendicular to the Curtain, which will be done by taking the lines IE, IF, equal to the lines IC, ID: And then the Flanks DE, CF, will become greater than in the foregoing Polygon, and consequently better. *Plate 21. Fig. 47.*

Count Pagan's *Manner of* Fortifying.

COunt *Pagan*'s Method of Fortifying a Polygon is quite contrary to the foregoing, and also better; for instead of making the Angle of the Flank acute, as *Errard* does, he makes it obtuse, that the Flank may discover more Ground, making it perpendicular to the Line of Defence of the opposite Bastion, that from such a Flank one may the better rase and defend the Face of the said opposite Bastion, which is the weakest Part of all the Fortification, and the first attack'd. This Count makes the Line of Defence always rasant, as *Errard* does; but he does not value making the Angle of the Bastion a right one.

This Author establishes Three Sorts of Fortification, the Great, the Mean, and the Little: And as he also Fortifies inwards, he makes the outward side of 200 Toises in the Great Fortification, of 180 in the Mean, and 160 in the Little, and allows 60 Toises for the Face in the Great one, 55 Toises in the Mean, and 50 in the Little one. Lastly, the Perpendicular, which determines the Rasant-line, is every where of 30 Toises, except in the Square, where it is only of 27 for the Great Fortification, and of 24 for the Mean, and for the Little. But to the Practice. *Plate 21. Fig. 48.*

Having suppos'd the outward side AB of 200 Toises, for the Great, of 180 for the Mean, and 160 for the Little Fortification, divide it into Two equal Parts, at the point C, and draw to its point C from the Center

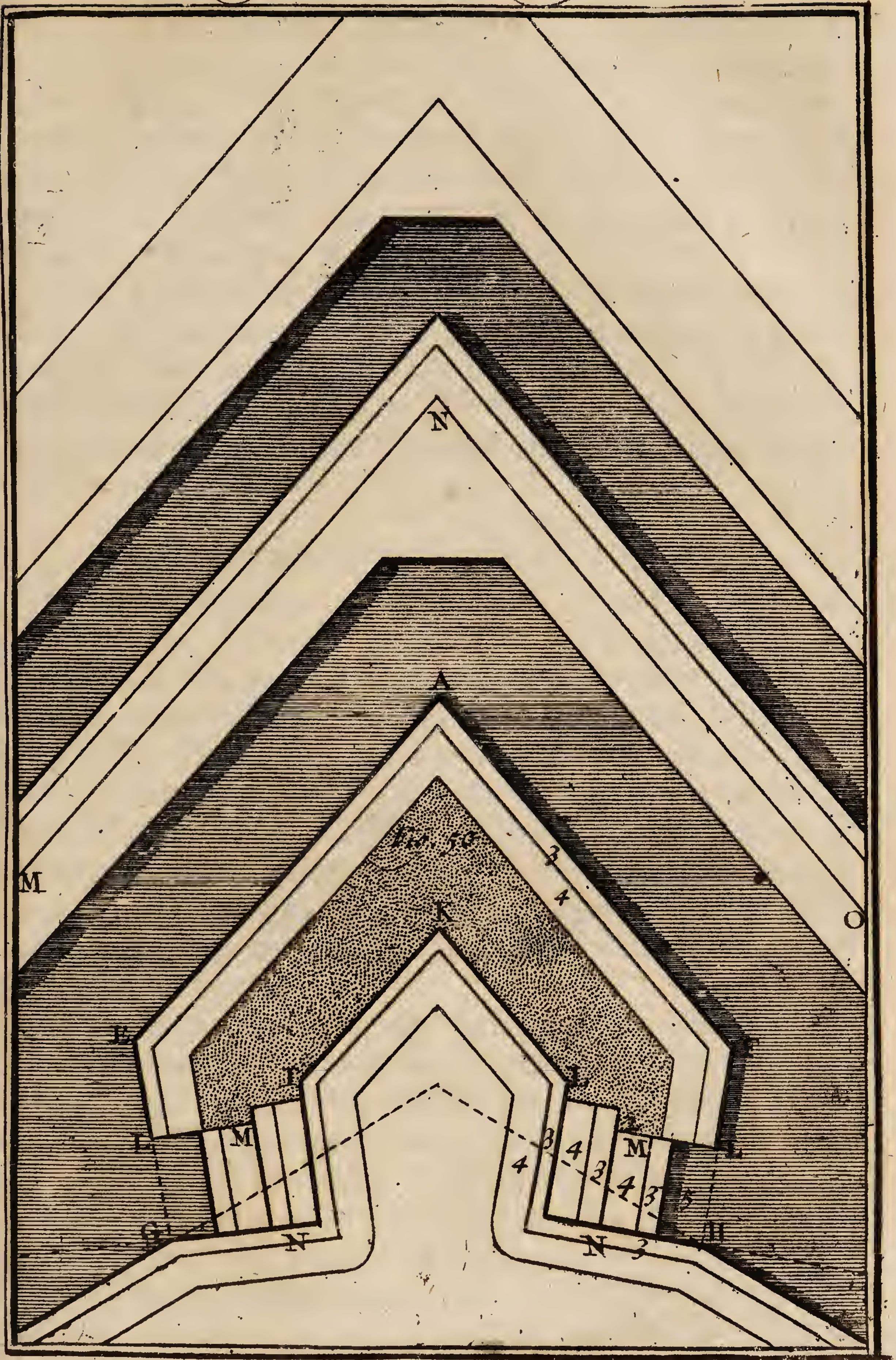
ter O, the perpendicular CD of 30 Toiſes in all the Polygons, except in the Square, where you ſhall only make it of 27 Toiſes for the Great, and 24 for the Mean, and for the Little Fortification. Draw thro' D the Lines of Defence ADH, BDG, to take upon them the Faces AE, BF, of 60 Toiſes each in the Great Fortification, of 55 in the Mean, as here, and of 50 in the Little. Laſtly, draw the Flanks EG, FH, perpendicular to the Raſant-lines BG, AH, and join the Curtain GH.

Plate 22. *Fig.* 50. For the Caſemates and Orillons, divide the Flanks EG, FH, each into Two equal Parts at L, and the lines EL, FL, will be the Flanks of the Orillons; which the Author has made Square. Lengthen the Line of Defence into the Baſtion towards N, and thro' the point L, draw LM parallel to the ſaid Line of Defence for the depth of the Caſemate, in which the Author has Three Batteries, where the Numbers mark'd in the Figure ſhew how many Toiſes the Parapets and Terre-plains of theſe Batteries are of, each of which muſt be capable of containing 3 Pieces of Cannon, at leaſt. The height of the firſt is 2 Toiſes above the bottom of the Ditch, that of the Middle-one 4, and the Third 6, being as high as the Rampart; which is 3 Toiſes above, as the bottom of the Ditch is 3 Toiſes below, the Level of the Field.

Plate 21. *Fig.* 49. This will be eaſily known by looking upon the Profill of thoſe Batteries, which we have deſcrib'd along the the Wall of the Demi-gorge, upon the lengthen'd part of the Line of Defence: Where the line AB repreſents the Terre-plain of the Rampart, and of the firſt Battery, of which CD is the Parapet: The line EF repreſents the Terre-plain of the Second or Middle Battery, of which GH is the Parapet: The line IK repreſents the Terre-plain of the Third and Loweſt Battery, whoſe Parapet is LM: Laſtly, the line NO repreſents the bottom of the Ditch.

Plate 22. *Fig.* 50. This *Count* adds in the Baſtion another leſs Baſtion IKL, with its Rampart and Parapet, which ſerves for a Retrenchment, whoſe Earth is taken from the dry Ditch hard by. The Counterſcarp of the great Ditch

is


N
A
M
O
Fig. 53
K
E
F
L
M
G
H
N

is parallel to the Face of the Baſtion, or to the Line of Defence, and beyond that Ditch, he adds a Counter-guard MNO, whoſe Ditch is 12 Toiſes broad, and Rampart 7, taking in the Parapet of 3 Toiſes: Pretending that a Place Fortified with theſe Three Ditches, and theſe Three Parapets defended by the Cannon, is Three Times as Strong as ordinary Places.

This Learned Author makes alſo upon the re-entrant Angle of the Counterſcarp, a double Ravelin, ſo plac'd as to be defended by the Face of the Baſtion, and by the Counter-guard, which is alſo flank'd by the Ravelin, as one may ſee in the Treatiſe of Fortification which he has publiſh'd.

Remarks upon Count Pagan's *Fortification.*

THIS Diviſion, which Count *Pagan* makes into Great, Mean, and Little Fortification, ſeems of very little uſe, at leaſt in Regular Fortification: Therefore in the Practice I ſhall keep to the Mean, which allows 180 Toiſes for the outward ſide of the Polygon, where it happens that the Line of Defence is of about 126 Toiſes, as you will ſee in the Calculation that we ſhall give of it, after we have ſaid ſomething concerning the Advantages and Diſadvantages of this manner of Fortifying, which has been a Model to all the others ſince it has been publiſh'd.

The Author conſidering that the chief Parts which defend powerfully, and very much retard the taking of Places, are the Flanks, the Ramparts, and the Ditches, makes his Flanks very great, and his Gorges very long, to make Three Batteries in them, in each of which he puts 4 Pieces of Cannon, Three of which are well enough cover'd, not to be eaſily damaged by the Beſiegers Counter-Batteries, who will ſuffer very much from them when they appear, and endeavour to make Lodgments in the ruins of the Breaches, where the Cannons of the Batteries fire *de Revers*, or in the Rear.

The Two Ditches with the Two Ramparts, that the Author makes, according to his Maxims, not only hin-

Plate 22. Fig. 50. der the Enemies from paſſing the Ditch eaſily, but alſo cuts them off from the uſe of Fourneaux, and from Lodging themſelves upon the ruines of Breaches: And beſides, when the Beſieger has taken the firſt Rampart, he will find himſelf Embaraſs'd in the Second Ditch, being almoſt in the middle of the Beſieg'd, who can eaſily fire upon him on all ſides, and deſtroy him with Bombs, Granado's, Gauderons, Mines and Fourneaux, *&c.*

Laſtly, The Ditch is as well defended as may be, from the Flank, which being perpendicular to the Line of Defence, flanks fully the Face of the oppoſite Baſtion: And the greatneſs of the Demi-gorge ſhortens the Line of Defence, ſo that the High and the Mean Places will never be out of Musket-ſhot, at leaſt the Mean, which conſequently ought to be preferr'd to the Great Fortification.

Theſe Advantages, and ſeveral others, make a great many approve of this Manner of Fortifying, but it is not to be approv'd in every thing, becauſe of the Diſadvantages which (I think) it has; firſt, becauſe the Faces being too long, the Enemy may make great Breaches in them, which are very troubleſome to the Beſieg'd, when they go about to repair them, leaſt the Enemy ſhou'd make a Lodgment in them, eſpecially when the Defenders are but few.

The Caſemates and Flanks are too much expos'd to the Counter-Batteries of the Beſiegers, who may eaſily diſmount the Cannons, then paſs the Ditch, and hide themſelves in the Breach, without fearing the Musket-ſhot of the oppoſite Flanks, which in the Great Fortification are too diſtant: Beſides, the Three hidden Cannons are not ſufficient to hinder the Enemies from lodging upon the ruines of the Breach, becauſe while the Pieces are charging, they may bury themſelves in it, and raiſe *Epaulements* or Shoulderings to ſecure themſelves.

Flanks perpendicular to the Raſant-line, which this Count made thus for this Reaſon, (*viz.*) that ſuch as make

make them otherwiſe don't conſider, that whatever ſees is alſo ſeen from what it ſees, may be rejected for this Reaſon: For tho' it is true that what ſees is ſeen, yet one ſhou'd conſider that what ſees a great Space, is alſo ſeen of that great Space, in which the Beſiegers may find advantageous Places to raiſe Batteries, and deſtroy thoſe which they will ſee in Flanks, which are too much expos'd. Beſides it is not neceſſary that the Cannons of the Flanks ſhou'd diſcover ſo much of the Field, their uſe being only to defend the oppoſite Baſtion's Face, and Counterſcarp, and the Ditch; for the reſt may be defended from ſeveral other Places.

Laſtly, The retrench'd Baſtion which increaſes the Expence by one third Part, tho' it is but a Retrenchment, and an empty Baſtion, may be taken as eaſily as the firſt, by means of a ſecond Mine, and by its loſs cauſe that of the Place, the Beſieg'd not being able to hinder it, or to raiſe any Retrenchment in that empty Baſtion, in order to Capitulate with more Advantage.

Theſe Reaſons ſeem weighty enough to ſhew, that tho' this manner of Fortifying be not blameable in moſt of its Maxims, yet I am not to blame that I have not follow'd it; it is true, that by retaining what is Good, and altering for the Better what is Diſadvantageous in it, one might certainly render it a very perfect way of Fortifying.

For Example, That the Caſemates may be longer and better Cover'd, inſtead of drawing the *Retirade* of the Flank LM parallel to the Line of Defence, one may draw it from the Angle of the oppoſite Baſtion, as we have done elſewhere: And becauſe the Flank'd angle begins to be obtuſe in the Heptagon, and that it is very much open in the Polygons of more ſides, which I think a conſiderable Fault; one may correct that Angle, by making it a right one, by means of a Semi-circle deſcrib'd upon the Two Flanks of the Baſtion, as we do in our Second Method, which will, indeed, make the Faces a little longer, but that fault is recompens'd by a Right-flank'd-angle, a cloſer Tenaille, and a Second-

flank upon the Curtain, which very much increaſes the Defence of the Place. Laſtly, One may draw the Flank from the Center of the Place, according to our Method, and then it will be leſs expos'd, and ſomething greater, which gives it Two conſiderable Advantages.

The Calculation of the Angles and Lines according to Count Pagan's *Deſign.*

Plate 21. *Fig.* 48. THE outward ſide AB belonging to an Hexagon, the Angle AOB of the Center will be of 60 Degrees, and the Angle ABK of the Polygon of 120. If you ſuppoſe the outward ſide AB of 180 Toiſes, ſuch as it muſt be in the Mean Fortification, which we follow here, its half AC, or BC will be of 90 Toiſes, the perpendicular CD of 30, and the Face AE of 55.

By means of theſe Angles, and theſe Lines, thus known, one may by Computation eaſily know the other Angles, and the other Lines; and firſt the Angle *diminué* CAD, by the following Analogy in the Triangle ACD rectangular at C:

As the ſide AC	90
To the whole Sine	100000
So the ſide CD	30
To the Tangent of the Angle CAD	33333

which you will find to be of 18. 26'. which being taken from half the Angle of the Polygon, or from 60 Degrees, 41. 34'. will remain for OAH half of the Flank'd-angle, wherefore the Flank'd-angle will be 83. 8'. to which if AOB the Angle of the Center, or 60 Degrees be added, the ſum will be 143. 8'. for the Flanking-angle ADB.

To know the Flank EG, one muſt firſt find the Hypotenuſe DE of the rectangular Triangle DGE, finding firſt the Tenaille AD, in the rectangular Triangle ACD, by this Analogy:

As

As the whole Sine 100000 *Plate 21. Fig. 48.*
To the Secant of the Angle diminué CAD 105408
So AC half the outward side AB 90
To the Tenaille AD 94. 5

which will be of 94 Toises, and about 5 Foot, from which taking the Face AE, which is of 55 Toises, 39 Toises and 5 Foot will remain for the Hypotenuse DE, and in the rectangular Triangle DGE, whose Angle EDG is equal to double the Angle *diminué*, and consequently of 36. 52'. you may find the Flank EG by this Analogy:

As the whole Sine 100000
To the Hypotenuse DE 39. 5
So the Sine of double the Angle diminué 59995
To the Flank EG 23. 4.

which we shall find to be of 23 Toises, and about 4 Foot.

If to GDE double the Angle *diminué*, that is, if to 36. 52'. the Angle DGE, or 90 Degrees be added, you will have 126. 52'. for the Angle of the Epaule AEG; and if to the Angle *diminué* FGH, which has been found to be of 18. 26'. 90 Degrees, or the Right-angle EGD be added, you will have 108. 26'. for EGH the Angle of the Flank.

To know the Rasant-line AH, or BG, first find out the line DG, or DH, which our Author calls *Complement*, by the following Analogy in the rectangular Triangle DGE:

As the whole Sine 100000
To the Hypotenuse DE 39. 5
So the Sine of the Angle of the Epaule E 80003
To the Complement DG 31. 3.

which we shall find to be of 31 Toises, and about 3 Foot, which being added to the Tenaille AD, or BD, which has been found to be of 94 Toises, and 5 Foot, you will have 126 Toises, and 2 Foot for the Rasant-line AH, or BG.

Plate 21. Fig. 48. By means of the Complement, DG, or DH, thus known to be of 31 Toises, and 3 Foot, you may find out the Curtain GH, by the following Analogy in the Isosceles Triangle GDH:

As the Sine of the Angle diminué DGH	31620
To the Complement DG	31. 3.
So the Sine of double the same Angle diminué	59995
To the Curtain GH	59. 3.

which will be of 59 Toises, and about 3 Foot, and which will serve us to know the lengthen'd Curtain LH, by the following Analogy in the obliquangled Triangle ALH:

As the Sine of half the Angle of the Polygon L	86602
To the Rasant-line AH	126. 2
So the Sine of half the Flank'd-angle LAH	66349
To the lengthen'd Curtain LH	96. 4.

which is of 96 Toises, and about 4 Foot, from which taking the Curtain GH, which has been found to be of 59 Toises, and 3 Foot, you will have 37 Toises and 1 Foot for the Demi-gorge LG, or HM, which being added to the said lengthen'd Curtain LH, or GM, that is, to 96 Toises and 4 Foot, you will have 133 Toises, and 5 Foot for the inward side LM.

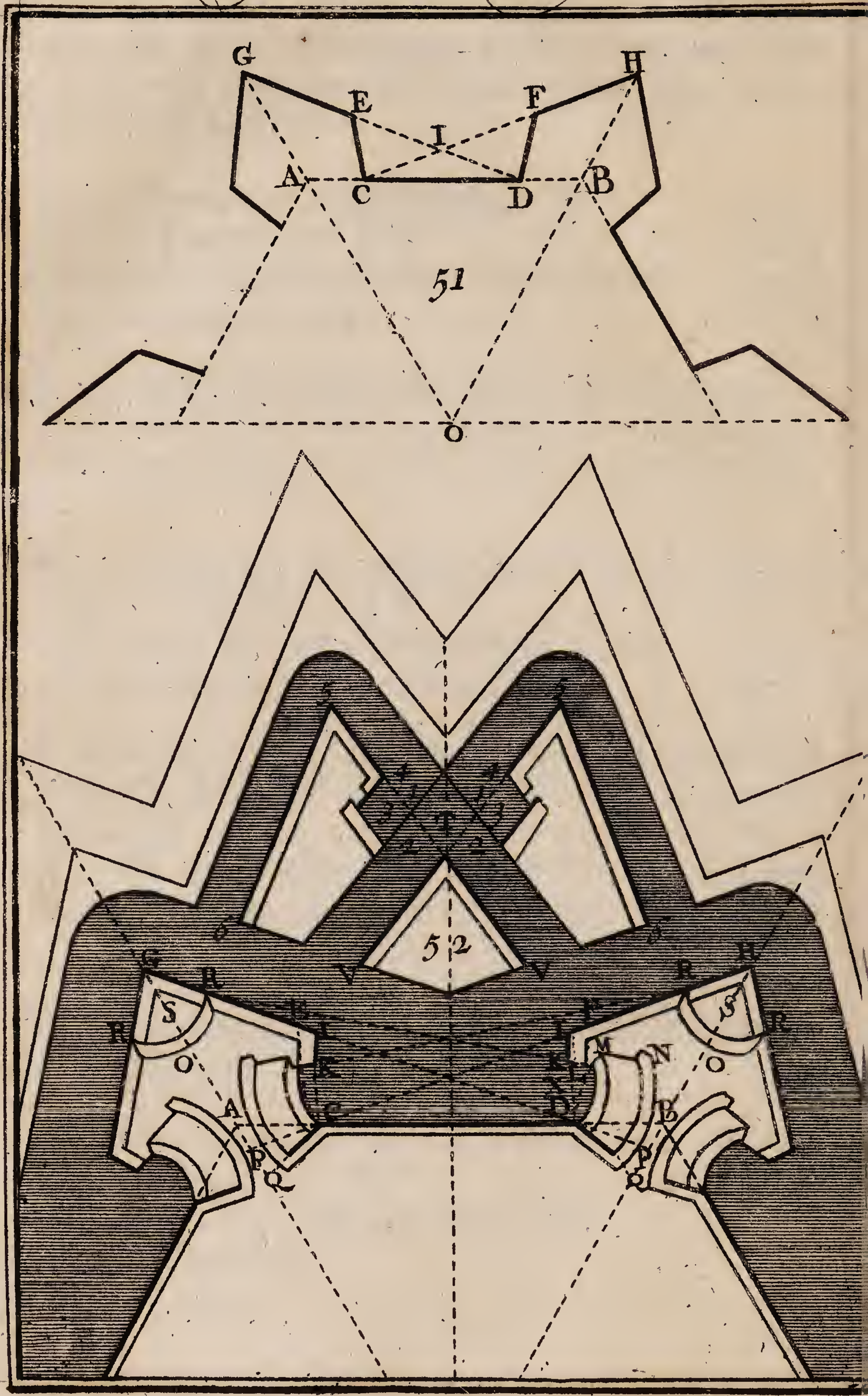
Lastly, you may find the Capital AL by this Analogy in the obliquangled Triangle ALH:

As the Sine of half the Angle of the Polygon L	86602
To the Rasant-line AH	126. 2.
So the Sine of the Angle diminué AHL	31620
To the Capital AL	46

of about 46 Toises.

Mr. Bombelle's *Manner of* Fortifying.

MR. *Bombelle* Fortifies outwards, and after Count *Pagan*'s Way, establishes Three Sorts of Fortification, the Little Royal, the Mean, and the Great Royal.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Royal. The inward side of the Little Royal is of 60 * Rods, or 120 Toises, that of the Mean of 70 Rods, or 140 Toises, and that of the Great Royal of 80 Rods, or 160 Toises. They are all Fortified one Way, which is the following:

The side of your Polygon being determin'd for your Great, Mean, or Little Royal, as AB, which here belongs to an Hexagon, allow the fifth Part of it to the Demi-gorges AC, BD, and the fourth Part to the Flanks CE, DF, which must each of them make with the Curtain CB an Angle of 100 Degrees, then you will have the Points of the Bastions G, H, by the Rasant-line, CH, DG, and the whole operation is ended. *Plate* 23. *Fig.* 51.

The Author adds beyond the Ditch, which he makes 12 Rods, or 24 Toises broad, a Ravelin, whose Point T is found by describing from the ends A, B, of the inward side AB, Two Arches, with the distance of the lengthen'd Curtain AD, or BC, whose Faces TV, tend to I the ends of the Square Orillons, wherewith the Author covers his Round-flanks or Casemates, which he thus describes: *Fig.* 52.

Draw upon the ends C, D, of the Curtain CD, the Two Flanks CI, DI, perpendicular, which will be terminated at I, by the Two Rasant-lines CH, DG, and will terminate the Faces GI, HI, which thus will be increas'd by all the space EI, or FI, which the Author allows that he may have more room in the Bastions, and make Two cover'd Flanks in them, thus:

Having drawn the Two lines GR, HR, each equal to one Third of the Faces GI, HI, and the lines IK equal also to a third Part of the Flanks CI, DI, draw from the point R, which terminates GR the third Part of the

---

* *Tho' by a* Rod, (English *Measure) we mean* 16 *Foot and a half, yet Mr.* Ozanam *only understands* 12 Paris *Feet, or Two Toises, which are* 12 *Foot* 9 *Inches and a half of* English *Measure; for the* French *Toise is Six of our Feet* 5 *Inches* 3 *quarters, tho' in the Marginal Note,* Page 5, *we have call'd it a Measure of Six Feet, not thinking such an exactness requisite in* Fortification, *there being little more than* 47 *Foot difference in a Line of Defence of* 120 *Toises.*

Plate 23. Fig. 52. Face GI, thro' the point K the line RKL, which will be terminated at L, by the line DL, perpendicular to the Rasant DG, which must be lengthen'd till it touches the Radius of the Polygon in some point, as in P, to take upon the same Radius the distance PQ, which will terminate the Two Cover'd-flanks, whose common Center X will be found by describing from the ends D, L, of the line DL Two Arches, with a distance equal to three quarters of the said DL.

The rest may be easily understood by a sight of the Figure: Wherefore we shall only say, that upon DL lengthen'd, the line LM has been taken two Rods long, and that from M, to the line KL has been drawn the parallel MN 12 Rods long, *&c.* You must observe that the Parapets are every where Two Rods, or 4 Toises broad.

The Casemates, which our Author adds at the Point of the Bastion, and which, in time of need, may serve as a Retrenchment, is describ'd by taking upon the Capital of the Bastion, the line GS, or HS equal to half of the line GR, or HR, and describing from the point S for your Center, thro' the points R, an Arch for the Casemate, whose Parapet is 4 Toises broad, and which is something lower towards the Point of the Bastion than the rest of the Bastion, and the way to it lies thro' a Passage made in some part of the Bastion, as at O.

We have forgot to say, that our Author covers his Ravelins with a Counter-Guard, which he makes beyond the Ditch of the Ravelin, which he makes 8 Rods, or 16 Toises broad, which is the third Part of the breadth of the great Ditch. This Counter-Guard is thus describ'd:

Lengthen TV the Face of the Ravelin beyond the Ditch towards 5, so that the line 1, 5, may be equal to the said Face TV, and at the point 5, with the line 1, 5, make an Angle of 60 degrees, by means of the line 5, 6. Make the lines 1 2, 1 4, each equal to the third Part of 1 5, and finish the Rhombs 1 2 3 4, and the Counter-Guard

Guard will be perfected, to which add on the outside a Ditch, and on the inside a Parapet, as in the Ravelin.

Remarks upon Mr. Bombelle's *Fortification.*

THE Way of making round cover'd Flanks has been found out by the *Italians*, and our Author very judiciously uses them, that they may be more capacious, and fitter to resist the shock of the Enemy's Cannon: But they seem to be something too round, because by their Convexity they take up too much room towards the Center of the Bastions, and leave too little space between the Two High-flanks.

As his Flanks are very great he is in the right to neglect a Second-flank upon the Curtain; but as he makes them of the same bigness in all the Polygons, to me they seem too long in the Square and in the Pentagon, and especially in the Square, where the Face of the Bastion becomes too long, and the Flank'd-angle too acute, being of no more than 48 Degrees, because the Angle *diminué* is every where of 20. 56'. as you will see in

The Calculation of the Angles and Lines, according to Mr. Bombelle's *Design.*

THIS Figure being a Demi-Hexagon, the Angle of the Center AOB will be of 60 Degrees, and the Angle of the Polygon of 120. The Angle of the Flank ECD having been made of 100 Degrees, will serve to find out the other Angles, and first the Angle *diminué* CDE, in the Triangle CDE, in which, besides the Angle of the Flank ECD, the Flank EC and the Curtain CD is known: For if 140 Toises be given to the inward side AB, as the Author does in the Mean Fortification, the Demi-gorge AC will be of 28 Toises, the Curtain CD of 84, and the Flank EC of 35. *Fig.* 51.

Therefore to find out the Angle *diminué* CDE, take from 180 Degrees the known Angle ECD, and the remainder will be 8 Degrees for the sum of the Two other Angles E,D, of the Triangle ECD, whose half will consequently be 40 Degrees: Then you may find half of their difference by this Analogy:

As

Plate 23. Fig. 51.

As the sum of the sides CD, CE,	119
To their difference	49
So the Tangent of half the sum of the Angles	83910
To the Tangent of half their difference	34551

which will be of 19. 4′. which being taken from half the sum of the Angles, or 40 Degrees, the remainder will be 20. 56′. for the Angle *diminuè* CDE, which being taken from half the Angle of the Polygon, or 60 Degrees, 39. 4′. will remain for half the Flank'd-angle AGD, wherefore the whole Flank'd-angle will be of 78. 8′.

If to the Curtain CD, which is of 84 Toises, 28 Toises be added for the Demi-gorge AC, or BD, you will have 112 Toises for the lengthen'd Curtain AD or BC, and in the obliquangled Triangle ADG you may know the Line of Defence DG, and the Capital AG, by these Two Analogies:

As the Sine of the Angle AGD	63022
To its opposite side AD	112
So the Sine of the Angle GAD	86602
To its opposite side DG	153.5

which you will find of 153 Toises, and about 5 Foot.

As the Sine of the Angle AGD	63022
To its opposite side AD	112
So the Sine of the Angle ADG	35728
To its opposite side AG	63. 3.

of 63 Toises, and about 3 Foot.

To know the Face GE, first find out the line DE by the following Analogy in the obliquangled Triangle ECD:

As the Sine of the Angle CDE	35728
To its opposite side CE	35
So the Sine of the Angle ECD	98381
To its opposite side DE	92. 3.

which

which will be of 92 Toises, and about 3 Foot, and which being taken from the Rasant-line DG, which we have found to be of 153 Toises, and 5 Foot, 61 Toises and 2 Foot will be left for the Face GE. The rest will be known, as in the following Method.

Mr. Blondel's *Manner of* Fortifying.

MR. *Blondel* Fortifies inwards as Count *Pagan* does; but he begins by the Angle *diminué*, which he finds out by taking a Right-angle, or 90 degrees from the Angle of the Polygon, and always adding 15 degrees to the third Part of the remainder. But this Angle according to his Principle may be found out, with less difficulty, without knowing the Angle of the Polygon, (*viz.*) by dividing 120 degrees by the member of the sides of the Polygon, and always substracting the Quotient from 45 degrees: Or, else an easier Way, by taking from 40 degrees the third Part of the Angle of the Center. Thus will this Angle *diminué* be of 15 degrees in the Square, 21 in the Pentagon, 25 in the Hexagon, and it will increase gradually in other Polygons, as far as the Right-line, on which it will be of 45 degrees.

By means of this Angle thus found, the Angle of the Bastion will be known to be of 60 degrees in the Square, 66 in the Pentagon, 70 in the Hexagon, and that it increases gradually in all the other Polygons as far as the Right-line, where it is of 90 degrees. *Plate* 24. *Fig.* 53.

The Flanking-angle in the Square is of 150 degrees, of 138 in the Pentagon, of 130 in the Hexagon, and decreases gradually in all the other Polygons as far as the Right-line, where it is but of 90 degrees.

As the Author is perswaded that the Line of Defence ought never to be longer than 140, nor shorter than 120 Toises, in Places that are call'd Royal; he has for that Reason Two Suppositions which he calls Manners, the first of which, being the Great one, allows 200 Toises for the outward side in all the Polygons, which makes the Line of Defence every where of 140, ac-

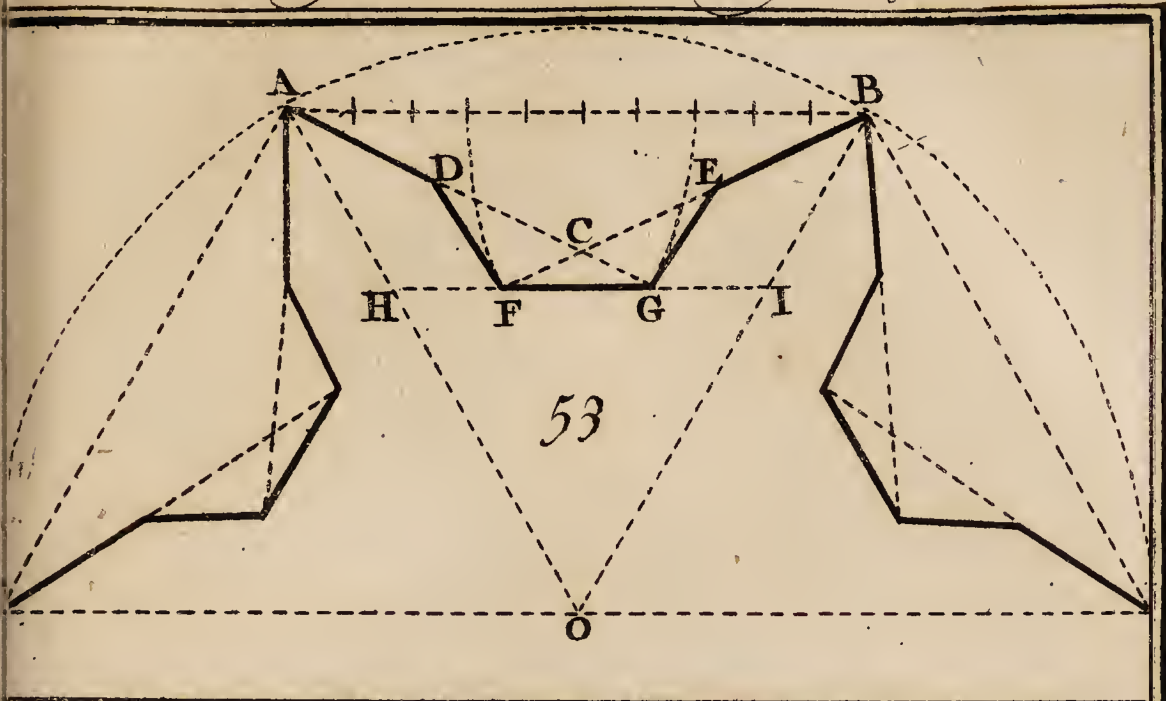
cord-

cording to his usual Way of Fortifying, which is to allow 7 tenths of the outward side for the Line of Defence, and half the Tenaille for the Face. The Second or Little one, makes the outward side every where of 170 Toises, which renders the Line of Defence a little shorter than 120 Toises: By these Terms he includes all that can be Fortified; because to extend the outward side farther makes the Defence useless, the Flanks being too distant, and to make it shorter than in his Little Fortification, lessens the Flank, and increases the number of the Bastions, and the Expence to no purpose.

Plate 24. *Fig.* 53. Let AB be the outward side of an Hexagon. At its ends A,B, make the Two Angles *diminuez* ABC, BAC, of 25 degrees each, such as they ought to be in the Hexagon, with the Two Lines of Defence AG, BF, which will be terminated at the points F,G, by making each of them of 7 tenths of the outward side AB. Divide the Tenailles AC, BC, each into Two equal Parts at D,E, to have the Faces AD, BE, and draw the Flanks DF, EG, with the Curtain FG.

Plate 25. *Fig.* 55. One may easily understand by this Figure, what the Author adds to his Fortification to render its Defence extremely good. First, he takes upon the Flanks DF, EG, the lines DH, EH, of 10 Toises each, for the bigness of each square Orillon, and the rest is employ'd in the Cover'd-flank, which he takes inwards the space of 5 or 6 Toises, and this Retirement is of use to him to make the Curtains of the Bastions longer in Polygons of many sides, and to give Curtains to those, that being upon a Right-line, have none, or very short ones; and in such a case he retires his Flanks inwards, as much as 20 Toises on each side, that he may have a Curtain more than 20 Toises long. The Retirade of the Flank is measur'd upon a Right-line, drawn from the point H to the Angle of the opposite Bastion. He makes 3 Batteries within the Casemate, as Count *Pagan* does, allowing 3 Toises for the breadth of each Parapet, and 5 for each Platform. The Plain of the low

Bat-


A
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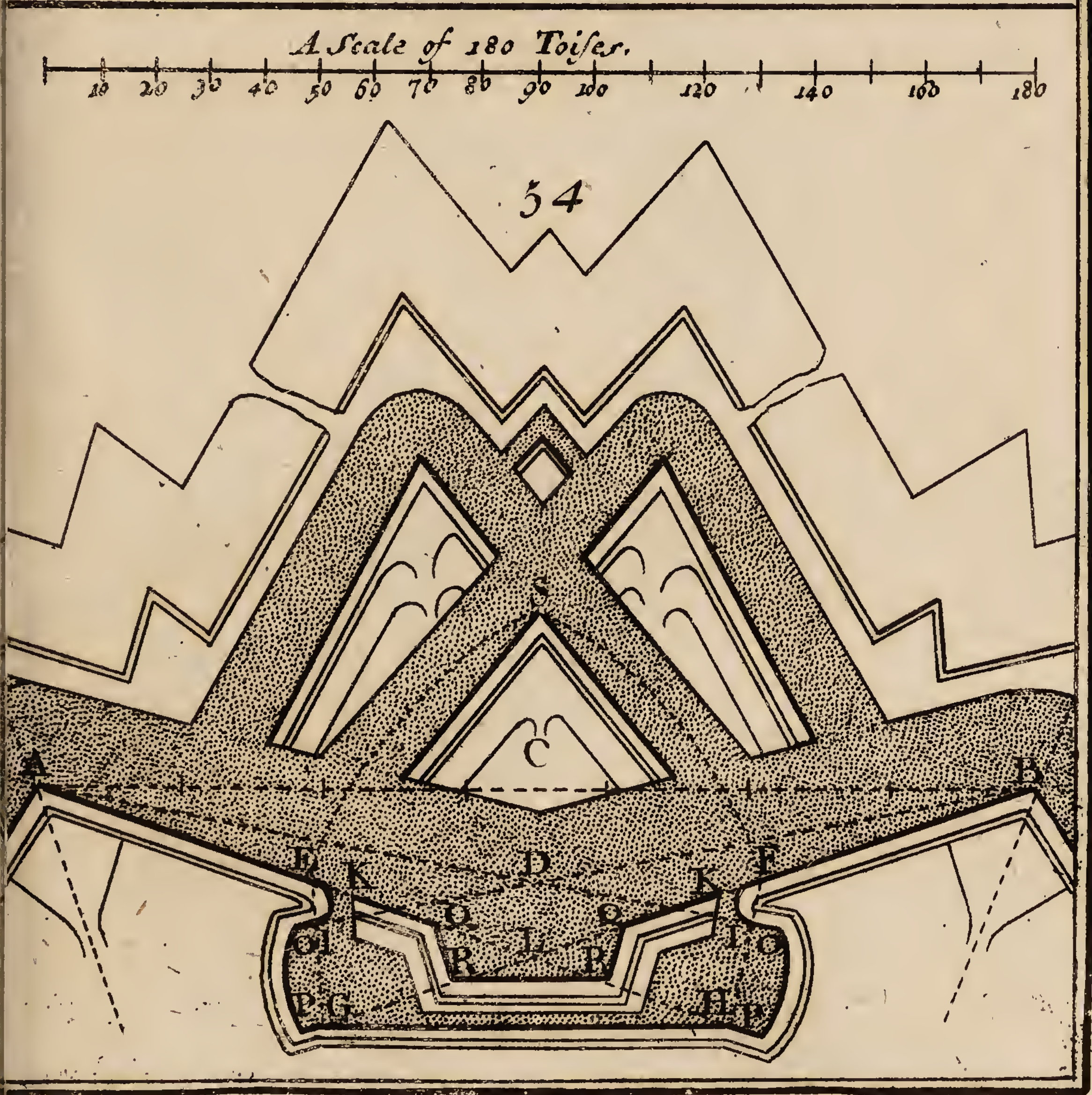
A Scale of 180 Toises.
10 20 30 40 50 60 70 80 90 100 120 140 160 180
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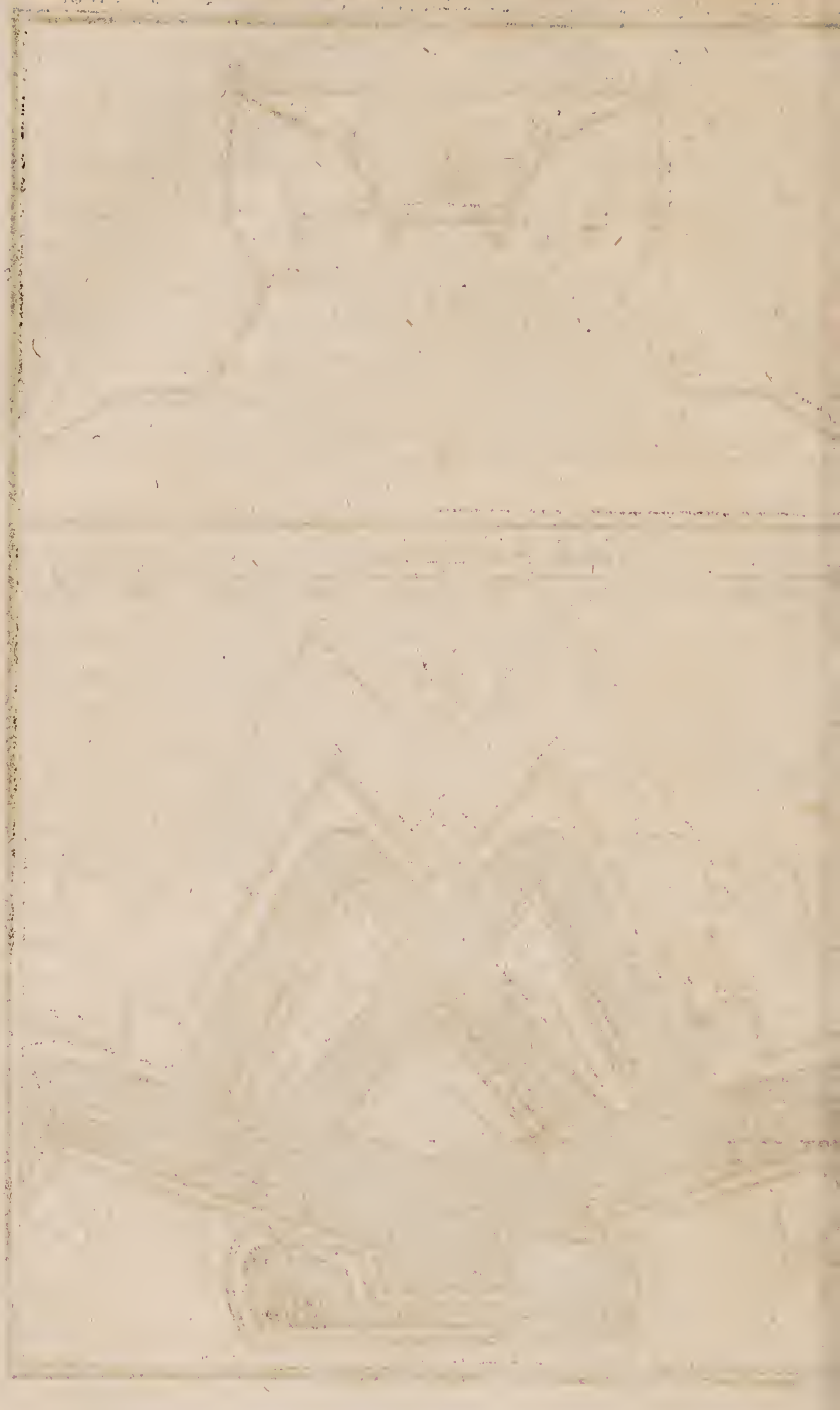


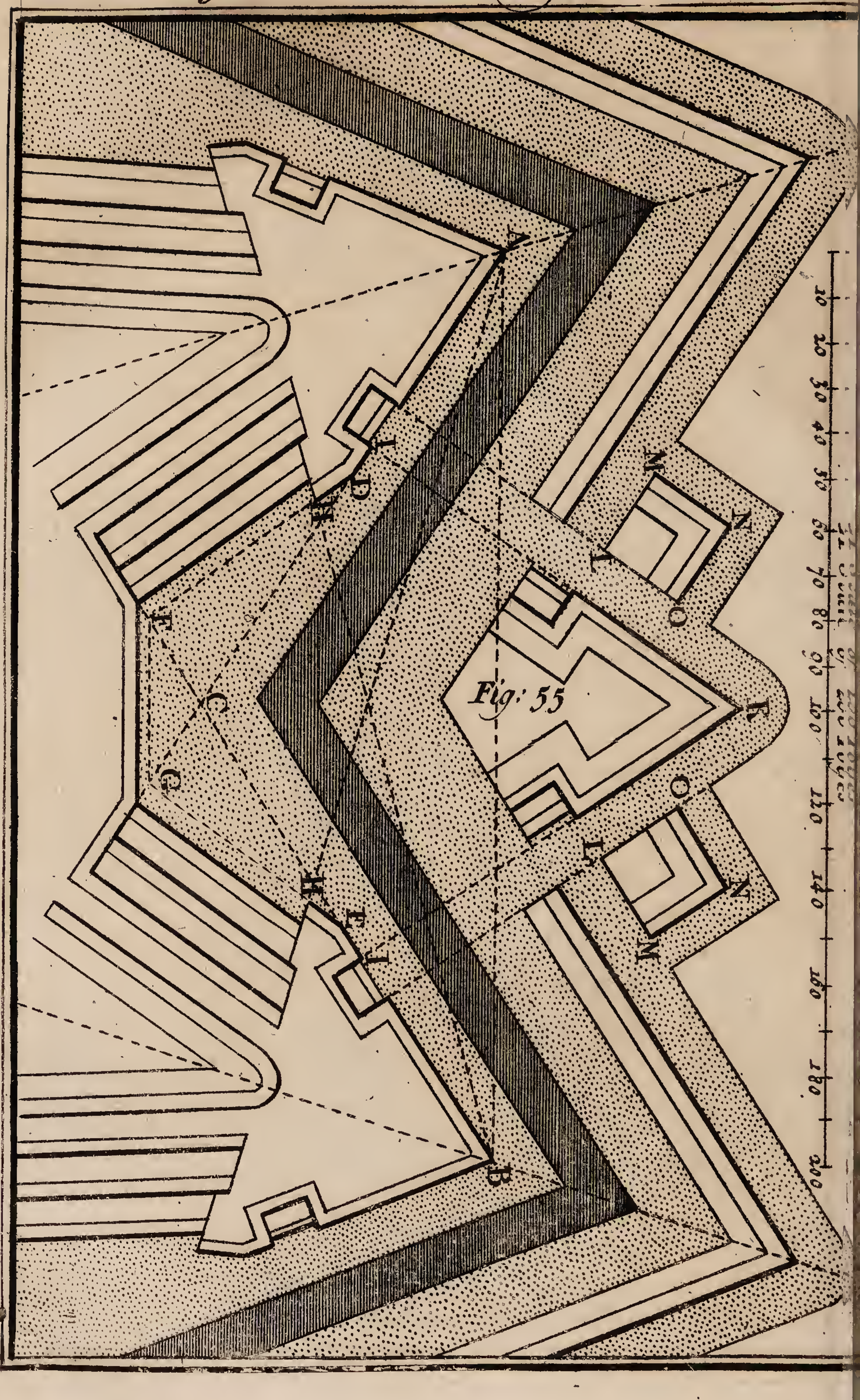


Fig: 55
10 20 30 40 50 60 70 80 90 100 120 140 160 180 200

Battery is from 9 to 12 Foot above the bottom of the Ditch, that of the Middle from 18 to 24, and that of the Highest Battery from 27 to 36, or level with the Rampart. *Plate* 25. *Fig.* 55.

These 3 Batteries are terminated towards the Demi-gorge, upon the lengthen'd Line of Defence, and towards the Orillon, upon the line drawn from the Angle of the opposite Bastion thro' the end of the same Orillon. The Parapet of the Low Battery is 9 or 10 Foot high, 6 or 7 in the Middle-one, and 3 and a half in the Highest, for which reason his Two first must have Embrasures.

As there is a great deal of room between the Two High-flanks in each Bastion, in those Spaces the Author erects Cavaliers, whose Figure you see, each side of which will be able to hold 12 Pieces of Cannon at least. These Cavaliers and Batteries must be made of the Earth taken out of the great Ditch, whose breadth is equal to the length of the Flank DF, or EG, so that the Angle of the Counterscarp is made about the Middle of the outward side AB.

The Author makes a Half-Moon, or Counter-Guard, at the Point of each Bastion, parallel to its Faces, of solid Mason's-Work, without Earth, and every where Countermin'd. Its breadth is 3 or 4 Toises in all; that is, with the Parapet, which is made but 8 or 10 Foot broad. This Counter-Guard is erected in the great Ditch, 10 or 12 Toises off of the Counterscarp, and that distance becomes its Ditch. The chief use of this Work is to hide from the Enemy the Batteries of the Low-flank, opposite to it, and its narrowness hinders the Enemy from setting their Cannon in it when they have taken it.

In a Right-line from that the Author adds, over against the Angle of the Counterscarp, a Ravelin, whose Angle K is found by the intersection of Two Arches, describ'd from the Angles of the Epaule D,E, with the Distance D,E, and whose Faces tend to the Two points I, 6 Toises distant from the Epaules, and are bounded upon the lengthen'd Lines of the Counter-Guards.

The

Plate 25. Fig. 55. The Ditch of this Ravelin muſt be 10 Toiſes broad, and that it may be the better defended, the Author takes in the Face of the Baſtion, beyond the point I, a ſpace that may ſee it, which conſequently muſt be likewiſe of 10 Toiſes, where he makes a low Battery of 4 or 5 Foot, and another on the inſide, the ſame height as the Parapet of the Place. The Plain of this Battery is level with the Second Battery of the Flank, that is, from 18 to 24 Foot above the bottom of the Ditch.

This Ravelin not only covers the Epaules and Orillons of each Baſtion, but alſo defends the Ditch of the Counter-guard, becauſe the Author takes as much of its Face as can diſcover all that Ditch to make two Batteries in, a high one and a low one, juſt as he does in the Faces of the Baſtions. He allows no more Terre-plain for the Ravelin then juſt enough for the recoiling of the Cannon of the Batteries, and leaves all the reſt of the inſide empty, the more eaſily to countermine the Rampart and hinder the Enemy from making a Lodgment in it after he has forc'd it.

Beſides, the Author adds a *Cunette* in his great Ditch, about 7 or 8 Toiſes broad, which he carries on all round, to prevent the low Flanks, which ſeem eaſy of acceſs, from being come at. One might again make a narrower Cunette in the Ditches of the Out-works, if they are 8 or 10 Toiſes broad, and eſpecially where Batteries have been made in the Faces of the Half-Moons or Ravelins.

That the Batteries of the Face of each Baſtion, which defend the Ditch of the Ravelin may be better cover'd, the Author adds in the Angle of the Counterſcarp of the Ravelin a *Lunette*, whoſe figure is a Lozenge, allowing 20 Toiſes for each of its Sides, *&c.*

Remarks upon Mr. Blondel's *Fortification.*

THO' this Manner of Fortifying be extremely well invented, yet it is too expenſive, as well for the Conſtruction of the Ditch, which the Author

is

is obliged to make very wide and deep, that it may afford Earth for the Rampart, and for all the Batteries of the Flanks and Faces of the Baſtions, as for the Quantity of Ammunition and the number of Gunners and Officers of Artillery, which a Place thus Fortified ought to have, and of the Out-works that it muſt have to cover the Flanks, which are too much expos'd.

Beſides, the 4 Batteries of the Flank are ſo long, and ſo cloſe together, that the Enemy may fill them with Bombs in a little time, and having once broken them with his Cannon, they may ſerve him as ſo many ſteps to mount, in the Aſſault. Then the Cavaliers, which are between the two high Flanks ſo fill up the Baſtion where they are, that it is hard to retrench there in time of need. Several more faults may be found with this Manner of Fortifying, which my Profeſſion and the Author's great Fame cauſe me to paſs over.

The Calculation of the Angles and Lines of a Polygon, Fortified according to Mr. Blondel's *Deſign.*

IF you allow 200 Toiſes for the outward Side AB, the Line of defence AG, or BF will be of 140 Toiſes: and becauſe this Figure is a Demi-Hexagon, where the Angle of the Center is of 60 degrees, and the Angle of the Polygon of 120, the Angle *diminué* BAG, or ABF, will be of 25 degrees, the flanking-Angle of 130, and the Flank'd-angle of 70; and conſequently half the Flank'd-angle FBO, or OAG, of 35. *Plate 24. Fig. 53.*

Thus all the Angles are known, except the Angle of the Flank DFG, and the Angle of the Epaule, which together with the other Lines may be thus found:

Firſt, To find out the Face AD, or BE, the Tenaille AC, BC, which is its double, muſt be found by this Analogy in the Iſoſceles Triangle ACB:

As the Sine of the Angle ACB	76604
To its oppoſite ſide AB	200
So the Sine of the Angle ABC	42262
To its oppoſite ſide AC	110

which

which we ſhall find to be of 110 Toiſes, whoſe half will be 55 Toiſes for the Face AD, which being taken from the Raſant-line AG, which is of 140 Toiſes, 85 Toiſes will remain for the line DG, from which if the Face AD, or the line CD be taken, that is, 55 Toiſes, the remainder will be 30 Toiſes for the Complement CG, or CF; and the Curtain FG may be found in the Iſoſceles Triangle FCG, by this Analogy:

As the Sine of the Angle FGC	42262
To its oppoſite Side FC	30
So the Sine of the Angle FCG	76604
To its oppoſite Side FG	54.2

which will be of 54 Toiſes and about two Foot, and which with the Side DG, which we have found to be of 85 Toiſes, will ſerve to find the Angle of the Flank DFG, in the obliquangled Triangle DFG, thus:

From 180 degrees take the Angle *diminuè* DGF, or 25 degrees, and the remainder will be 155 degrees for the Sum of the two other Angles, whoſe half is 77.30.' Then make this Analogy:

As the Sum of the Sides DG, FG	85.0
To their difference	54.2
So the Tangent of half the Sum of the Angles	451070
To the Tangent of half their difference	162739

which will be of 58 Degrees, and about 26 Minutes, which being added to half the Sum of the Angles, or to 77 degrees, you will have 135. 26'. for the Angle of the Flank DFG, to which adding FGD the Angle *diminué*, or 25 degrees, you will have the Angle of the Epaule ADF of 160.26'.

If you wou'd know the Flank FD, it muſt be by this following Analogy in the ſame obliquangled Triangle DFG:

As the Sine of the Angle DFG	69966	*Plate* 24.
To its opposite Side DG	85	*Fig.* 53.
So the Sine of the Angle FGD	42262	
To its opposite Side FD	51. 2.	

which will be of 51 Toises, and about 2 Foot.

To know the Demi-gorge FH, or GI, we must first find the lengthen'd Curtain GH, in the obliquangled Triangle AGH, by this Analogy:

As the Sine of the Angle AHG	86602
To its opposite Side AG	140
So the Sine of the Angle GAH	57357
To its opposite Side GH	92.4

which will be of 92 Toises, and about 4 Foot, from which taking the Curtain FG, which we have found to be of 54 Toises and 2 Foot, the remainder will be 38 Toises and 2. Foot for the Demi-gorge FH, or GI, after which the inward side HI will appear to be of 147 Toises, *&c.*

Mr. Vauban's *Manner of* Fortifying.

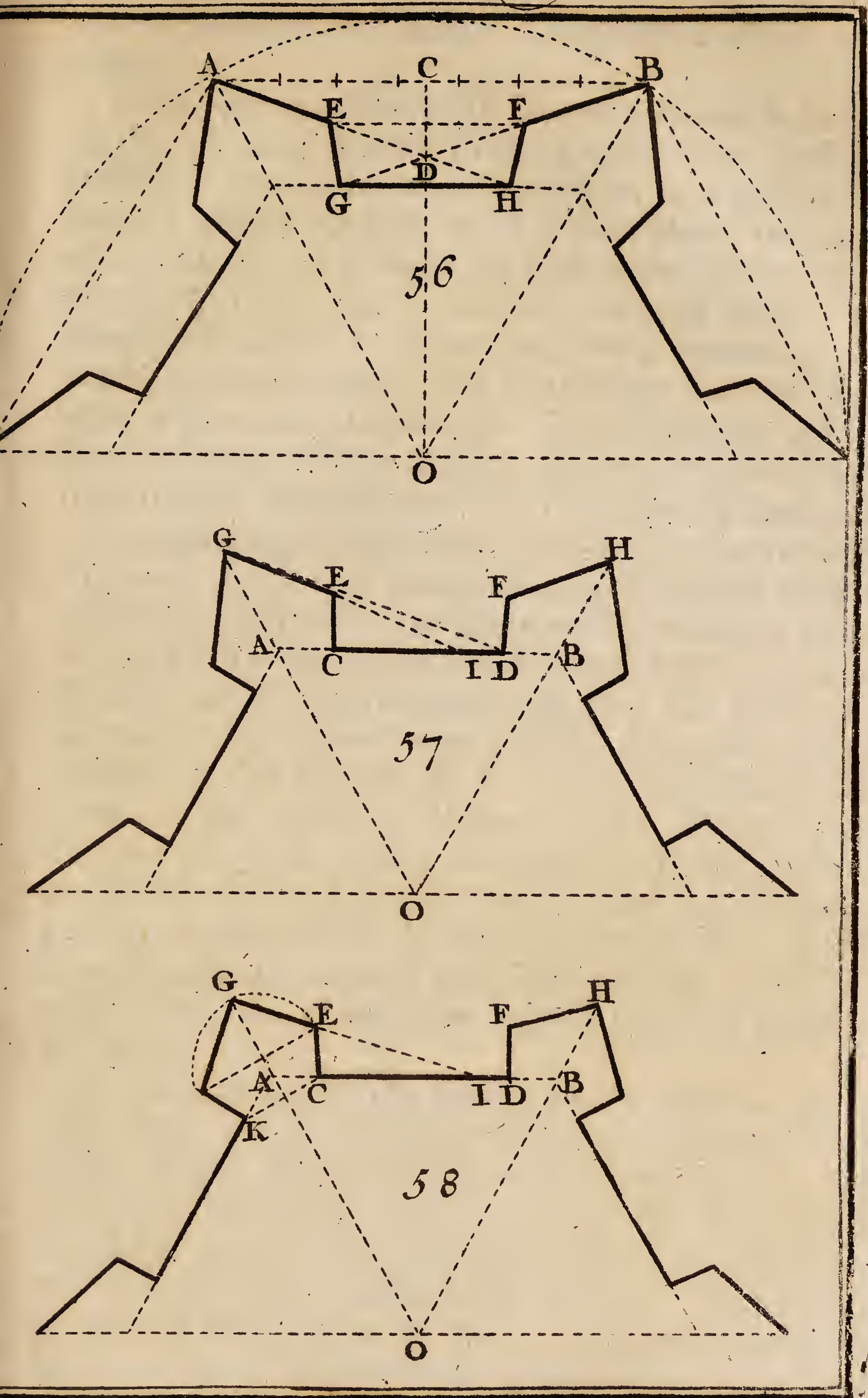
MR. *Vauban* Fortifies inwards, and begins by the Rasant-Lines, as Count *Pagan* does, but he does not make his Faces so long, nor his Flanks so short, and so much expos'd to the Enemy's Batteries; and as they are very great, they supply the want of Second Flanks, which are not of so much Moment, and may at any time be added without changing the Flanks or Curtains, as you have seen in our second and third Method, when the Angle of the Bastion becomes too obtuse.

Having, as in Count *Pagan*'s Fortification, divided the outward side AB, into two equal parts at C, and let fall from C the perpendicular CD, equal to the eighth part of AB for the Square, to the seventh for the Pentagon, and to the sixth for the Hexagon, as here, and all the other Polygons; draw from the ends A, B, thro' the point D, the Lines of defence ADH, BDG, which will be terminated at H, G, thus: *Plate* 26. *Fig.* 56.

Plate 26. Fig. 56. Divide the outward ſide AB into Seven equal Parts, and allow two of them for the Faces AE, BF, and make the lines EH, FG, each equal to the line EF, to have the Curtain GH, and the Flanks EG, FH; and if the like be done every where, you will have the Maſter-line of the Curtains and Baſtions, to which on the inſide muſt be added a Rampart about 12 Toiſes broad, with its Parapet of 3, and on the outſide a Ditch 16 or 18 Toiſes broad over-againſt the Flank'd-angle, and 20 Toiſes broad over-againſt the Epaule. Not forgetting a cover'd way of 5 Toiſes in breadth, and an Eſplanade of 20.

Plate 24. Fig. 54. For the Caſemates and Orillons, take the lines EI, FI, each equal to the third part of the Flanks EG, FG, and draw thro' the points I, from the oppoſite Flank'd Angles A, B, the Revers, or back-part of the Orillon IO of 5 Toiſes: take alſo upon the lengthen'd Lines of defence the *Enfoncement*, or depth of the Caſemate GP, HP, of 5 Toiſes, then the concave Flank OP will be deſcrib'd as we have taught in our Method. As for the Orillon, it muſt be deſcrib'd by drawing upon the lines EI, FI an Arch, whoſe Diameter muſt be a little leſs than the line EI, or FI, ſo that its Convexity may not exceed the Line of Defence, as we have alſo taught in our Method.

The Author adds a *Tenaillon*, or *reinforc'd Tenaille* in the Ditch, that is, a Tenaille with Flanks, thus deſcrib'd. Take upon the Lines of defence AH, BG, the lines EK, FK, of about 3 Toiſes each, and draw from the points K, lines parallel to the Flanks EG, FH, to terminate the Tenaille, whoſe Faces KQ are each as long as half the lines KL, and whoſe Flanks QR are perpendicular to the Lines of defence. Add within this Work a Rampart, which muſt be but 5 Toiſes broad towards the Curtain, and 7 broad towards the Flanks and Faces. As for the Parapet it ought to be no leſs than 3 Toiſes broad. As this Work is made for ſcarce any other uſe, than the defence of the Ditch,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Ditch, methinks a ſimple Tenaille, as FGH, wou'd ſuffice. *Plate* 44. *Fig.* 94.

To Scoure the Ditch ſtill better, and oppoſe its being paſs'd, the Author adds in the Ditch, over-againſt the middle of the Tenaille, a Caponiere, or a double Way, cover'd with a Parapet, rais'd 3 Foot above the bottom of the Ditch; it is about 12 Foot broad, perpendicular to the Curtain, and Paliſſado'd on both ſides. This Work is ſo much the better, becauſe it commands without being commanded, and is a Paſſage for the Musketeers to go to the Out-works. Thus there are 4 Flanks for the defence of the Ditch, that of the Place, that of the Orillon, and thoſe of the Tenaille and Caponiere. *Plate* 24. *Fig.* 54.

To deſcribe the Half-Moon, which is at the re-entrant Angle of the Counterſcarp, deſcribe from the ends GH, of the Curtain, thro' the Epaules EF, with the diſtance of 70 Toiſes, ſuppoſing the outward ſide of 200, two Arches, whoſe interſection will give the point of the Ravelin at S, whoſe Faces muſt be drawn towards the Angles of the Epaules E, F. This Half-Moon is to have on the inſide a Rampart of 10 Toiſes, with a Parapet as uſual, and on the outſide a Ditch parallel to its Faces, and 12 Toiſes broad: and to hinder the paſſing of this Ditch, Places of Arms muſt be made in it perpendicular to the Face of the Ravelin, and cover'd with a Parapet three Foot higher than the bottom of the Ditch.

In the Center of the ſaid Ravelin muſt be made a retrench'd *Corps de Guard*, that is, a *Reduct*, or Redoubt, whoſe Walls muſt have Battlements, or *Meurtrieres*, to be cover'd and fire upon the Enemy with Muskets, and to retire in, when one is preſs'd ſo hard as not to be able to ſuſtain the Aſſault. This Ravelin is cover'd with a ſort of Counter-guard, or *great Lunette* 10 or 12 Toiſes broad towards the great Ditch, and 25 or 30 towards the Ditch of the Half-Moon, having ſuch a Ditch as the Half-Moon has, in which as well as in the Terre-plain of the Half-Moon, and its Rampart, when it has one, a Traverſe or Parapet

Plate 24. Fig. 54. is rais'd 5 or $5\frac{1}{2}$ Foot high, leaving close to these Traverses a space of 2 or 3 Foot for Files of Souldiers.

To cover the Half-Moon the better, a little Ravelin or *Lunette* is added in the re-entrant Angle of the Counterscarp of the two great Lunettes, having a Ditch 3 Toises over, and as deep as that of the Half-Moon, with a Parapet but no Rampart.

The double line which we have added upon the Glacis of the Esplanade, over-against the Flank'd-angle of the great Counter-guard, represents a Cut about 10 Foot wide, made in the Parapet, and call'd *Sortie*, (Sally) because it is a Way to Sally out at, and receive Succour. These *Sorties*, or Ways, are shut up with Oaken rails.

The Calculation of the Angles and Lines of a Polygon Fortified after Mr. Vauban's *Method.*

Plate 26. Fig. 56. THE Line AB, being the side of an Hexagon, the Angle of the Center AOB will be of 60 degrees, and the Angle of the Polygon of 120. If you make the outward Side AB of 180 Toises, its half AC, or BC, will be of 90 Toises, and the perpendicular CD of 30. The Face AE, or BF, will be of 51 Toises and about 2 Foot, and therefore in the practice it is usually suppos'd of 50 Toises, and we shall suppose it so here, for the sake of an even number.

The Angle *diminu'* CAD, or CBD, will be of 18.26'. as in Count *Pagan*'s Fortification, and consequently the Flank'd-Angle of 83. 8'. and the Flanking-angle of 143. 8'. The line AD will be also as in Count *Pagan*'s way of 94 Toises, and about 5 Foot, from which taking the Face AE, or 50 Toises, the remainder will be 44 Toises and five Foot for the line DE, or DF, and one may in the Triangle EDF, know the line EF by this Analogy:

As the Sine of the Angle EFD	31620
To its opposite side DE	44. 5.
So the Sine of the Angle EDF	59995
To its opposite side EF	85.

Thus

Thus the line EF, or its equal EH, will be of about 85 Toises, to which if you add 50 Toises for the Face AE, you will have 135 Toises for the Line of Defence AH, from which if DE be taken, which has been found to be of 44 Toises and 5 Foot, 40 Toises and 1 Foot will remain for the Complement DH, or DG, and in the Isosceles Triangle GDH, one may know the Curtain GH, by this Analogy: *Plate 26. Fig. 56.*

As the Sine of the Angle DGH	31620
To its opposite side DH	40. 1
So the Sine of the Angle GDH	59995
To its opposite side GH	76. 5.

which will be of 76 Toises, and about 5 Foot, which one might also have found without the knowledge of the Angles, by comparing together the two Similar Isosceles Triangles GDH, EDF, *&c.*

Because EFG is an Isosceles Triangle, if from 180 degrees the Angle EFG be taken, which is equal to the Angle *diminué*, that we have found to be of 18. 26'. half the remainder will be 80. 45'. for the Angle at the Base EFG, to which if the Angle *diminué* FGH, or 18. 26'. be added, the Sum will be 99. 11'. for the Angle of the Flank EGH, to which if the Angle *diminué* GHE, or 18. 26'. be once more added, you will have 117. 37'. for the Angle of the Epaule AEG.

The Flank EG is found in the Triangle EDG, by this Analogy:

As the Sine of the Angle EGD	98700
To its opposite side DE	44. 5.
So the Sine of the Angle EDG	59995
To its opposite side EG	27

which will be of about 27 Toises. The rest may be known, as has been taught in Mr. *Blondel*'s Fortification.

Sardi's Italian *Method of* Fortifying.

THO' the foregoing Methods are the beſt, eſpecially the laſt mention'd, (*viz.*) Mr. *Vauban*'s, which, by reaſon of its plainneſs and eaſineſs, is to be preferr'd to all that went before, which may in general be call'd the *French Manner*, becauſe their Authors are *French* Men; yet it is not well to be ignorant of the other, that one may be able to judge of them, and make uſe of them upon occaſion, becauſe one may always find ſomething good in them.

The *Italians*, who do not much matter making the Angle of the Baſtion right or obtuſe, rather chooſing to have it acute, that they may have a Second Flank upon the Curtain, have ſeveral Ways of Fortifying, taught by ſeveral Authors of their own Nation, amongſt which I have choſen *Sardi*, as thinking his Method better than that of the other *Italian* Authors.

Fig. 57. Having ſuppos'd the inward Side AB of 800 Geometrical Paces or Feet, allow 150 for the Demi-gorges AC, BD, and as much for the Flanks CE, DF, which muſt be perpendicular to the Curtain CD, whoſe eighth part DI will be taken for the Second Flank, ſo that I will be the point of the Raſant-line IG, which upon the lengthen'd Radius AO, will give the Point of the Baſtion at G, and doing ſo all round will end the operation.

Plate 27. *Fig.* 59. For the Caſemates and Orillons, take upon the Flanks CE, DF, upon the Demi-gorges AC, AD, and upon the lengthen'd Faces HE, HF, the lines CO, DO, CI, DI, EK, FK, each equal to the third part of a Flank, or of a Demi-gorge, and draw to the Flanks CE, DF, thro' the points K, I, the indefinite parallels LM, KQ, lengthen the Flanks CE, DF, towards N, in ſuch manner that the lines CN, DN, be of 15 Foot each, and draw thro' the points N, the lines MN parallel to the Demi-gorges AC, AD.

Draw the lines OP of 10 Foot each, and the lines IL of 70, and draw the right-line LP, and the line LM will be the beginning of the low Place, which will

be capable of 3 Pieces of Cannon. If you wou'd have a Square Orillon, its Front KQ will be terminated at Q by a right-line drawn from the point O, thro' the middle of the Face of the opposite Bastion: and if you wou'd have it round, with a Radius of 50 Foot, describe an Arch upon the line KQ. *Plate* 27. *Fig*. 59.

The Author adds square Cavaliers upon the middle of the Curtain, whose Faces are parallel to the Parapet of the Rampart, and 30 Foot distant from that Parapet, where he places 7 Pieces of Cannon, three of which play upon the Field, and the other four upon the two next Bastions, two on each side, to flank such Breaches as the Enemy may make in the Face of those Bastions, and keep him from giving Assault.

Remarks upon Sardi's *Fortification.*

As the *Italians* affect to have the Angle of the Bastion acute, that the Faces of the same side of the Place may defend one another with advantage, and serve for Flanks, when their Casemates and Flanks are broken: and that the Second Flank upon the Curtain is too little to make the Flank'd Angle always acute; it seems that He, from whom we have taken this Construction, has not well explain'd the Authors Mind.

Therefore to render this Method of Fortifying more perfect, one shou'd not give the Square or Pentagon any Second Flank, because their Angles are not great enough: But one might give one to the Hexagon and the other Polygons, to encrease according to the number of the Bastions, making it equal to the 8th part of the Curtain for the Hexagon, to the 7th for the Heptagon, to the 6th for the Octogon, to the 5th for the Enneagon, to the 4th for the Decagon, to the 3d for the Hendecagon, and to half for the Dodecagon, *&c.*

The Calculation of the Angles and Lines according to Sardi's *Design.*

Plate 26. Fig. 57. BEcause this Figure is a Demi-Hexagon, the Angle of the Center AOB will be of 60 degrees, and the Angle of the Figure of 120: And as the Flank is perpendicular to the Curtain, the Angle of the Flank ECD, or CDF, will be of 90 degrees. Moreover, the inward Side AB having been suppos'd of 800 Paces, and having allow'd 150 for each Demi-gorge AC, BD, as also as much for the Flanks EC, DF, the Curtain CD will be of 500 Paces, and the Second Flank ID, which is the 8th part of it, of 62 ½ Paces, and consequently the Complement CI of 437 ½ Paces.

By means of these Angles, and these Lines thus known, you may easily find out the other Angles and Lines, and first the Angle *diminuè* CIE, in the rectangular Triangle ECI, by this Analogy:

As the side CI	437½
To the side CE	150
So the whole Sine	100000
To the Tangent of the Angle diminuè CIE	34286

which will be of 18 degrees and about 55 minutes, to which if you add the Angle of the Flank of 90 degrees, you will have 108.55'. for the Angle of the Epaule CEG. But if the said Angle *diminuè* 18. 55'. be taken from half the Angle of the Polygon, or 60 degrees, the remainder will be 41.5'. for half the Flank'd-angle, which consequently will be of 82. 10'.

If to the line CI, which is of 437 ½ Paces, the Demi-gorge AC, or 150 Paces be added, you will have 587½ Paces for the length of the line AI, and in the oblique-angled Triangle AIG, one may know the rasant-line GI, and the Capital AG, by the two following Analogies:

As the Sine of the Angle AGI	65716
To its opposite side AI	587½
So the Sine of the Angle GAI	86602
To its opposide side GI	774

which

which will be of about 774 Foot.

As the Sine of the Angle AGI	65716
To its opposite side AI	587½
So the Sine of the Angle AIG	32419
To its opposite side AG	289

which will be of about 289 Foot.

If to the Curtain CD, which is of 500 Foot, the Demi-gorge AC, or 150 Foot be added, the sum will be 650 Foot for the lengthen'd Curtain AD, which will be a means to find out the length of the fichant Line DG, finding first the Angle ADG by this Analogy: *Plate 26. Fig. 57.*

As the sum of the two sides AD, AG	939
To their difference	361
So the Tangent of a quarter of the Angle of the Polygon	57735
To another Tangent	22196

to which 12 degrees, and about 31 minutes answer in the Tables, which being taken from a quarter of the Angle of the Polygon, or 30 degrees, the remainder will be 17.29'. for the Angle ADG, by means of which one may know the fichant Line DG, by this following Analogy, in the Triangle AGD:

As the Sine of the Angle ADG	30043
To its opposite side AG	289
So the Sine of the Angle GAD	86602
To its opposite side GD	833

which will be of about 833 Geometrical Paces.

The Chevalier de Ville's French *Method of* Fortifying.

THE *Chevalier de Ville* makes his Flanks perpendicular to the Curtain, and equal to the Demi-gorges, as *Sardi* does, but less, (*viz.*) the Sixth part only of the inward side, and in the Square and Pentagon, he determines the Flank'd-angle by a Rasant-line, but in the other Polygons he makes it a right-one, by means *Plate 62. Fig. 58.*

Plate 26. Fig. 58. means of a Semi-circle describ'd upon a right-line, which joyns the two Epaules of a Bastion, as you have seen in our Second Method, and then there is a Second Flank ID upon the Curtain CD, which will increase as the number of the sides of the Polygon does. The length of the Capital AG, is in such a case found equal to the Gorge-line CK, which gives an easier Method to find the Points G,H, of the Bastion.

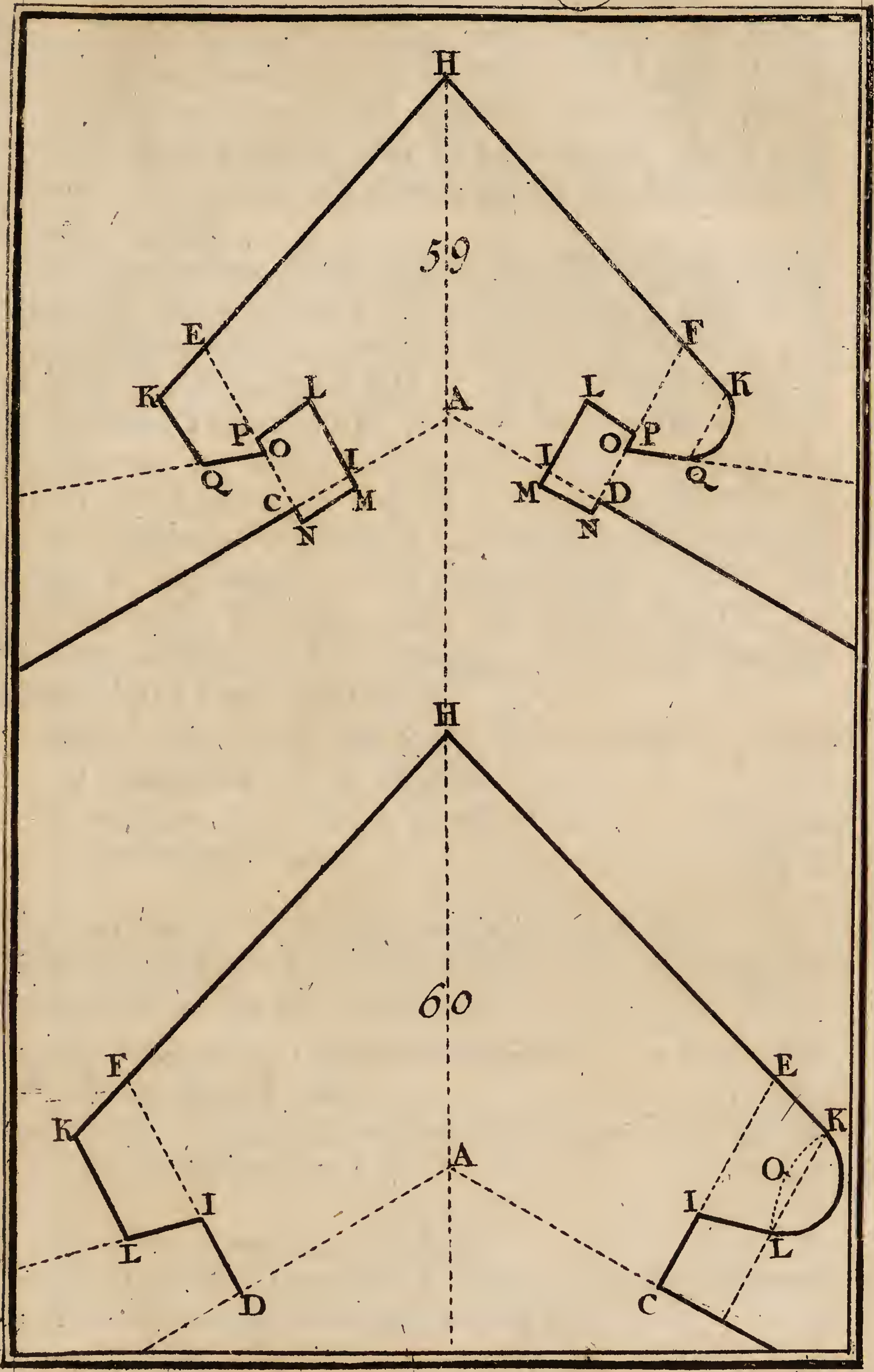
Plate 27. Fig. 60. As for the Casemates and Orillons, take upon the Flanks CE,DF, and upon the lengthen'd Faces HE,HF, the lines CI,DI,EK,FK, each equal to a third part of the Flank, or of the Demi-gorge, and draw thro' the points K, to the Flanks CE, DF, the parallel lines KL, which will be terminated at L, by a right-line drawn from I to the Angle of the opposite Bastion, and this line KL will be the Front of the Orillon, when it is made square, but if you wou'd have it round, from K, and L, with the distance KL, describe two Arches which here intersect at M, and from that point M thro' the same points KL, describe the Arch KOL, whose middle point O will be the Center of the Orillon.

Remarks upon the Fortification of the Chevalier de Ville.

AS the Flanks and Demi-gorges are but the Sixth part of the inward side, that makes the Curtains too long, and the Bastions too little, and uncapable of Retrenchments. Wherefore the Author increases his Faces to make his cover'd Flanks, which I think also too little, as having scarce room in them to make a tolerable Battery. The Second Flank upon the Curtain becomes so great in great Polygons, that it is hard to have the Ditch defended by the whole Flank, unless it be made very wide before the Curtains, which renders the Expence excessive.

The Calculation of the Angles and Lines, according to the Chevalier de Ville's *Design.*

Plate 26. Fig. 58. BEcause this Figure is a Demi Hexagon, the Angle of the Center AOB will be of 60 degrees, and the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Angle of the Polygon of 120: and because the Flank is here perpendicular to the Curtain, the Angle of the Flank ECD will be of 90 degrees, and consequently half the Flank'd-angle AGI of 45 degrees, the Angle *diminué* AIG of 15, and the Angle of the Epaule CEG of 105 degrees. *Plate* 26. *Fig.* 58.

Making the inward side AB of 120 Toises, the Demi-gorge AC, and the Flank CE will be of 20 Toises each, and consequently the Curtain CD of 80 Toises, and the lengthen'd Curtain AD of 100. The Angles of the Isosceles Triangle AKC are also known, for if from 180 degrees be taken the Angle KAC of 120 degrees, half of the Remainder will be 30 degrees, for each of the two Angles at the Base K,C, which is always equal to half of the Angle of the Center AOB. Thus one may find in the Triangle AKC, the Gorge-line CK, or the Capital AG, by this Analogy, which our Author seems to have been ignorant of:

As the Sine of half the Angle of the Center	50000
To the Flank of the Bastion	20
So the Sine of the Angle of the Center	86602
To the Capital AG	34.4.

which will be of 34 Toises, and about 4 Foot.

You may find the Rasant-line GI, in the obliquangled Triangle AIG, by this Analogy:

As the Sine of the Angle diminué AIG	25882
To the Capital AG	34.4
So the Sine of half the Angle of the Polygon OAB	86602
To the Rasant-line GI	116.4.

of 116 Toises, and about 4 Foot.

In the same obliquangled Triangle AIG may be found the line AI, by the following Analogy:

As the Sine of the Angle diminué AIG	25882
To the Capital AG	34.4
So the Sine of half the Flank'd-Angle AGI	70711
To the Line AI	94.4

which

Plate 26. Fig. 58. which will be of 94 Toises, and about 4 Foot, from which taking the Demi-gorge AC of 20 Toises, 74 Toises and 4 Foot will remain for the Complement CI, which being taken from the Curtain CD, or from 80 Toises, the remainder will be 5 Toises and 2 Foot for the Second Flank ID, which may be also found, by finding the Complement CI in the rectangular Triangle ECI independently from the Capital AG, which will prove that Capital to be equal to the Gorge-line, whose Geometrical demonstration is very easy.

As the whole Sine	100000
To the Tangent of the Angle CEI	373205
So the Flank EC	20
To the Complement CI	74.4.

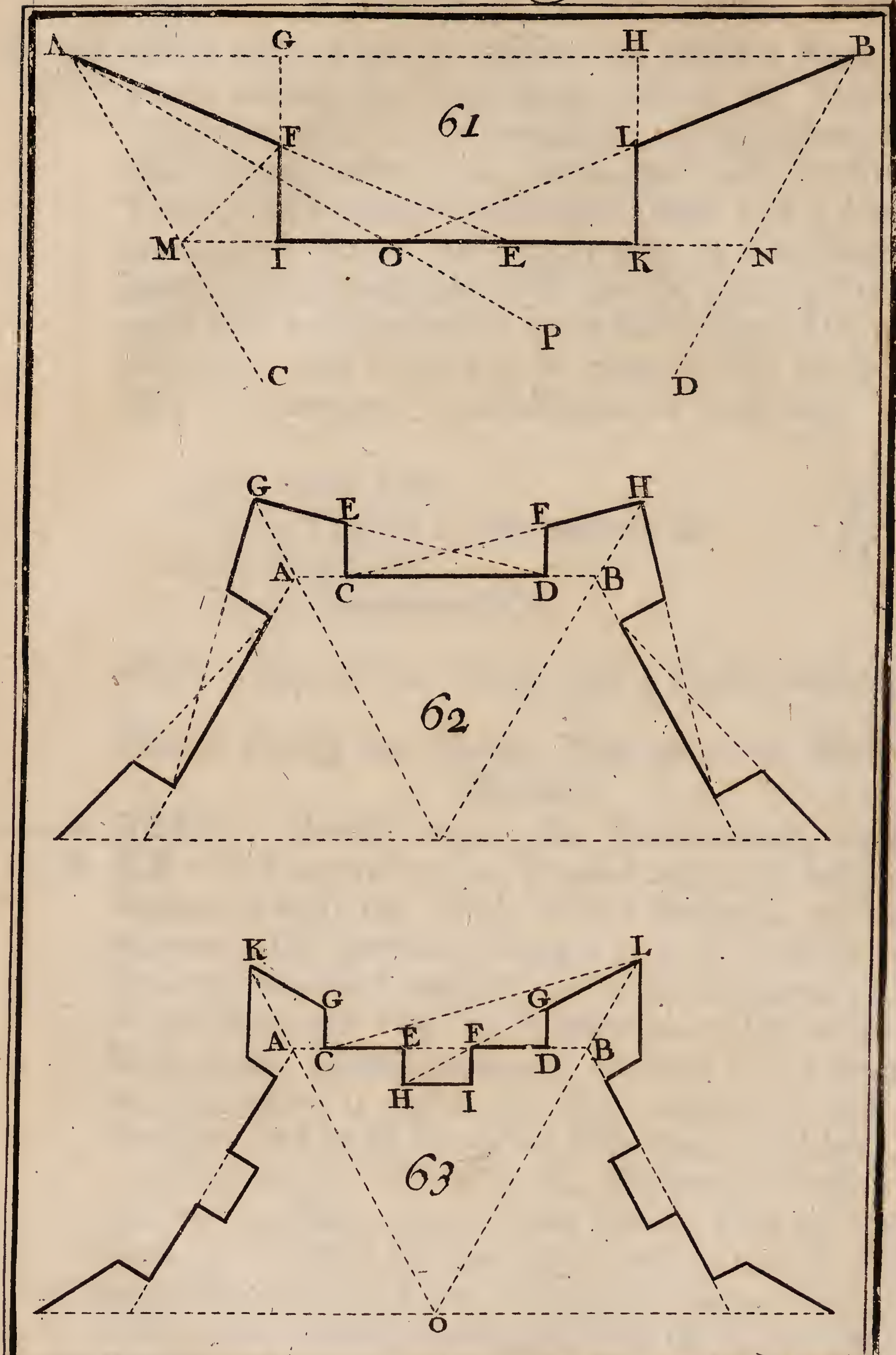
which will be of 74 Toises, and 4 Foot as before, *&c.*

How to Fortify the Dutch *Way, after the Method of* Marolois.

Plate 28. Fig. 61. BEfore *Marolois* begins his Fortification, he finds the Aperture of the Flank'd-angle, by adding 15 degrees to half the Angle of the Polygon, whence it happens, that the Flank'd-angle is of 60 degrees in the Square, 69 in the Pentagon, 75 in the Hexagon, 79.17'. in the Heptagon: and as it becomes obtuse after the Dodecagon, by this continual addition of 15 degrees, he only makes it right, or of 90 degrees in the Dodecagon, and in all the other following Polygons.

He allows 36 * Rods, or 72 Toises for the Curtain IK, and 24 Rods, or 48 Toises for the Face of the Bastion AF, or BL; so that the Curtain is in a *Sesquialteral Ratio* to the Face, that is, as 3 to 2. Lastly, he makes the Flank-forming-angle FMI, of 40 degrees, when he wou'd only have a single Flank, that the Demi-gorge IM may be to the Flank IF, as about 6 to 5: But when he wou'd have a cover'd Flank, he makes

** The same Measure as before-mention'd in Mr.* Bombelle's *Fortification.*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the Flank-forming-angle FMI of 35 degrees only, and then the Demi-gorge IM is to the Flank IF, as about 7 to 5. *Plate* 28. *Fig.* 61.

Let AB be the outward side, which is indefinite towards B. At A make the Angle BAC equal to half the Angle of the Polygon, and having divided it into two equal parts by the line AP, at the point A, with this line AP make the Angle PAE of 7. 30'. by the Rasant-line AE, upon which the Face AF of 30 Toises must be taken to have the Epaule at F, from which point the indefinite perpendicular GI must be drawn upon the outward side AB, which will be terminated at I, making at the point F, with the line FI, the Angle FIM of 50 degrees, and drawing thro' the point M, where the line FM meets the Radius AC, the indefinite line MN parallel to the outward side AB, which will be the inward side of the Polygon, when it is terminated at N, which will be done thus:

Having made the Curtain IK of 72 Toises, and the line GH as long, joyn KH, to take upon it the Flank KL equal to the Flank IF, and having made the Demi-gorge KN equal to the Demi-gorge IM, and HB, equal to AG, draw the Face BL, and the Capital BN, which being lengthen'd, will meet the Capital AM also lengthen'd, in a point which will be the Center of the Place. The rest is easily ended.

Remarks upon the Dutch *Fortification.*

THis Way of Fortifying teaches us an easy Method of working upon the Ground, where one cannot describe a regular Polygon by means of a Circle; for one may draw such a Polygon upon the Ground, with the Master-line of the Curtains and Bastions, making first upon the Ground the Angle of the Polygon, equal to that which is describ'd upon Paper, and by ending the rest, as has been just taught. *Plate* 28. *Fig.* 61.

There are other kinds of *Dutch* Fortifications, which we shall not here speak of, because they are not worth it: for tho' several have believ'd the *Dutch* For-

Plate 28. *Fig.* 61. Fortification to be the beſt, becauſe the War has laſted ſo long in that Country, which muſt have made them Skilful in that Art, by long Experience, and that to reſiſt a great Prince, they have endeavour'd to excell other Nations in it; yet the ſame Experience has ſhown in the Wars of 1672, 1673, *&c.* that moſt of their beſt Places have been taken in 3 Weeks, and wou'd have been taken much ſooner, but for the Number of their Out-Works, which ſince that time has much leſſen'd the reputation of their Strength, and has render'd deſpicable the way that they are Fortified, eſpecially becauſe they have no Caſemates, or very ſmall ones.

As in all their Methods they have affected to have a ſecond Flank upon the Curtain, and that the Counterſcarp has been made parallel to the Faces of the Baſtions, this conſiderable fault happens, (*viz.*) that the Flank, which is the chief defending Part, does not diſcover all the Ditch, becauſe the Counterſcarp being parallel to the Face of the Baſtion, when there is a Second Flank, the lengthning of the outward ſide of the Ditch wou'd often meet the Curtain, whereas it ought to end at the Angle of the Epaule, whence the Enemies may be lodg'd in the Ditch without fear of Flank'd-ſhot, becauſe the Counterſcarp covers them againſt the Flank, and that being only ſeen of the Second-flank which is quickly ruin'd, the entrance of the Ditch is made eaſy to the Beſiegers.

The Calculation of the Angles and Lines of a Polygon, Fortified according to Marolois's *Method.*

THE outward ſide AB belonging here to an Hexagon, the Angle of the Center will be of 60 degrees, and the Angle of the Polygon of 120, and conſequently CAB half of the Angle of the Polygon of 60, and half the Flank'd-angle (*viz.*) CAE of 37. 30'. which being taken from CAB, half the Angle of the Polygon, or 60 degrees, 22. 30'. will remain for the Angle *diminué* EAB.

Be-

Because the Angle of the Flank FIK is a right one, or of 90 degrees, if to these 90 degrees FEI the Angle *diminuë*, or 22. 30′. be added, their sum 112. 30′. will be the Angle of the Epaule AFI, from which if IFM be taken, which is an Angle of 50 degrees, 62. 30′. will remain for the Angle AFM, wherefore the Angle AFM, will be precisely of 80 degrees. Thus are all the Angles known. *Plate 28. Fig. 61.*

There are but two lines known, (*viz.*) the Face AF of 48 Toises, and the Curtain IK of 72, by means of which and of the known Angles, the other lines may be known, and first the Capital AM, and the line MF, by the two following Analogies in the oblique-angled Triangle AMF:

As the Sine of the Angle AMF	98480
To the Face AF	48
So the Sine of the Angle AFM	88701
To the Capital AM	43.1

which will be of 43 Toises, and about 1 Foot.

As the Sine of the Angle AMF	98480
To its opposite side AF	48
So the Sine of the Angle FAM	60876
To its opposite side FM	29.4

of 29 Toises, and about 4 Foot, which will be of use to us in finding out the Flank IF, and the Demi-gorge MI, by the two following Analogies in the rectangular Triangle MIF:

As the whole Sine	100000
To the line FM	29. 4
So the Sine of the Flank-forming-angle IMF	64279
To the Flank IF	19

which will be of about 19 Toises.

As the whole Sine	100000
To the line MF	29. 4
So the Sine of the Angle IFM	76604
To the Demi-gorge MI	22. 4.

which

Plate 28. Fig. 61. which will be of 22 Toises, and about 4 Foot, whose double, or 45 Toises, and 2 Foot, being added to the Curtain IK, which is of 72 Toises, the sum will be 117 Toises, and 2 Foot, for the inward side MN.

To find the outward side AB, the line AG must first be found in the Triangle AGF rectangular at G, by this Analogy:

As the whole Sine	100000
To the Sine of the Angle AFG	92388
So the Face AF	48
To the line AG	44. 2.

which will be of 44 Toises, and about two Foot, whose double, or 88 Toises and 4 Foot, being added to the line GH, which is of 72 Toises, the Sum will be 160 Toises and 4 Foot for the outward side AB.

To know the Second-flank EK, the Complement EI must be first known, by the following Analogy, in the rectangular Triangle EIF:

As the whole Sine	100000
To the Tangent of the Angle EFI	241421
So the Flank IF	19
To the Complement IE	45.5

of 45 Toises, and about 5 Foot, and which being taken from the Curtain IK, or from 72 Toises, the remainder will be 26 Toises and 1 Foot for the Second-flank EK, or IO, *&c.*

Of the Spanish *Fortification.*

Plate 28. Fig. 62. THE *Spaniards*, who like obtuse Flank'd-angles, neglect a Second-flank upon the Curtain, always making their Fortification with a Rasant Defence, that is, never having any fichant Line of Defence, not minding whether the Angle of the Bastion be right, acute, or obtuse. Their Manner of Fortifying, except the Second-flanks, and the Flank'd-angle being right, is the same with that of the *Chevalier de Ville*, which for that reason has been call'd the *Compos'd Draught*, be-

because it is compounded of the *Italian* and *Spanish*. The Demi-gorges AC, BD, and the perpendicular Flanks CE, DF, must be the Sixth part of the inward side AB, then the Point of the Bastion is found by a rasant-line of Defence.

I shall not explain this Manner of Fortifying any more, because it is easy to understand by what has already been said, and by this Figure, which represents a Demi-Hexagon, where the Angle of the Bastion begins to be obtuse, and continues to be so more and more, as the Polygons increase. As the *Spaniards* have no Fortification within Harquebuse or Fuzil-shot, and that the least is only within Musket-shot, (*viz.*) 120 Toises, according to that measure the length of the Demi-gorges, Flanks, and Curtains must be taken, supposing the inward side AB of 120, to compute all the other Lines and Angles according to that Supposition, which will be easily done in imitation of the former Calculations.

Of the Reinforc'd Order.

TO lessen the Number of Bastions, which wou'd be requisite in a large Compass, if we wou'd make the geat Line of Defence within Musket-shot, a way has been invented to Fortify a long Side, usually suppos'd of 160 Toises, by a Curtain retir'd inwards, that the little Line of Defence might be within Musket-shot, as HL: and this Manner of Fortifying has been call'd *Ordre Renforcé*, (Reinforc'd Order) concerning which several *Italian* and *Spanish* Authors have wrote large Treatises: but without taking any notice of what they have said, I shall only explain it in a few Words, as I found it at the End of Father *Bourdin*'s Fortification. *Plate 28. Fig. 63.*

Divide the inward side AB into eight equal parts, and allow one for each of the Demi-gorges AC, BD, and the same to each of the Flanks CG, DG, which must be perpendicular to the inward side AB. Make each of the Curtains CE, DF, of two of those parts, and the

retir'd Flanks EH, FI, which must also be perpendicular to the inward side AB, of one of them a-piece. Lastly, draw the retir'd Curtain HI, and draw from H thro' F, the line HL, which passing thro' G, the end of the Flank DG will meet the lengthen'd Radius OB at L, which will be the Point of the Bastion.

Supposing the inward side AB of 160 Toises, each Flank and Demi-gorge will be of 20 Toises, and each Curtain of 40: and to know the length of the little Line of Defence HL, the Aperture of the Angle *diminué* IHF, or DFG must be found, whose Tangent is equal to the Sine 50000 of an Arch of 30 degrees. If then that Sine be found in the Tables of the Tangents, 26 degrees and about 34 minutes will answer to it for the quantity of the Angle *diminué* BFL, which being taken from the Angles ABO, which in the Hexagon is of 60 degrees, the remainder will be 33. 26′. for BLF half of the flank'd-angle, and in the obliquangled Triangle FBL, you may know the line FL by this Analogy:

As the Sine of the Angle BLF	55097
To its opposite side BF	60
So the Sine of the Angle FBL	86602
To its opposite side FL	94.2.

which will be of 94 Toises, and about 2 Foot.

The line HF will be found by the following Analogy, in the rectangular Triangle HIF:

As the whole Sine	100000
To the Secant of the Angle diminué IHF	111804
So the Curtain HI	40
To the line HF	44.4.

of 44 Toises, and about 4 Foot, to which if 94 Toises and 2 Foot be added, which is the line FL, the Sum will be 139 Toises for the Rasant-line HL.

The FOURTH PART.

OF

Irregular Fortification.

HAving taught how to Fortify Regular Places, it naturally follows to ſhew how to Fortify Irregular ones; I mean the moſt common, becauſe the opportunity of building new Places in a free piece of Ground is very rare, and there is oftner occaſion to Fortify the old ones, which are almoſt all irregular.

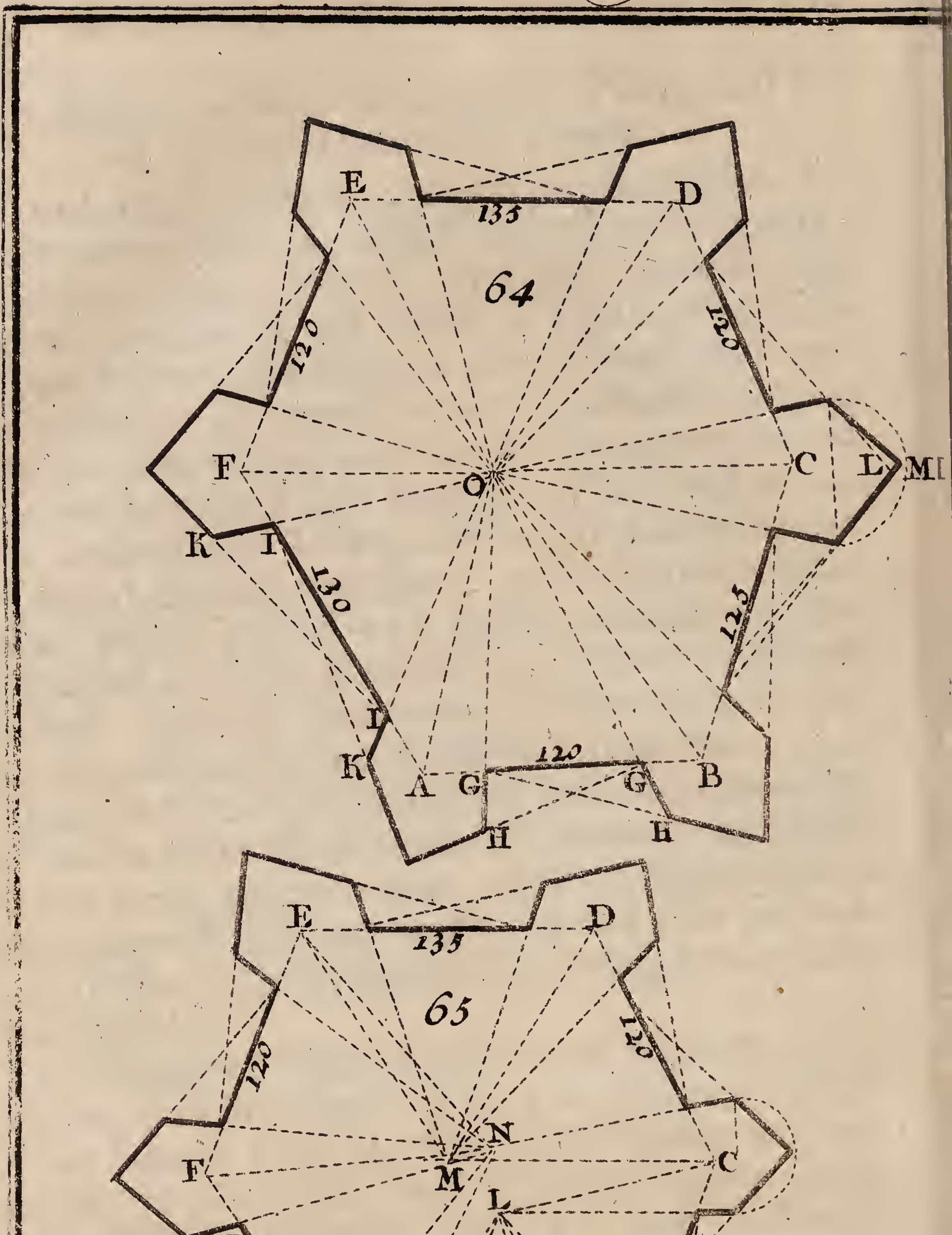
To Fortify an Irregular Place, and give it a good Defence, we muſt be aſſiſted by Regular Fortification, which will be a Rule and Foundation for the Irregular; and always have in Mind the general Maxims, which we have explain'd in the beginning of this Treatiſe, and eſpecially that which teaches us, that the great Line of Defence ought not to exceed the fartheſt reach of Musket-ſhot, that is, 150 Toiſes at moſt.

Irregular Places muſt be render'd as regular as may be, keeping almoſt the ſame Compaſs of Ground, as has been alſo taught at the beginning of this Treatiſe: and if that can't be done, by reaſon of their being encompaſs'd with Precipices, Rivers, Hills, or Mountains, as it ſometimes happens in Towns built near the Sea upon firm Ground, in Iſlands, or upon the ſide of a Mountain, with great or little Walls Moated round, or without Out-Works, and Fortified with round or ſquare Towers; the following Rules muſt be obſerv'd, to make their Fortification compleat.

How to Fortify outwards an Irregular Place, that has all its Angles and Sides Regular.

WE call *Regular Angles*, Saliant Angles which are not acute, that is, which are right or obtuse; acute Angles being reckon'd *Irregular*, because they cannot be Fortified by the Rules of regular Fortification: and *Regular sides* such as are not longer than 150 Toises, which is the utmost extent of Musket-shot, as we have said else-where, which makes the great Line of Defence not to exceed that length, and also such as are not shorter than 100 Toises, which will not make the great Line of Defence too short; reckoning those sides *Irregular*, which exceed 150 Toises, because the Bastions erected at their Ends are too distant to defend each other: and also such as are shorter than of 100 Toises, because their Line of Defence is too short; for since Muskets shoot 120 Toises and farther, vigorously, it is useless to make the Bastions any nearer; and even hurtful to make them so near together, because two Bastions cannot Flank one another when they are too near, by reason of the difficulty of shooting downwards at a little distance.

All the foregoing Methods may be of use in Fortifying such an irregular Plan; but here we shall only make use of our Second Method, where the Flanks are not so great as in the First, which might render some *Irregular Bastions* deform'd, that is, such Bastions, whose Flanks, and whose Faces are not equal to one another, and other Bastions too little, which wou'd make us deviate from the Maxim which orders the strength to be equally distributed, that is, that a Place must be every where equally Fortified, because when it is equally strong, the Enemy has no reason to attack it in one place sooner than another: yet the inequality of the Bastions is not much to be minded, if it is not very great; for the Beauty of their Equality is only seen upon Paper, but not from the Field, where you cannot at one time discover all the parts of a Place.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The First Manner.

LET the irregular Hexagon ABCDEF, whose Angles as well as Sides are regular, be of as many Toises of a side, as you see mark'd in the Figure. First, because this irregular Plan is almost regular, being only too long for its breadth, and that but a little, you may easily Fortify it, finding the Center by approximation, as at O, which is that of a Circle, whose circumference passes thro' AC, E, the three most distant points. Draw from that Center O, right lines to all the Angles of the Polygon, which you must look upon as the Radii of a regular Polygon, and Measure all the Angles of the Center, to know what regular Polygons they belong to, or which they come nearest to, that you may Fortify the sides opposite to them agreeably to such Polygons. *Plate 29 Fig. 64.*

Having thus found the Angle of the Center AOB to be of about 51 degrees, which is pretty near that of an Heptagon, as will appear if you divide 360 by 51, Fortify its opposite side AB, as in the Heptagon, without changing any thing, because that side is exactly of 120 Toises; that is, make the Demi-gorges AG, BG, of 27 Toises each, and the Flanks GH, which must be drawn from the Center O, of 24 Toises each.

Likewise knowing that the Angle of the Center AOF being of 77 degrees, comes pretty near to that of a Pentagon, the side AF must be Fortified as in the Pentagon, so that the Demi-gorge AI shou'd be of 25 Toises, and the Flank IK of 20, if the side AF was of 120, but as it is longer, (*viz.*) of 130 Toises, the Demi-gorges and Flanks must be made proportionably greater, which you may do by the Rule of Three direct, saying, if 120 gives 25 for the Demi-gorge, what will 130 give? and the Quotient will be 27 Toises for each Demi-gorge, AI, or FI: and likewise —— 120: 20 for the Flank : : 130: and the Answer will be about 22 for each of the Flanks IK.

After the same Method, must the other Flanks and

Demi-gorges be determin'd upon the other sides; then the Angles of the Bastions must be found by the intersection of Rasant-lines, and the whole Polygon will be well defended, and if it happens at any time that the Flank'd-angle is obtuse, as at L, it may be made right if you will, by means of a Semi-circle describ'd upon the line which joyns the Epaules, dividing that Semi-circle into two equal parts to have equal Faces, which cannot always be, because it may happen that the defence is wanting on one side, which you may know to be so, when the Face of the Bastion being lengthen'd, instead of meeting the Curtain will cut the Flank and Demi-gorge; in such a case, instead of having the Point of the Bastion in the middle of the Semi-circle, you must have it in that part of the Semi-circle which is cut by the Rasant-line, drawn from that side where the defence is wanting.

As a Flank is of very little use, when it is less than of 20 Toises, and that by our Method it may become such a one, namely, when the Angle of the Center is great, and its opposite side little, it will be better in such a case to make it of 20 Toises, unless that renders the Angle of the Bastion too acute: for tho' we have establish'd a Method of Fortifying, it is not necessary to adhere too strictly to it, as a mighty Mystery; for to Fortify a Place well, and become a Skilful Engineer, it is enough to make good use of one's reason, in applying the Rules of Fortification, it being allow'd to vary from those Rules several ways, according to different occasions: for it is impossible to Fortify an irregular Place as if it was altogether regular.

Thus instead of adhering to a right Flank'd-angle at M, which is not of great Moment, you may make it acute by increasing the Flanks a little, and especially the least of them, more or less, according as that Angle will be more or less acute, and it will always be tolerable, if it be not less than of 60 degrees. Likewise instead of Fortifying the propos'd Hexagon as has been taught, the three or four different Manners following may be us'd. *The*

The Second Manner.

THIS Second Manner is more perfect, and more general than the foregoing, because it may be us'd in a Polygon more irregular than the last, that is, much longer than it is broad. Yet we shall apply it here to the same Hexagon ABCDEF, to shew how much it excels the first, for you will see that the side AF will be better Fortified by the Second Manner than by the first, where the Flanks are more oblique, and consequently more expos'd to the Enemies Batteries. This is the Practice of it: *Fig. 65.*

Having taken any two joyning sides as AB, BC, and considering them as the sides of two regular Polygons inscrib'd in the same Circle, find L the Center of that Circle, which is the same as goes thro' the three points A, B, C, to know the Aperture of the two Angles of the Center, ALB, BLC, and determine the Flanks and Demi-gorges upon the sides AB, BC, as has been taught in the foregoing Method. Find also M the common Center of the two next sides CD, DE, and likewise N the common Center of the two last sides EF, AF, and end the rest as before taught.

When the propos'd Polygon has an uneven number of sides, it will happen at last that there will only remain one side, whose Center must be found; which will be done if you consider it as the side of a regular Polygon, whose Center will be found by the intersection of two right lines, which divide into two equal parts the Angles of the ends of that side, as we shall say more particularly in

The Third Manner.

THIS third Manner does not seem so perfect as the foregoing, but a great deal easier: for all you have to do is to divide into two equal parts, all the Angles of the propos'd Polygon by right lines, of which such as go from the ends of the same side, will in their intersection give the Center of a regular Polygon, one of whose sides the included side will be, and it must *Plate 30. Fig. 66.*

be Fortified agreeably with the Angle of the Center.

Having then divided each of the Angles A, B, of the Polygon into two equal Angles by the lines AL, BL, the point L of their interſection will be the Center of a regular Polygon, of which the line AB is a ſide, to be Fortified agreeably with the Angle of the Center ALB, as has been taught in our firſt Manner. Likewiſe the Angle C is to be divided into two equal parts by the line CM, which will cut the foregoing BL in a point, as M, which you muſt look upon as the Center of a regular Polygon, of which BC is a ſide, to Fortify it according to the Angle of the Center BMC, and ſo for the other ſides.

The Fourth Manner.

TO render the foregoing Manner more Perfect, and as Compleat as poſſible, we have here added a fourth Manner, which will eaſily ſhew us the Way to apply our fourth Method to Irregular Fortification: But without dwelling any longer upon it, we will ſtill follow the deſign of our ſecond Method, becauſe it makes the Flanks bigger.

Fig. 67. Having then divided one of the Angles of the Polygon, as A, into Two equal Parts, by the line AL, divide the ſide AB into Two equal Parts alſo, by LM perpendicular to that ſide, which here cuts the line AL at L, which you muſt take for the Center of a Regular Polygon, whoſe Angle of the Center has ALM for its half, according to which, the Demi-gorge AG, and the Flank GH, which muſt be drawn from the Center L, will be determin'd the firſt Way.

Having alſo divided the Angle of the Polygon B into Two equal Parts, by the line BP, it will meet the perpendicular ML, at P, which muſt be taken for the Center of a Regular Polygon, whoſe Angle of the Center has BPM for its half, according to which muſt be determin'd the length of the Demi-gorge BG, and of the Flank GH, which muſt be drawn from the Center P.

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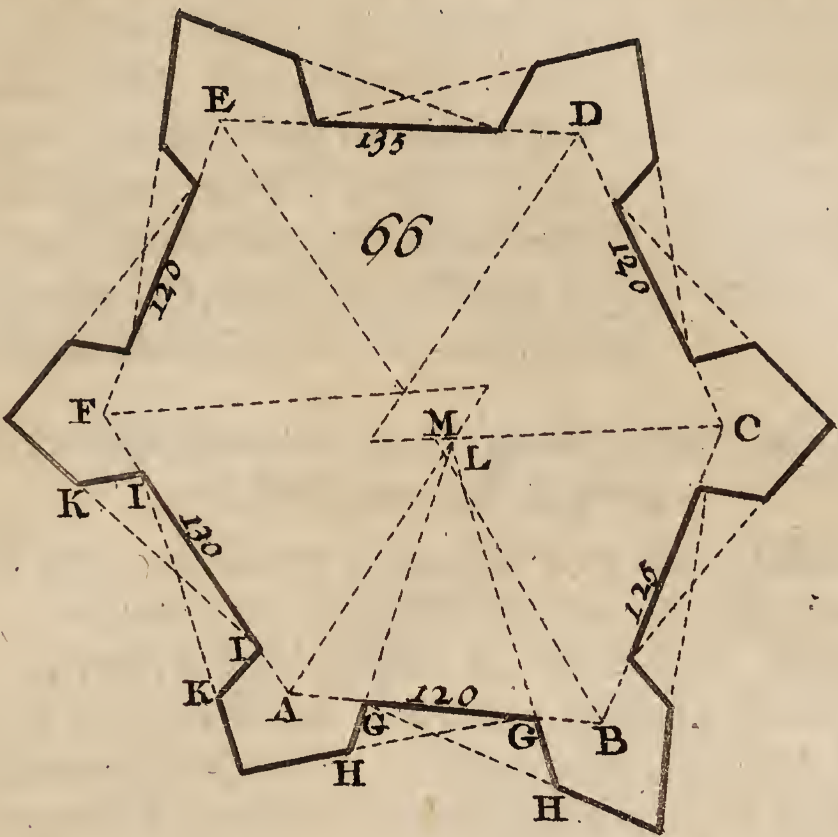
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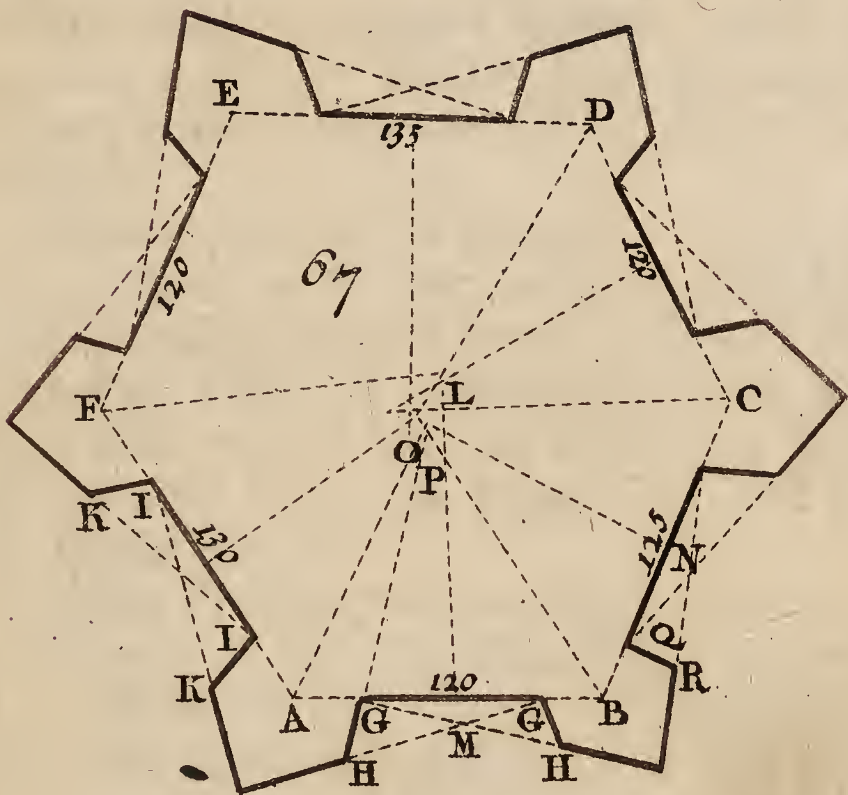
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Likewiſe having divided the ſide BC into Two equal Parts at N, and from that point having drawn its perpendicular NO, the point O, where that Perpendicular meets the line BP muſt be taken for the Center of a Regular Polygon, whoſe Angle of the Center has BON for its half, according to which muſt be determin'd the Demi-gorge BQ, and the Flank QR, which muſt be drawn from the Center O; and ſo on.

How to Fortify inwards an Irregular Place, whoſe Angles and Sides are all Regular.

WHEN we have bounded a regular ſide between 100 and 150 Toiſes, an inward ſide was meant; for an outward ſide ought to be longer, becauſe the Points of the Baſtions are more diſtant from one another than their Centers. That therefore all the parts of a Polygon Fortified inwards may have a juſt Proportion, its ſides ſhall be eſteem'd *Regular*, when they are between 160 and 200 Toiſes, becauſe according to theſe limits a Line of Defence will neither be too long, nor too ſhort.

The different Methods of Fortifying inwards a Regular Plan, which have been taught in the foregoing Part, may be eaſily apply'd to an Irregular one: But without loſing Time in mentioning thoſe Methods, which are now out of uſe, we ſhall only apply Mr. *Vauban*'s Method here, as being the beſt, and that which is follow'd now.

An Irregular Plan may be Fortified inwards as many different Ways as we have taught to Fortify it outwards, the beſt of which may be choſen: Wherefore, not to repeat the ſame thing over again, we ſhall give another Manner of Fortifying, which may be as a fifth Way of Fortifying inwards, as we ſhall always do in the Sequel. Therefore without minding of the quantity of the Angles, or length of the Sides of the propos'd Polygon ABCDEF, if they be but Regular, it may be Fortified according to Mr. *Vauban*'s Method, thus: *Plate* 31. *Fig.* 68.

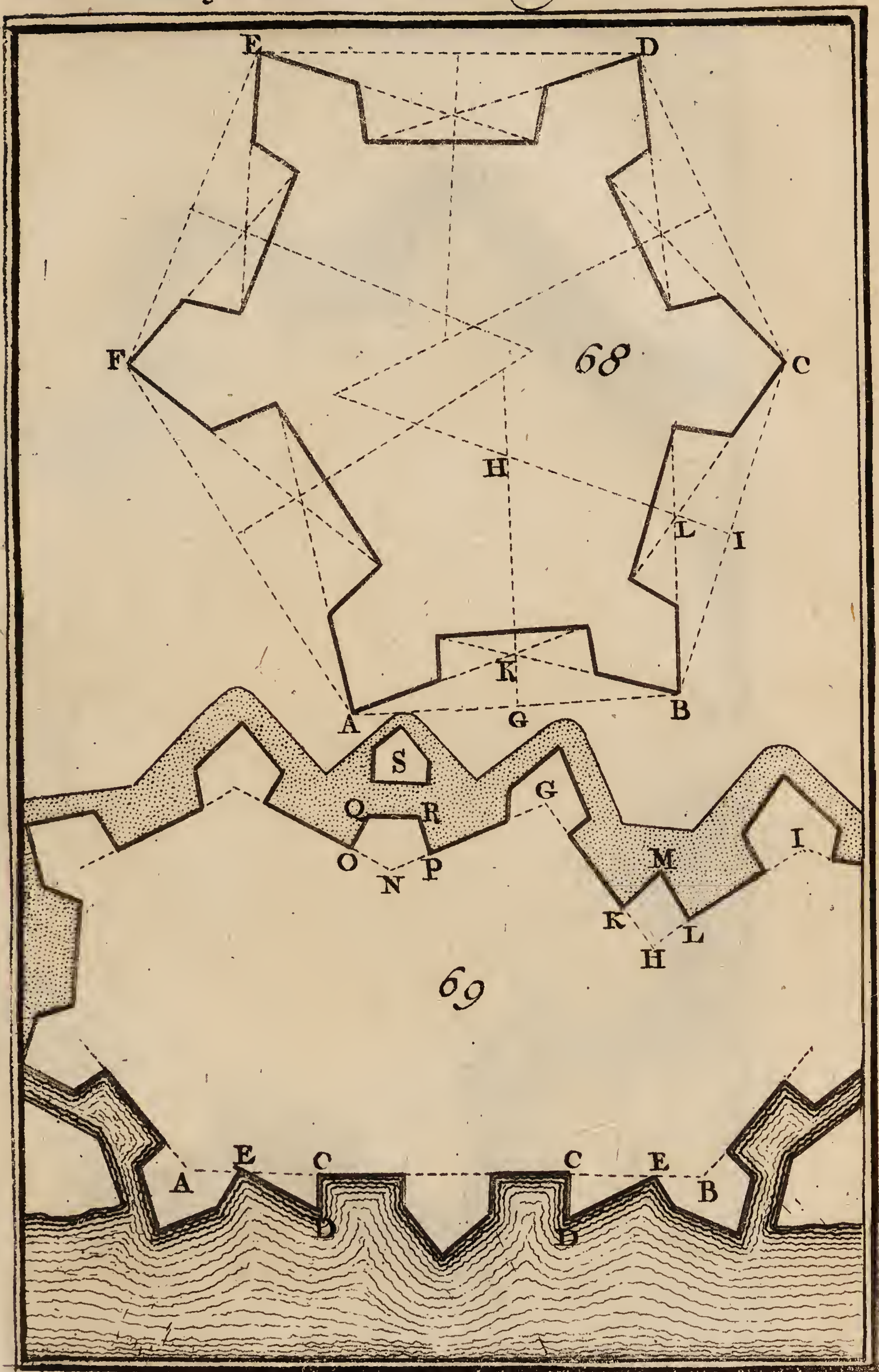
Plate 31. *Fig.* 69. Having divided the side AB into Two equal Parts, by its perpendicular GH, and likewise the next side BC, by its perpendicular IH, take the point H, where these Two Perpendiculars intersect, for the Center of a Regular Polygon, one of whose Angles is ABC, and GHI the Angle of the Center, which Angle you must measure, or else ABC that of the Polygon, to determine according to its Aperture, the length of the lines GK, IL, which by Mr. *Vauban*'s Method, must be each a sixth part of the sides AB, BC, or a third part of their halves BG, BI, if the Angle belongs to an Hexagon, or any greater Polygon.

How to Fortify an Irregular Side.

Plate 32. *Fig.* 70. TO Fortify outwards the side AB, which I suppose of 240 Toises, which is twice as far as the usual reach of a Musket, because the Bastions at the ends A,B, are too distant to defend one another, the side AB must be divided into two equal parts at C, to make at the middle point C a *flat Bastion*, call'd *Moineau*, allowing 30 Toises for each of its Demi-gorges CD, CE, and as much to the Flanks EF, DF, which must be perpendicular to the Curtains; and if you wou'd have the Angle of that Bastion a Right-one, the Capital GC must be perpendicular and equal to the Gorge DE.

The side HI being but of 90 Toises, the Demi-gorges HK, IL, have been made but of 15 Toises each, that there might remain 60 Toises, at least, for the Curtain, which ought scarce to be less: And if that side had been yet shorter, the Demi gorges HK, IL, must have been still less, and they must have been infinitely small, that is, the side HI must have been taken for a Curtain, if it had been but of 60 Toises, or something less: But if it shou'd be extremely short, then it must of necessity be chang'd, because the Bastions, at its ends, wou'd be too near together.

Because the side MN is not long enough to receive a flat Bastion in the middle, being but of 180 Toises, and that it is too long to have only Bastions at its ends to de-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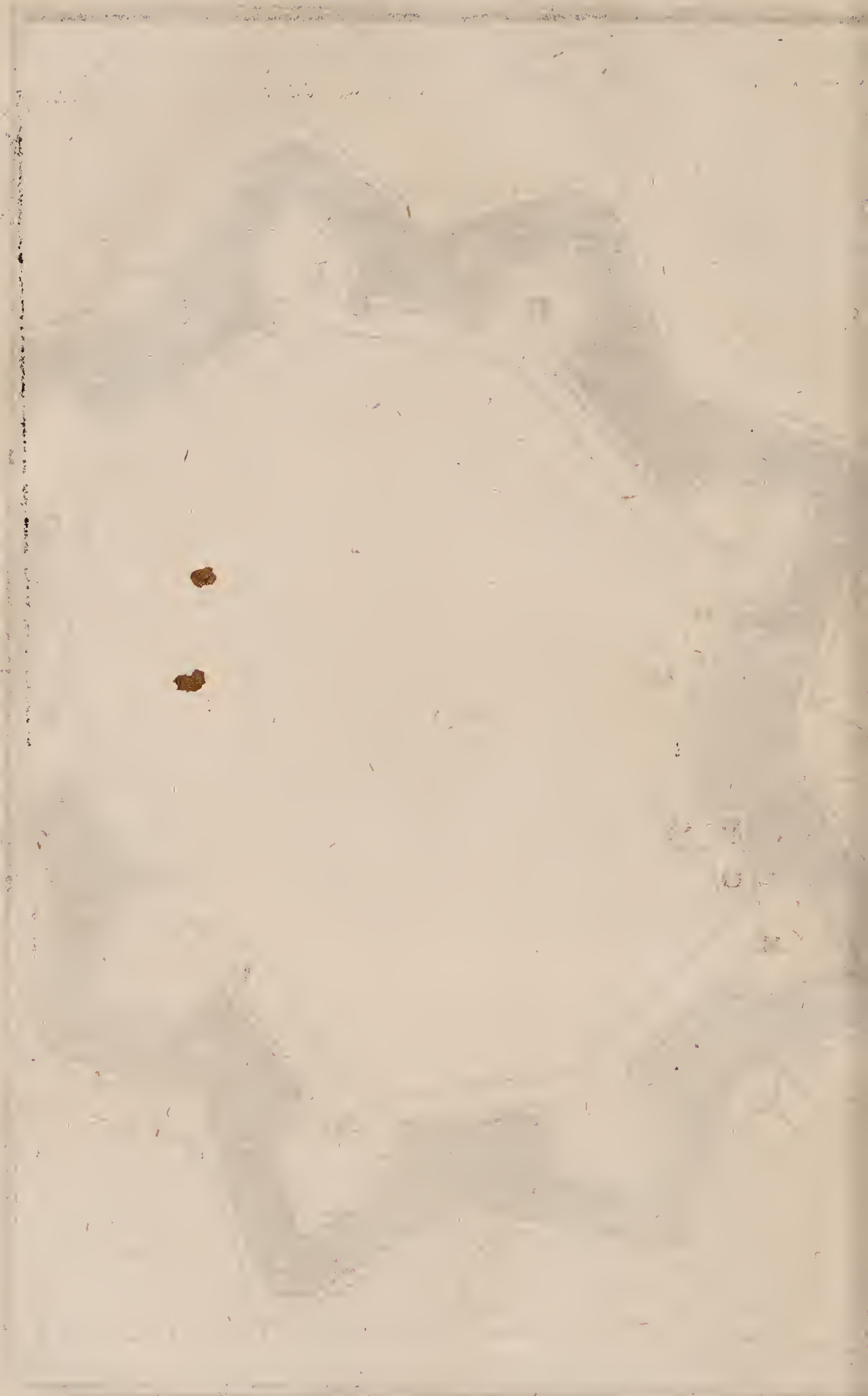


defend one another, we have Fortified it with a Curtain retir'd inwards, as in the Re-inforc'd Order, makeing the Curtains OP equal to the Demi-gorges MO, NO, or a little longer, according to the length of the ſide MN, and drawing upon the ſaid Curtain the Perpendicular Flanks PQ of 20 Toiſes each. *Plate* 32. *Fig.* 70.

One might have made a flat Baſtion in the middle of that ſide; but as it would in ſuch a caſe be too near the Baſtion on each ſide, you muſt take back the other Two Baſtions, if the other Two ſides will permit it, that is, if they are long enough to receive the whole Gorge, ſo that the Flanks, which are at O, may be at M, N, and perpendicular to the ſide MN, as if the ſide MN was to be taken for a Curtain: And then ſuch a Baſtion is call'd *Difform Baſtion*, becauſe it is Deform'd, in oppoſition to *Uniform* and *Beautiful* Baſtions, which, indeed, are the beſt flank'd, and of the beſt defence.

If the ſide to be Fortified is of a very great length, you muſt build upon it ſeveral flat Baſtions, or to avoid Expence, only Demi-Baſtions, 60 or 80 Toiſes from one another, in the Shape of a Horn-Work: Or elſe you may uſe Curtains retir'd inwards, with a flat Baſtion in the middle, as upon the ſide IL, if we are not by it forc'd to pull down Houſes; otherwiſe, when ſuch a ſide has an advantageous Situation, as when it is upon the ſide of a River, as AB, it will ſuffice to fortify it with *Redents*, call'd alſo *Saw-Works*, becauſe they are made like the Teeth of a Saw, as CDE, allowing at moſt 25 Toiſes for the Flank CD, and 60 at leaſt for the breadth CE. *Plate* 33. *Fig.* 71. *Plate* 31. *Fig.* 69.

The Quality of the Angles at the ends of the long ſide, oblige us ſometimes to fortify ſuch a ſide another way, and all that depends upon Judgment and Experience; eſpecially when you are well skill'd in Fortifying an Irregular angle, as we are going to teach, tho' we ſhall take no notice of ſeveral particulars, which are better underſtood by Experience and good Senſe, than a long Diſcourſe.

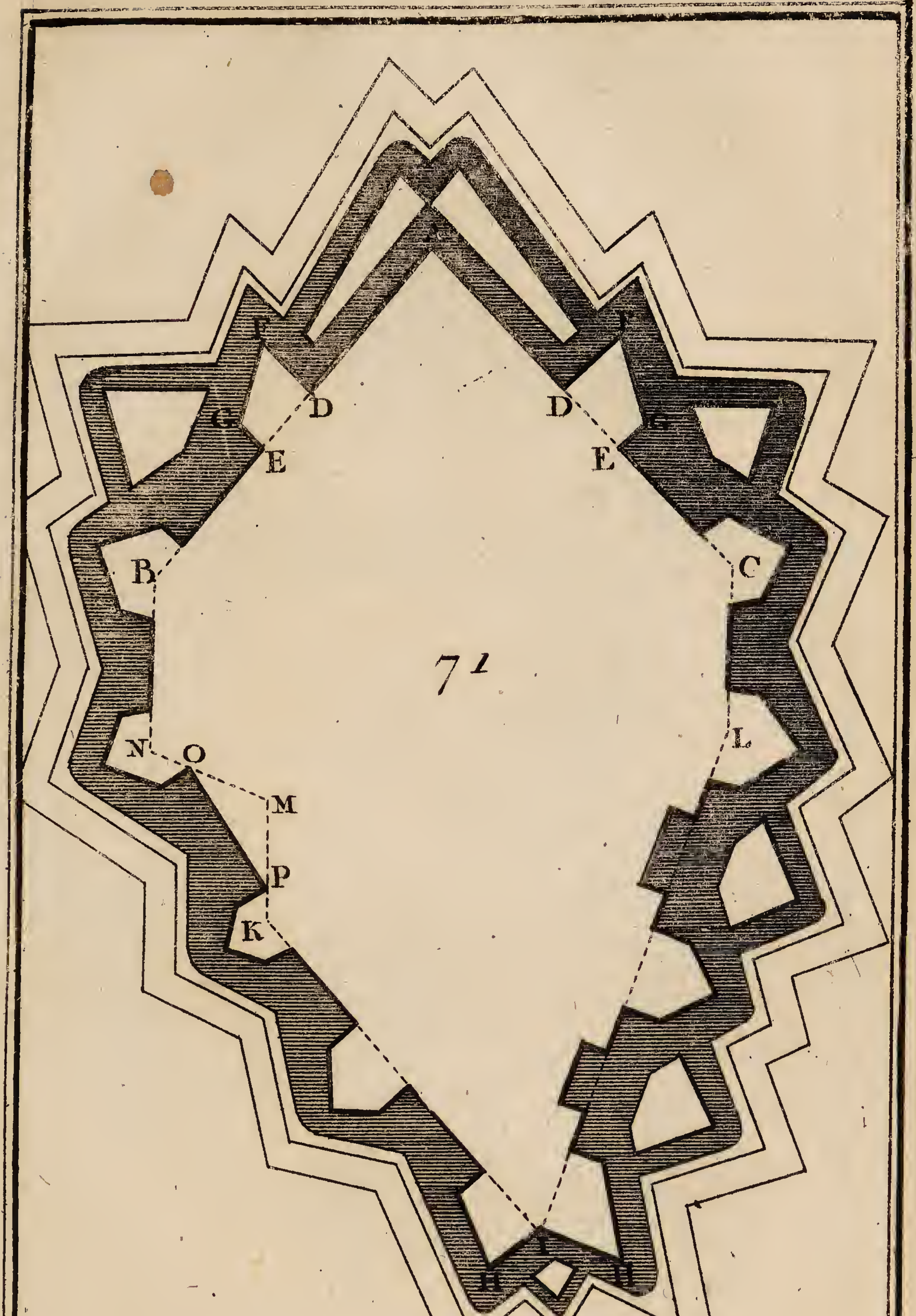
How

How to Fortify an Irregular Angle.

Plate 33. *Fig.* 71. FIRST, if it be a Saliant-angle, and greater than of 60 degrees, as A, which being acute, can't receive a Beautiful and Uniform Baſtion, becauſe the Angle of this Baſtion wou'd neceſſarily be too acute; it muſt be left as it is, being open enough to reſiſt the Enemy's Cannon: and that it may be well defended, inſtead of making upon the middle of the Two ſides AB, AC, (which are too long) flat Baſtions, whoſe Faces towards the Angle wou'd be ill flank'd, Demi-Baſtions muſt be erected, whoſe Gorges DE, muſt be 50 or 60, the Little-flank EG 25, and the Great-flank DF, 40 or 50 Toiſes long, which muſt be perpendicular upon AD, thence to defend the better the Angle A, which we have here cover'd with a Counter-guard, whoſe defence is from DF.

If the Saliant-angle be too acute, as I, it may be Fortified with a *Cut-Baſtion*, call'd alſo *Baſtion with a Tenaille*; that is, with Two Demi-Baſtions like the former, in ſuch manner that the longeſt Flanks, which make a Re-entrant-angle at I, may be perpendicular to the ſides IK, IL, which make the acute Angle KIL: And becauſe the Re-entrant-angle HIH is very much expos'd; and that the Tenaille or long ſides IH can ſcarce defend one another, it ought to be cover'd with a little Lunette, whoſe each Demi-gorge is of 10, and whoſe Face is of 15 Toiſes, that it may be defended from the Tenaille HI. This ſuppoſes that the ſides IK, IL cannot any way be ſhortned, when they are very long, as here.

Tho' this Angle I thus ſeems ill Fortified, yet one ought to conſider, that the ſuppoſition, which we have made, that nothing can be taken from the ſides IK, IL, may proceed from ſuch an advantageous Situation, as renders the Angle I hard of acceſs, and exempt from the danger of Mining; wherefore, in ſuch a caſe, it is well to profit of the advantage of that Situation, without any need of fortifying that Angle with ſo much precaution.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caution. Otherwife part of the fides IK, IL muft be taken off, to make in it a reinforc'd Tenaille, which may be cover'd with a good Ravelin: But yet this way we err againft the Maxim, which imports that a greater fpace in a Place is preferable to a lefs, becaufe it is capable of containing more Defenders, and of Retrenchments, in time of need. *Plate* 33. *Fig.* 71.

Secondly, To Fortify a Re-entrant-angle, as M, whofe lines MK, MN, are too fhort to receive a Platform at M, if poffible MO, MP muft be taken off to augment the capacity of the Place, in fuch manner that NO and KP, the lines which are left, be no longer than 30 or 40 Toifes, which, in this cafe, may be turn'd into Demi-gorges for the Baftions, which will be made at the Angles N, K; but care muft be taken, leaft the line OP, which muft be the Curtain, be too long, fo that the Two Baftions which muft be made at N,K, may defend one another, which will happen when the Angle M is very open, and the fides MN, MK, of about 100 Toifes.

One may Fortify the fame Re-entrant-angle KMN by a Curtain, retir'd inwards, thus: Lengthen the lines MK, MN, which I fuppofe too fhort, to O, and P, fo that the lines MO, MP, be of about 20 Toifes each, and when you have joyn'd the Curtain OP, draw from O, the Flank OQ of 20 Toifes, and perpendicular to MO, and likewife from the point P, the Flank PR of 20 Toifes, and perpendicular to the line PN, and draw the lines NQ, KR, to take upon it the Demi-gorges KS, NS, of about 24 Toifes each, and end the reft, as you fee in the Figure. *Plate* 34. *Fig* 72.

Such a Re-entrant-angle as A, may be Fortified yet otherwife, and more fimply, by a reinforc'd Tenaille, taking upon the lengthen'd fides the lines AB, AC, AD, AE, of 20 Toifes each, and drawing the Curtain BC, and the Two Flanks BE, CD, which are perpendicular to that Curtain. Some from the Angle A defcribe part of a Circle inwards, towards the Place, defcribing that Circle, or round Wall, thro' the Four points E,B,C,D.

One

Plate 34. Fig. 72. One might alſo make a round Curtain outwards, as EFD, or inſtead of a round Curtain, you may have a ſtreight one, which paſſing thro' the Two points D,E, will be well enough defended from the Baſtion on each ſide.

Plate 31. Fig. 69. When the ſides of the Re-entrant-angle are of a reaſonable length, as GH, IH, they may be Fortified by a Platform HLMK, allowing 24 or 25 Toiſes for each of the Demi-gorges HK, HL, and drawing from K and L, the Two Flanks KM, LM, perpendicular to the ſides GH, HI, which by their interſection will give at M the Angle of the Platform, which will never be too acute, when the Re-entrant-angle H is no greater than of 120 Degrees.

This way have we Fortified a Re-entrant-angle N, except that becauſe it is very open, which wou'd render the Angle of the Platform too weak, and the Flanks too long, we have theſe Flanks OQ, PR, only of 30 Toiſes each, and we have joyn'd the Face QR, which will be always defended enough by the Space which it takes up in the Re-entrant-angle: And to defend it the better, it will be well to make before it a *Detach'd Baſtion*, as S, whoſe each Flank and Demi-gorge muſt be of about 20 Toiſes.

A Place, whoſe Angles and Sides are all Regular, is call'd *well-condition'd*, (in *French* Bien-conditionée) and one whoſe Angles and Sides are not all Regular, is call'd *Ill-condition'd*; (in *French* Mal-conditionée) where you muſt obſerve, that when the foregoing Rules cannot be follow'd without ſome conſiderable Diſadvantage, the ſides of the Figure muſt be chang'd, by increaſing or diminiſhing it according to the Situation of the Ground.

How to Fortify an Irregular Place, leaving the old Enceinte, and making a new one.

THE old Enceinte of Places is kept, when it is encompaſs'd with a Ditch and a good Rampart, which may ſerve in the place where the Curtains are to be, ſo that only good Baſtions are to be made by the fore-

foregoing Rules, and by that means a good part of the Expence will be ſav'd; tho' care muſt be taken that the Baſtions be not too far aſunder, nor the Ramparts too high, for in ſuch a caſe they muſt be reduc'd to a reaſonable height, and the Earth taken from them may ſerve for ſome new Works, as for Example, for the conſtruction of ſome flat Baſtion, or Ravelin before the Curtain, to remedy the too great diſtance of Two Baſtions.

But if this old Enceinte is not ſuch a one, and the Place is only encompaſs'd with a little Ditch, and ſome old Walls with old-faſhion'd Towers, to avoid the expence of a new Enceinte, the Ditch muſt be made deeper, and broader, and the Earth taken from it muſt ſerve for the Rampart, whoſe *Chemiſe* the old Walls may be, filling the Towers with the ſame Earth, to make them ſerve as Cavaliers, when every one of them is encompaſs'd with a good Baſtion, or elſe they may be turn'd into Second Baſtions, after Count *Pagan*'s Manner, when they have a Saliant Angle towards the Enemy. Theſe Baſtion'd Towers may be both Magazines and Cavaliers.

This ſuppoſes the Town capacious enough to receive a Rampart within its Walls, but as it is difficult to carry Earth thither for the conſtruction of ſuch a Rampart, which thus made, cramps and leſſens the Town, and neceſſarily cauſes ſome Houſes to be pull'd down; it will be better to let the old Enceinte alone, and take compaſs in the Field to make a good Fortification of the Place, Regular if poſſible, or at leaſt, ſuch a new Enceinte or Polygon, whoſe Sides and Angles may be all Regular; for then the little Irregularity, which it will have, will not diminiſh its Strength. Its Baſtions will be large and few, a great many little ones being not ſo Advantageous as a few great ones, which require fewer Defenders in the Place, and are more capable of Retrenchments.

When you have not time to Fortify a Place as it ſhou'd be, or will not go to the Charge of it, or when

a conquer'd Place is to be Fortified to be of use in carrying on a War, or a Town that has a wide and deep Ditch, which shou'd be fill'd with Earth where the Bastions are to be joyn'd to the Place, which requires a great deal of Expence and Time, it is usually Fortified with good Out-Works, or Detach'd Pieces, built upon the edge of the Ditch, towards the Field, as Half-Moons and Ravelins, Horns and Crown-Works, wherein Cavaliers and Barbettes may be rais'd; or by Detach'd-Bastions and Demi-Bastions, which are usually Irregular from the Situation of the Place; tho that signifies little, if the Whole be well flank'd, and the Line of Defence be of a due length. *Maestrict*, which is reckon'd a strong Place, is Fortified after that Manner.

When you wou'd Fortify upon the Enceinte of the old Walls, which is chiefly done when a Town is Populous, and the Wall is thick enough to be a *Chemise* for the Rampart to be rais'd on its inside, you must recede from those old Walls, when there are Re-entrant-angles, to avoid making Platforms in them, the flat Bastions being of a better Defence: And you must endeavour to retain the Towers which happen to be in the middle of the Curtains, because when they are fill'd with Earth, they will be good Cavaliers, from which you may thunder into the Enemy's Works.

If the Town has Suburbs not Fortified, it is dangerous, least the Enemy shou'd make an Advantage of them, when he has taken them, by covering himself in his Approaches, and battering the Town from thence. Wherefore to deprive him of this Advantage, all the Houses must be pull'd down, as well as every thing that may cover the Enemies, and hide them from the sight of the Besieg'd, within Musket-shot and farther. You must also take from the Enemy, all that may favour him in his Trenches, as *Cavins*, which must be fill'd with Earth, and *Rideaux*, which must levell'd as much as possible.

If

If you wou'd preserve the Suburbs, and are not oblig'd to demolish it for fear of a Siege, in case that its Situation makes it rather defensive than troublesom to the Town, and that the Town can't well be without it, as being too little for the Inhabitants; the Suburbs must be taken into the Fortification, as in a Crown-Work, with several Bastions equally Fortified every where, so that the Fortification be as regular in one as in the other Tenaille: and if the two parts can't be joyn'd into one, that it make but one Body of the Place, Retrenchments and Redoubts must be made about the Suburbs in such manner, that each Piece may defend it self, and the one may not command the other, least the loss of the one shou'd occasion the loss of another; and when you wou'd build a Citadel about it, let it be in such a convenient Place as may command both the Town and Suburbs.

Where-ever the Ditches are dry, they must be digg'd as deep as may be, that the Bastions may receive the Earth digg'd out of such a Ditch, and be the fitter for the Besieged, who may in such a full Bastion fight for every Inch of Ground; but when Bastions are detach'd from the Body of the Place, it is better that they shou'd be empty, that the Enemy may be less able to cover himself in them, when he has taken them. The Ditches must be equally deep in all the Out-Works; but not so deep as the Ditch of the Town, when 'tis a dry Ditch.

All that we have said supposes the Town to be of no greater Compass than what is sufficient to make a good Place of War; but when it has a vast Enceinte, it will suffice to Fortify such a part of it as commands without being commanded, always upon the Avenues, but never in the middle of the City, that one may receive Succour, tho' the Enemy shou'd have taken some Houses: and to hinder him from being cover'd by them Cavaliers must be rais'd upon the Bastions of this new Fortification, at the end of which, towards the Field, a Citadel may be built, if the Ground is high enough,

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that the Befieg'd may retire thither, and hold out, if the firft Fortification fhou'd be taken.

How to Fortify a Place commanded by fome rifing Ground.

A Place over-look'd by a neighbouring Hill is faid to be *Commanded*, and fuch a Hill is call'd, a *Commanding Ground*, (*in French* Commandement) which is *Single* when it is 9 Foot; *Double* when it is 18 Foot; and *Triple* when it is 27 Foot high, and fo on, always reckoning 9 foot in height a Commanding Ground. There are other Commanding Grounds, which we fhall explain occafionally.

When an Eminence, which commands a Place, fees it and plays upon the back of it, as when it fees the Gorge of a Baftion as far as the Flank'd-angle, it is call'd a *Reverfe Commanding Ground*: and when a Height faces the Poft it plays upon, it is call'd, A *Front Commanding Ground*; and laftly, when it fees along a ftraight line and can fcour it, it is call'd, An *Enfilade Commanding Ground*, or *Curtain Commanding Ground.*

You muft Fortify your felf againft thofe Commanding Grounds by *Epaulements*, or earthen Parapets, to cover your felf, or elfe with Gabions and Cavaliers; and when you have them in your power, they muft be Fortified with Tenailles, *Fortins* or (Little Forts) Crowns, or Horns; thefe two laft Works having been chiefly invented for that purpofe.

To Fortify a Commanded Place, the Baftions muft be made folid, with Parapets higher than ordinary; the Baftions ought to be full which are oppofite to the Commanding Grounds, fo that they may command thofe very Eminences with Batteries and Cavaliers rais'd in them.

This fuppofes fuch a Height not too diftant, as within Musket-fhot, or within lefs than Cannon-fhot of the Place; for if it was but juft within Cannon-fhot or farther off, it ought not to be minded; for the Enemy from fuch a diftance cou'd not hurt the Place. When the Eminence is very near the Place, it muft be taken

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in with the Fortification of the Place, if poſſible; if not, with a Ravelin; and if a Ravelin is not capacious enough, greater Works muſt be us'd, as Horns or Crowns, as we have already ſaid.

If none of theſe things can be done; build a Fort, Caſtle or Citadel upon it, to hinder the Enemy from taking poſſeſſion of them; which I think very proper, tho' ſuch a Fort ſhou'd be very diſtant from the Place, as three quarters of a Mile: for if the Enemy neglects it, he will from thence be beaten upon the Rear, and if he attacks it, he cannot hinder the Town from being reliev'd; neither can he ſhut up ſuch a Fort in his Circumvallation, becauſe it wou'd be too great, and conſequently eaſy to be paſs'd, by ſuch Convoys as ſupply the Beſieged with Proviſions.

How to Fortify Towns Situate upon high Places.

SUCH Towns are either upon Mountains or Rocks. Thoſe that are upon Mountains have this advantage, that there is ſeldom any more than one or two ways to them, ſo that they need be Fortified but in one or two places: for when they have many Avenues, they ought to be better Guarded and Fortified, becauſe there is danger of Surpriſe from divers places. Thoſe that are upon Rocks are leſs acceſſible, becauſe their Enceinte is commonly Steep, either by Art, or Nature.

To Fortify thoſe upon Rocks, after having cut the Rampart in the Rock, a Parapet of Earth muſt be rais'd, and ſuch parts of the Rock as ſtand out too far muſt be cut down, and the hollow places fill'd up, that the Defenders may have a free Proſpect on every ſide. As the Ditch cannot be digg'd above four or five Foot deep, by reaſon of the hardneſs of the Stone, which wou'd render the charge exceſſive, it ought to be made very broad, and the Baſtions of the Town ought to be cover'd with Counter-guards ſhap'd like Demi-Baſtions, and with Half-Moons rais'd in the Ditch, that they may be defended from the Place, and command the Counterſcarp and all the Ground round about.

In Towns Situate upon Mountains, the Bastions, which may be Single or Double, must also be cover'd with Counter-guards, and the Curtains with good Ravelins to hinder Approaches: and because the Places are not always exempt from the danger of Mining, that inconveniency may be remedied by encompassing the Foot of the Mountain with a light Circumvallation, so digg'd that the Horse as well as Foot may be cover'd in it.

Those places of the Hill which the Enemy might make Lodgments in, must be Fortified with *Bonnettes*, which are Works of Earth made like a Ravelin, call'd also *Flecbes*. They have no Ditch, but only a Parapet 3 Foot high, border'd with a Palissade: and that its access may be harder to the Besiegers, you must add another Palissade 10 or 12 Paces beyond the first, and about 3 or 4 Foot out of the Ground.

If the Town is built upon the fall, and part of it at the Foot, of the Mountain: the upper part of the Town must be Fortified separately, so as to be detach'd from the lower; that if this be taken by the Enemy, the higher, which is the strongest, may be a place of Refuge to hold out in. In it must be the Magazines and Ammunition, and Cisterns to receive and preserve Rain-Water, when there is neither Wells nor Springs, as it often happens.

How to Fortify a Place Situate near a River.

Plate 34. Fig. 72. FIRST, if the River runs thro' the Town, it ought to come in and go out thro' a Curtain, which may be better open'd than a Bastion, which is not so well Flank'd: and if the River be narrow, it will suffice to make an Arch in the Curtain, and shut it up with a double Grate of Iron: and to keep the Enemy more at distance, a little Fort may be built upon Piles on that side where he is most fear'd, within Cannon-shot.

If the River be wider, yet not so much as to exceed in breadth the length of a Curtain, which is of about

70 Toises, a Bastion must be made on each side, that the passage may be the better Flank'd; and to Flank it still better, where it comes into, and where it goes out of the Town, Palissado's must be set; and to hinder Surprises, the Entrance must be shut up with an Iron Chain, sustain'd by little Boats or Logs of Wood, which every Night must lye quite cross the River.

If the breadth of the River exceeds the length of the Curtain, tho' not the reach of a Musket, a Demi-Bastion must be built on each side, whose Capital must be along the River, that these two Capitals may mutually defend one another with Muskets; for if whole Bastions shou'd be made, their opposite Faces towards the River cou'd not be defended, much less defend one another.

Lastly, if the breadth of the River very much exceeds the Line of Defence, and that its depth is such, that no Works can be advanc'd in it to Flank one another, by reason of their distance; such a Town must be Fortified as two separate Places, and on the right and left Redents must be made that Flank one another: and to prevent Surprises, you may build two Redoubts upon the opposite Banks, something high, to place Centries in them, who must get up with a long Ladder, which they are to draw up after them. *Plate* 31.

Fig. 69. Secondly, If the River only washes one of the Places, and that side be very long as AB, because such a place is hard of access, especially when the River is very broad, it will suffice to Fortify it with a Rampart only and Redents, to Scoure the River: it will not be amiss to have a Ravelin or flat Bastion in the middle, to put Cannon in, which will very well defend all the side AB, and to raise at each end A, B, a Cavalier to beat and discover the Avenues of the River.

Beyond the River, Works are generally made to Fortify the Bridge, when there is one, as a Ravelin, which must be defended by the Musket of the Place: and if the River be wider than the reach of Musket-shot, instead of a Ravelin, a Horn-Work or a Crown must be made, especially if there be Houses on the

other ſide of the Bridge, as well to ſecure the Inhabitants, as to guard the Bridge.

To Fortify the other ſide of the Place, towards the Land, according to the length of the great ſide AB, you muſt find out what Polygon will ſuit beſt with it, if the Ground will give you leave to make a regular Figure, which muſt be Fortified by the foregoing Methods, in ſuch manner that there may be two Baſtions or two Demi-Baſtions at the ends of the ſide AB, and that that Baſtion or Demi-Baſtion may take up all the Ground quite to the Water, the more eaſily to repel the Enemy along the River.

Becauſe the River-ſide ought to be well guarded, and that in ſuch a place there are uſually but Demi-Baſtions, it will be neceſſary to add ſome Out-Works, as Horn-Works cover'd with Ravelins, which in caſe of neceſſity may alſo have other Out-Works before them.

How to Fortify a Place built upon the Sea-ſide.

TOWNS built upon the Sea-ſide are Fortified towards the Land as uſual; towards the Sea they muſt be Fortified with a good Parapet, ſupported by a ſtrong Wall, and that ſuch a Parapet may defend it ſelf, it ought to be made in the ſhape of a Tenaille, Demi-Baſtion, or whole Baſtion, and Cavaliers or Platforms muſt be rais'd at ſeveral diſtances to keep off the Enemy: and the better to hinder Boats from advancing, artificial Shelves muſt be made under Water. Then, if poſſible, the Fortification of the Town muſt be ſo order'd, that the Entrance of the Harbour may be in the middle of a Curtain; that its Entrance, and its Way out may be defended by the oppoſite Flank of each Baſtion.

Ports are Fortified the ſame way as the Entance of Rivers, and better by Citadels that Flank their Entrance, and for want of a Citadel, the Shore muſt be Fortified with Cavaliers and Platforms, whoſe Artillery will hinder the Enemy from approaching, and Pirates from burning and pillaging ſuch Ships as are at Anchor.

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How to Fortify a Town Situate near a Lake.

WHEN a Town is Seated near a Lake, that has one end in the Enemy's Country; to avoid ſurpriſes the Fortification muſt be carried on as far as the Water of the Lake, and to hinder the Wall from being approach'd between the Lake and the Fortification of the Town, you muſt make a Falſe-Braye, ſupported by a little Wall: and if a River runs from the Lake thro' the Town, its Way in, and its Way out, muſt be block'd up, as we have ſaid before.

How to Fortify a Place Situate in an Iſland.

PLaces that are in an Iſland have no need of a regular Fortification, becauſe the Enemy can have no Stable Batteries, by reaſon of the continual motion of the Ships; and Baſtions will not be of leſs ſtrength for being acute; but inſtead of Baſtions, you may only have Redents, which defend one another, for tho' the Water is a Natural Fortification to ſuch a Place, yet it ought to be a little Fortified, leaſt it ſhou'd be ſurpris'd by an Enemy's Fleet.

The Advantages and Diſadvantages of the different Situations of a Place.

THO' what we ſhall ſay may indifferently belong to a Regular and an Irregular Place; yet we ſhall chiefly underſtand it of Irregular ones. Places are diſtinguiſh'd, either in reſpect to their Figure, which may be either a Square, or a Pentagon, or an Hexagon, *&c.* of which ſome may be irregular, as we have often ſaid: or in reſpect to their Situation; ſome being ſeated in a Dry and Hilly Country; others in a Plain, which may be Sandy, or of Clay, or Fenny; others in Vallies; and laſtly, others may be ſeated upon the edge of a Lake, or upon the Sea-ſhore, or upon a River-ſide, or in an Iſland, or any other place, encompaſs'd with Water. All theſe places have the following Advantages and Diſadvantages.

Concerning Places of a high Situation.

THE Advantages of a Place Situate upon a Mountain or a Rock, are the following. First, a good Air. Then it is almost proof against Mines. Its Fortification covers all the Top, and it Murders more than any other, because it sees the Works of the Enemy all round about, who in his approaches can raise no Work to command the Place, being oblig'd to make his Trenches very high, least he shou'd be seen from the Place, and the Parapet of his Batteries must be still higher, that the Cannon may recoil without being in danger: and whatever he does, he is always seen from the Defenders, whom he can't see. Mountains are of themselves so strong, that very little Art will help out Nature; so that they may be Fortified at little expence, there being no occasion for the Rampart or Parapet to be very high: and as there are usually but one or two Ways to them, they need be Fortified and Defended no where else. If the Place is inaccessible, the Enemy can't force it, and with a pretty deal of Provision it may hold out a long time with a small Garrison. Last of all; in Sallies, those of the Place have always the advantage of higher Ground.

The Disadvantages of a Place seated very high, are, first, that the Place is usually small, especially when it is Situate upon the Top of a Mountain or of a Rock, so that there is only room to build a few Castles, which are of little resistance: and if one may build any considerable Fort upon it, it is hard to make it regular, by reason of the odd Figure that Nature has commonly given to such places, which it is almost impossible with all our Art to make more regular. These Places generally want Earth to make the Fortifications, and their height facilitates the Enemy's Approaches, because it is hard for Musketeers to shoot downwards, when the Parapet is of a due thickness, and harder for the Cannon, which does more mischief shooting upwards than shooting downwards. Besides the difficulty of

of Carriage and Traffick, there is often a want of Water, which yet may be ſupplied by Ciſterns, which receive and preſerve Rain-Water. Whereas the Ditch in a dry Place ought to be deep, and narrow, here you are oblig'd to make it juſt the reverſe, that is, broader and leſs deep, at leaſt where the Ground is Rocky, by reaſon of the hardneſs of the Stone, and the expence in digging it out. Theſe Places are apt to be Scal'd, becauſe the Garriſon being ſmall and truſting to the ſtrength of the Place, often neglect to have a good Guard and Watch. Then laſt of all, the Earth being Sandy is unfit for Fortifications, becauſe unable to laſt long, as well by reaſon of the Rain, which quickly deſtroys it, as the Enemy's Cannon, whoſe ſhock can eaſily demoliſh it.

Places which are upon a deſcent, and conſequently commanded, tho' they ſtand high, are very defective and hard to Fortify ; this muſt be remedied by taking in, if poſſible, the Commanding Ground, or building a Caſtle or Citadel upon it, as we have ſaid before : and if you can do neither, you muſt cover your ſelf with ſeveral Pieces one before another, as Horn-Works, Crown-Works, *&c.*

Places Situate in Vallies, and encompaſs'd with Mountains are ſo defective, that it is better to leave than to endeavour to Fortify them ; for whatever Work you make, the Enemy from the Hills will deſtroy all with his Cannon, and thunder into the Town and the Field about it.

Concerning Places Seated in a Plain.

THE Advantages of a Town in a Plain, are firſt the extent of the Ground, which is free to make ſuch a Fortification as you pleaſe, and to make a regular Figure agreeable to the capacity of the Place, if the Ground be every where equal and uniform. As the Earth is in ſuch a place uſually ſtiff and marly, it will be very uſeful in Fortifications to make Ramparts, Parapets, Cavaliers and Out-Works, which may be made large and well Flank'd, there being no want of Earth

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to retrench with more eaſe upon occaſion. The Soil is fruitful round about, and uſually ſupplies the Place with neceſſaries, both for Man and Beaſt. Then ſuch a Place is fit for Commerce, eſpecially when built upon a navigable River.

The Diſadvantages of a Place built in a Plain, are firſt, that the Enemy enjoys the ſame Advantage with the Defenders, becauſe finding good Earth, he eaſily makes his Lines of Circumvallation, and Approaches. The fruitfulneſs of the Soil affords him Subſiſtence for his Army, and chiefly the Horſe, and he enjoys the Fruits of the Country better than thoſe who are ſhut up in the Place. And the Earth being a Clay, is the moſt fit for Mines againſt the Town, which is expos'd on all ſides.

Concerning Places built in Marſhy Ground.

PLaces that are in the Fenns, and have Water on ſeveral ſides, are very Strong, and have this Advantage, that they can be Attack'd but in a few Places, which may be well Fortified by ſeveral Works, rais'd before one another, in ſuch manner, that the greateſt be towards the Enemy, to hinder him from approaching. Beſides, the Place will want but a light Fortification, and a ſmall Garriſon, becauſe the Beſiegers can't eaſily make Batteries, or Lines of Approach, being oblig'd to fetch the Earth from another Place. Laſtly, ſuch Places are uſually exempt from Mines.

But they have theſe Diſadvantages. Firſt, that of a bad Air, and a great Expence when you have a mind to Fortify them well, becauſe there is no Earth for the Works, for in digging the Ditch you quickly come to Water, which obliges you to fetch the Earth a great Way, and the Foundation is ſo ſoft, that Baſtions are often ruin'd, and ſunk by their own weight, unleſs they are built upon good Piles, as at *Amſterdam*. Laſtly, the Enemy may eaſily ſhut up all the Paſſages, and the Fenns may be drain'd by the breaking ſome Canals, Banks, or Sluces, near the Town, which the Enemy may take

with

with eaſe, to the prejudice of the Place ; but when ſuch a thing is fear'd, the Fenns may be cover'd with Forts to repel the Enemy, and hinder him from undertaking any thing on that ſide.

There are ſome Places in *Zealand*, and part of *Holland*, where the Ground lies ſo low, that the Country may at pleaſure be laid under Water, by breaking the Banks, as was done at *Landrecy*, where a great part of the *Spaniſh* Army, that Beſieg'd it, was drown'd.

Concerning Places ſituate upon the Sea-Shore, or the Banks of a River.

PLaces ſituate upon the Water-ſide have this Advantage, that they do not require a ſtrong Garriſon, and that the part towards the Water may ſometimes be Fortified very Cheap. It may alſo very eaſily receive neceſſary Refreſhments, eſpecially if a Sea-Port, becauſe Naval Armies cannot always ſtay before the Place, for fear of Storms. The Enemy muſt have great Forces to Conquer ſuch a Place, becauſe he muſt neceſſarily Beſiege it both by Sea and Land. Laſtly, The Neighbourhood of the Sea or Rivers, makes a Town of Trade fitter for Arts, Husbandry, and the carriage of Wood, Victuals, and all Things neceſſary for the ſubſiſtence of the Inhabitants and Garriſon.

If theſe Places are like thoſe of *Holland* and *Zealand*, when the Enemy has taken the Banks, he may drown all the Country, to the great prejudice of the Inhabitants: And likewiſe, having taken thoſe Banks and Sluces which maſter the Courſe of the River, they may turn it into the Town, and drown it all, or the greateſt part of it. A River, or the Sea, affords the Enemy the ſame Advantage as it does the Beſieged, by the means of Boats ; for they have not only the Enemy to fear, who is at hand, but diſtant Forces too, who with an unexpected Fleet may take the Town. The Enemy with little trouble may ſhut up the River, by the means of Boats and Pontons, and thus prevent the Garriſon from a ſupply of Men or Proviſions on that ſide. Laſtly, it does

does not always happen, that such a Place may be Fortified conveniently and at little Charge, by reason of Bridges, which it must not want; and the Detach'd-Works, which must often be built on the outside of the River, to defend the Town that way; and several other Works, as Banks, Canals, Sluces, Mills, strong Palissado's, and other Things fit to withstand the violence of the Water.

Concerning Fortresses Erected in Islands.

FOrtresses built in Islands about Cannon-shot distant, or a little farther, from the Continent, have this Advantage, that they can't be Batter'd from the Land; but the Cannon of the Enemy's Ships may be very troublesome to them. The Inhabitants of an Island may easily hinder the Enemy from a Descent; but they may as easily be Surpriz'd by the Enemy's Fleet. Lastly, The Place may be Fortified with little Expence: But that supposes the Soil fit for Fortification, otherwise the Expence will be great; besides to resist the Tide, or the Stream of a River, Banks, Causeys, and Mounds of Earth must be rais'd.

How to choose a Place to be Fortified.

IF we consider the Advantages and Disadvantages just mention'd, proceeding from the Situation of a Place, it will be easy to choose the most Convenient Place to build a Fortress in.

Good Air and Water are chiefly to be chosen, because bad Air breeds the Plague, and bad Water the Feaver and Scurvy, which quickly destroy the Garrison of a Place Besieg'd.

As advantageous a Situation as possible, must be pitch'd upon, so that the Place may Command without being Commanded, that it may be hard to Besiege, and that in a Siege it may be Succour'd in spite of the Enemies: And last of all, the Earth must be good, and fit to Work with.

THE

The Fifth Part.

OF

Fortification Offensive.

BY *Fortification Offensive*, or the *Attack of Places*, is meant the Art of Besieging and Taking strong Places, which is either done by Famine or Force. When you wou'd take a Place by Famine, it is sufficient to *Bloccade* or *Invest* it, that is, to stop up all the Passages, to hinder its Convoys, and keep the Inhabitants in the Town, it being certain, that the greater their Number is, the sooner will the Provisions be consum'd. If you wou'd take it by Force, one cannot precisely tell which way it is to be Besieg'd, because all Places have not the same Situation, and the General, or Chief Engineer, who are not to be Contradicted, usually Determine that: But one may say in general, that the Approaches must be cover'd with Trenches, which are usually carried on to the very Foot of the Wall, where divers Assaults are given, as we shall say more particularly in the sequel.

We shall here briefly speak of the Way of carrying on a Siege before a Place, either to take it by main Force, or to reduce it by Famine. The General designing to make himself Master of it, must before he undertakes any thing, be well inform'd of the Advantages and Disadvantages of the Place, by a faithful Plan of the Fortress, and all the Country round about, for a Mile or Two, by a Man of Sense and Skill, who understands Fortifications well, that according as the Place is, he may judiciously begin his Siege, when he

has

has receiv'd Orders from his Sovereign, who in his Council of War has resolv'd to have the Place taken.

When you are about to lay Siege to a Place, consider whether there is any fear that an Army of the Enemies should Relieve it; or, if being Master of the Field you have nothing to fear. First, when a Succour of Men, or a Convoy of Provisions is fear'd, a Rampart, or Elevation of Earth, with Saliant and Re-entrant-angles must be made, which is call'd *Circumvallation*, in Lines which defend one another, call'd *Lines of Circumvallation*, and which Encompass the Place within Cannon-shot.

In the Second Case, that is, when nothing is fear'd, it is sufficient to dispose the Quarters of the Army in due order, and without losing time in making Forts, to fall about the *Trenches of Approach*, which are Hollow Ways cover'd with a Parapet, on that side towards the Place, which cover and bring on obliquely the Besieger's Attacks as far as the Counterscarp.

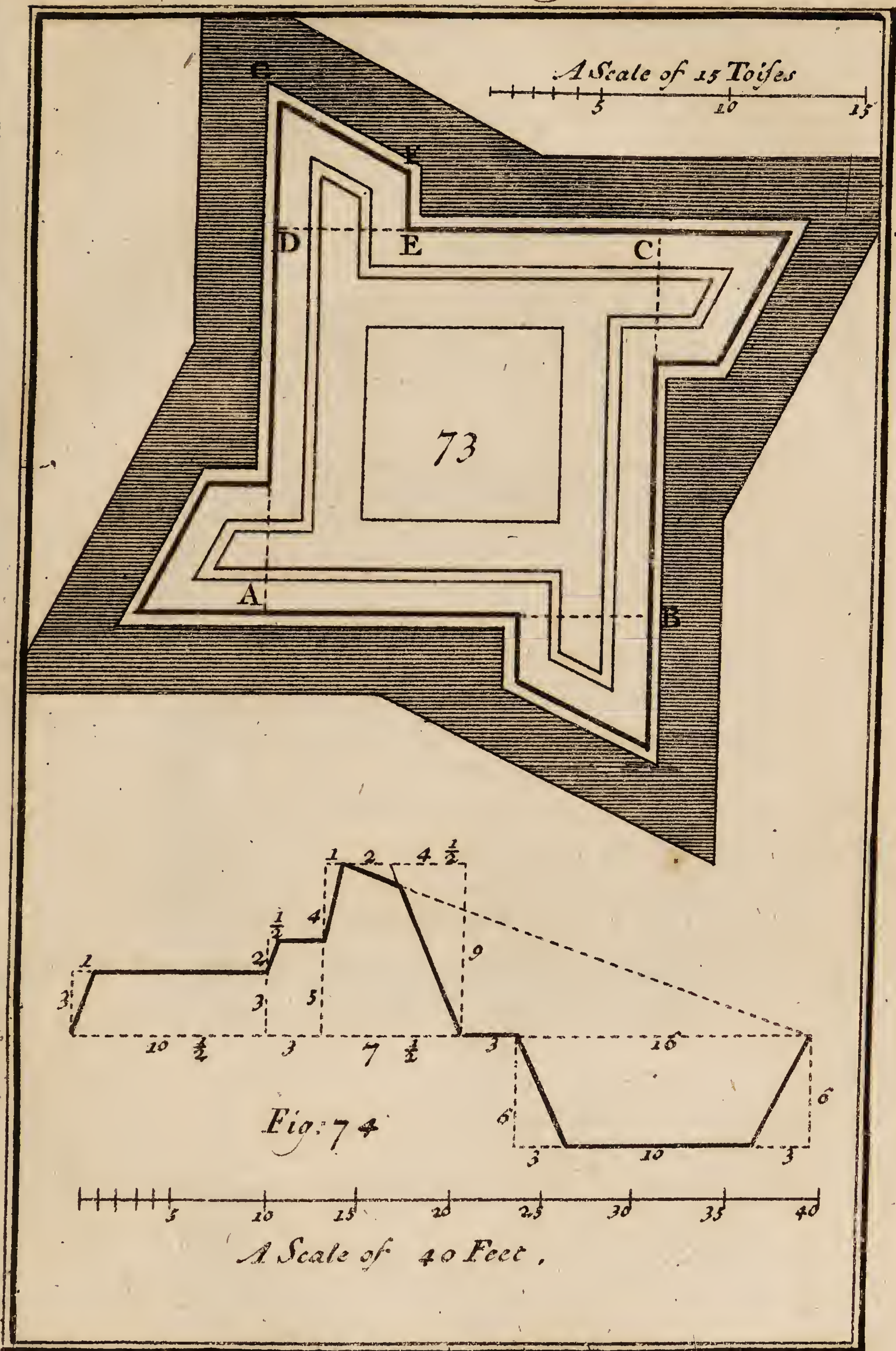
We shall speak more particularly of the Trenches of Approach, and Circumvallation, after we have said something of Field-Forts, to speak afterwards of Batteries, Galleries and Mines.

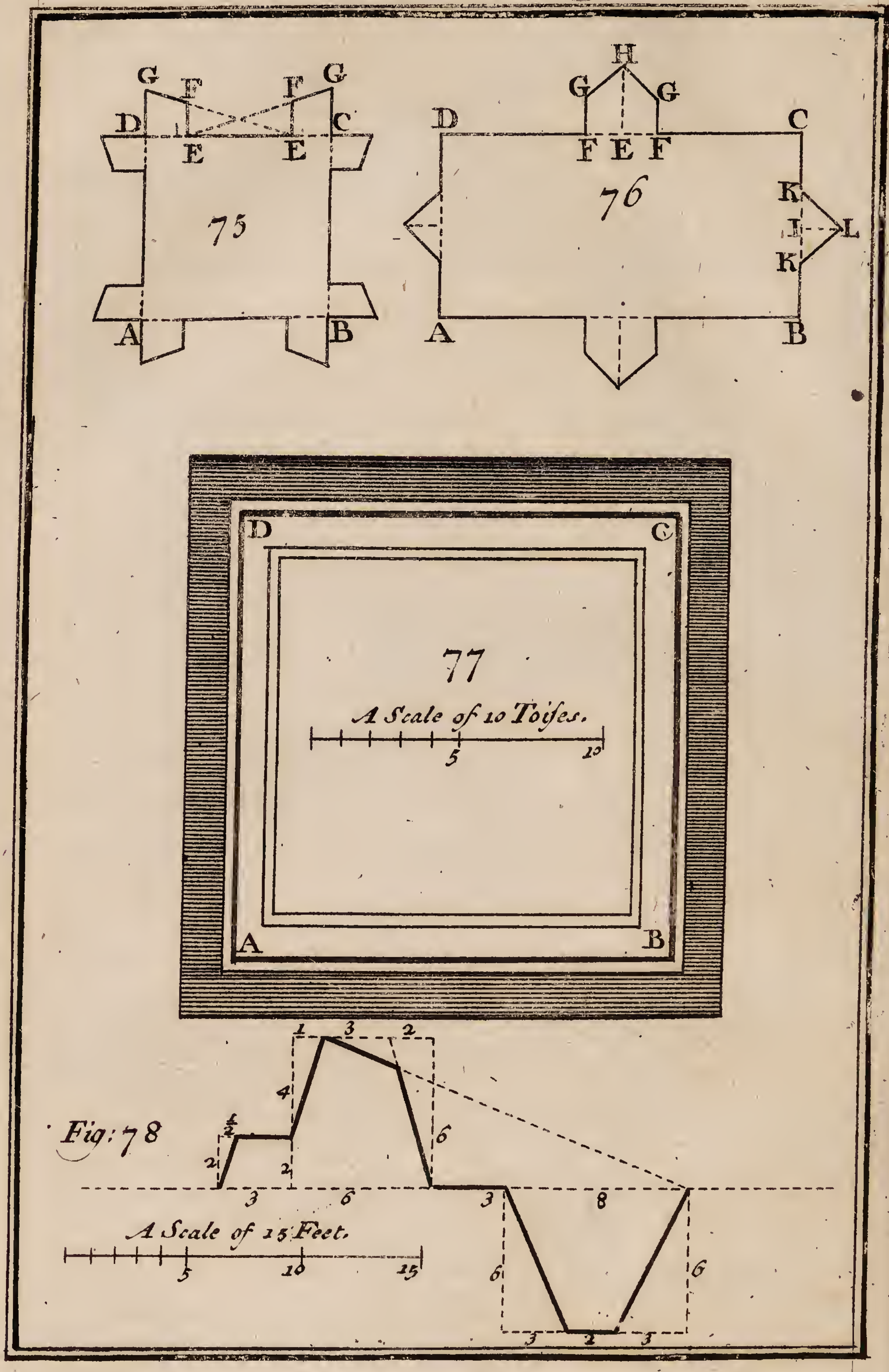
Of Field-Forts.

Field-Forts, call'd also *Fortins*, are not only useful for Sieges, but also for Fortified Places, because several Places, which are commanded by some Eminence, or which have their Avenues cut with Rivers, Bridges, or Rising-Grounds, may be secur'd by Field-Forts, erected upon, or near such Places. Therefore these Two Reasons oblige us to give you their Construction.

How to build a Fort with Demi-Bastions.

A Fort with Demi-Bastions is made in a Triangular, Square, or Pentagon Figure, and are all describ'd the same way, as you will see in the Square, which may be a Model for the Two others.


A Scale of 15 Toises
5 10 15
G
F
D
E
C
A
B
73
1 2 4 ½
½
4
2
9
1
3
3
5
10 ½ 3 7 ½ 3 16
6 6
3 10 3
Fig: 74
5 10 15 20 25 30 35 40
A Scale of 40 Feet.


G F F G
D C
E E
75
A B
H
G G
D C
F E F
76
K
I L
K
A B
D C
77
A Scale of 10 Toises.
5 10
A B
Fig: 78
A Scale of 15 Feet.
5 10 15

To describe, for Example, the Demi-Bastion EFG, lengthen the side AD towards G, in such manner that the excess DG be equal to the third Part of the said side AD. Make also the Demi-gorge DE equal to the third Part of the side DC, or to the Capital DG. Lastly, Draw at E the Flank EF perpendicular to DC, and equal to half the Demi-gorge DE, to have the Face GE. The same way must the other Three Demi-Bastions be made, and the Square will be Fortified, to which must be added a Berme and Ditch on the outside, and on the inside a Rampart and Parapet, with its Banquette, according to their breadth, which we have mark'd in the numbers which represent Feet in the Profil, which will be easily describ'd. *Plate* 35. *Fig.* 73.

The sides of such a Fortin are usually 15 or 20 Toises, when they are us'd in a Circumvallation; for these Forts are much larger, when they are put to other uses, as to possess some rising Ground, to stop an Army, to guard a Bridge, to cut off and hinder a Passage, *&c.* according to the necessity of the Places where they are to be Erected. The Square is the most convenient, and common of all Fortins, which is sometimes Fortified with double Demi-Bastions, or cut Bastions, as ABCD, allowing the fifth Part of the side for the Demi-gorges DE, CE, and as much for the perpendicular Flanks EF; or only the sixth Part, and drawing Rasant-lines, which where they meet the lengthen'd sides, will give you the Points G, *&c.* *Plate* 35. *Fig.* 74. *Plate* 36. *Fig.* 75.

If the Square be longer than it is broad, as A,B,C,D, whose length AB is twice the breadth BC, flat Bastions may be made upon the middle of the Two long sides AB, DC, allowing for the Demi-gorge EF, and the perpendicular Flank FG, the tenth Part of the same side, and making the Capital EH equal to the Gorge FF. You may do as much upon the middle of the little sides AD, BC, or only make the *Advance* KLK, allowing for its Demi-gorge KI, and its Capital IL, about the fifth Part of the same side, *&c.* *Fig.* 76.

How to describe a Redoubt.

Plate 36. Fig. 77. AS Redoubts, call'd also *Reducts*, are made in hast in time of War, in a Circumvallation for a Corps de Guard, and to secure the Circumvallation from a Contravallation, and to defend the Lines of Approach; they are made only square, without any Defence, as ABCD, whose sides may be of 10, 15 or 20 Toises, according to the Capacity of the Ground, and the Number of Men, which they are to hold. On the outside it must have a Berme and Ditch, and on the inside only a Parapet with its Banquette, according to the breadth which you see mark'd by the Numbers that express Feet in the Profil, which by help of those Numbers may be easily drawn. Fig. 78.

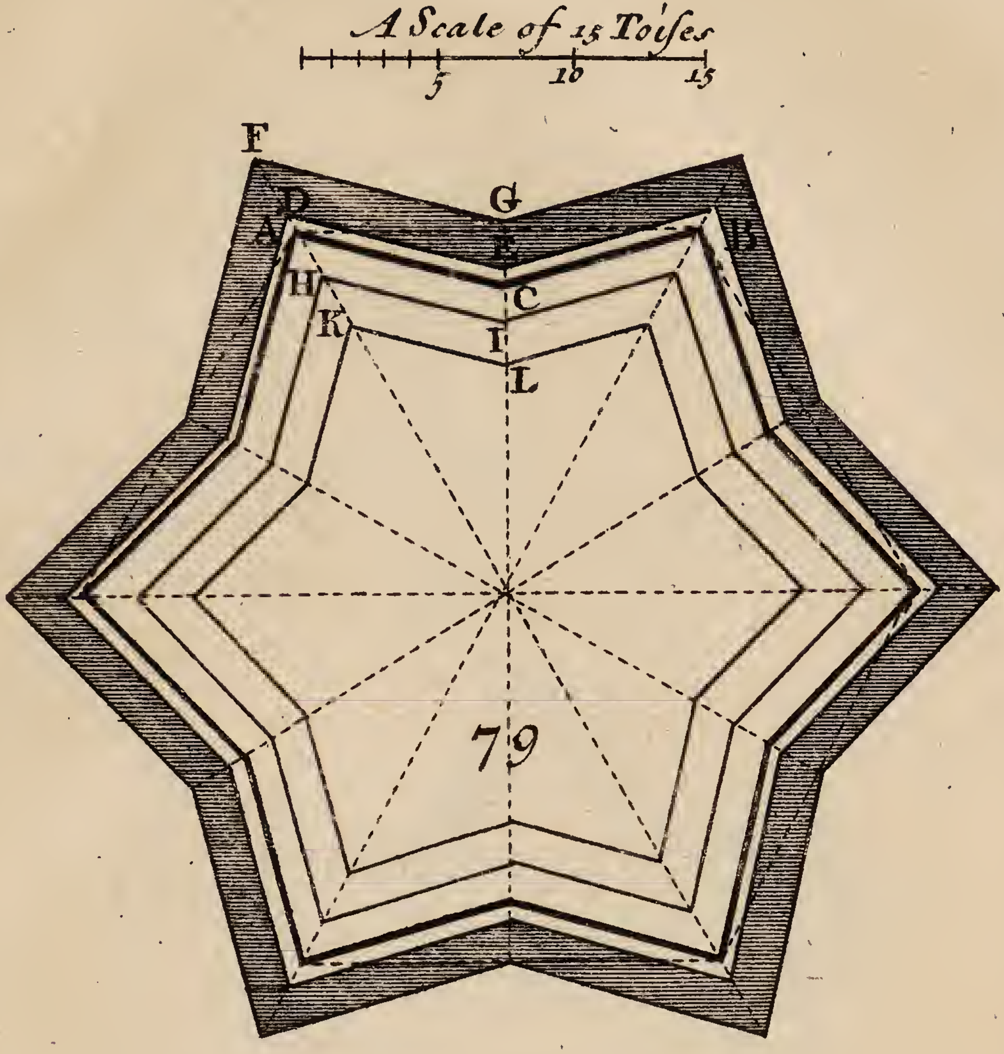
How to describe a Star-Fort.

WE call *Star-Fort*, or only *Star*, a kind of Redoubt, made of Saliant and Re-entrant Angles, which are not only us'd in a Circumvallation to secure its Enceinte, and fortify the Quarters of a Siege; but also to encompass some Post that you wou'd preserve, as to enclose little Castles, Churches, and other Places which are near Towns.

This Fortin has four, five, or six, and sometimes Plate 37. Fig. 79. seven or eight Points, or Saliant Angles, of 60 Degrees each in the Square, 80 in the Pentagon, and 90 in the other Polygons: And consequently its half KAC is of 30 Degrees in the Square, of 40 in the Pentagon, and of 45 in the other Polygons.

The line DE, which is parallel to the Tenaille AC, terminates the Berme, FG the Ditch, HI the Parapet with its Talu, and the line KL the space of the 3 Ban- Fig. 80. quettes, whose breadth will be known by the particular Scale of the Plan; but better upon the Profil, by the Number of Feet, which we have added to each line.

Tho' we have given different Profils for those Field-Forts, we are not oblig'd to stick to those Measures as necessary and invariable Proportions: For the Profils may


A Scale of 15 Toises
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15
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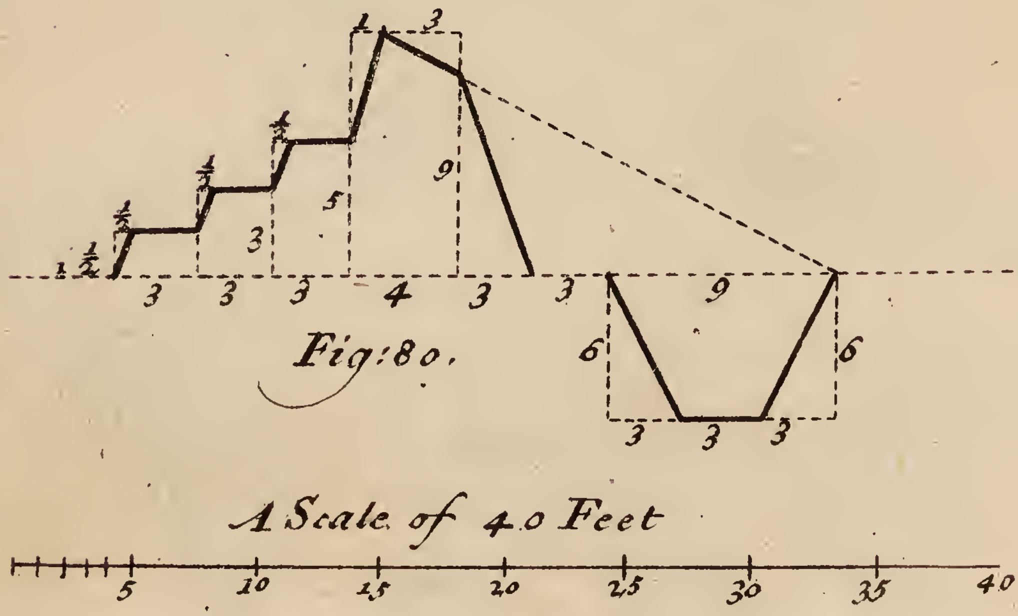
L
79


1
3
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9
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3
4
3
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A Scale of 40 Feet
5
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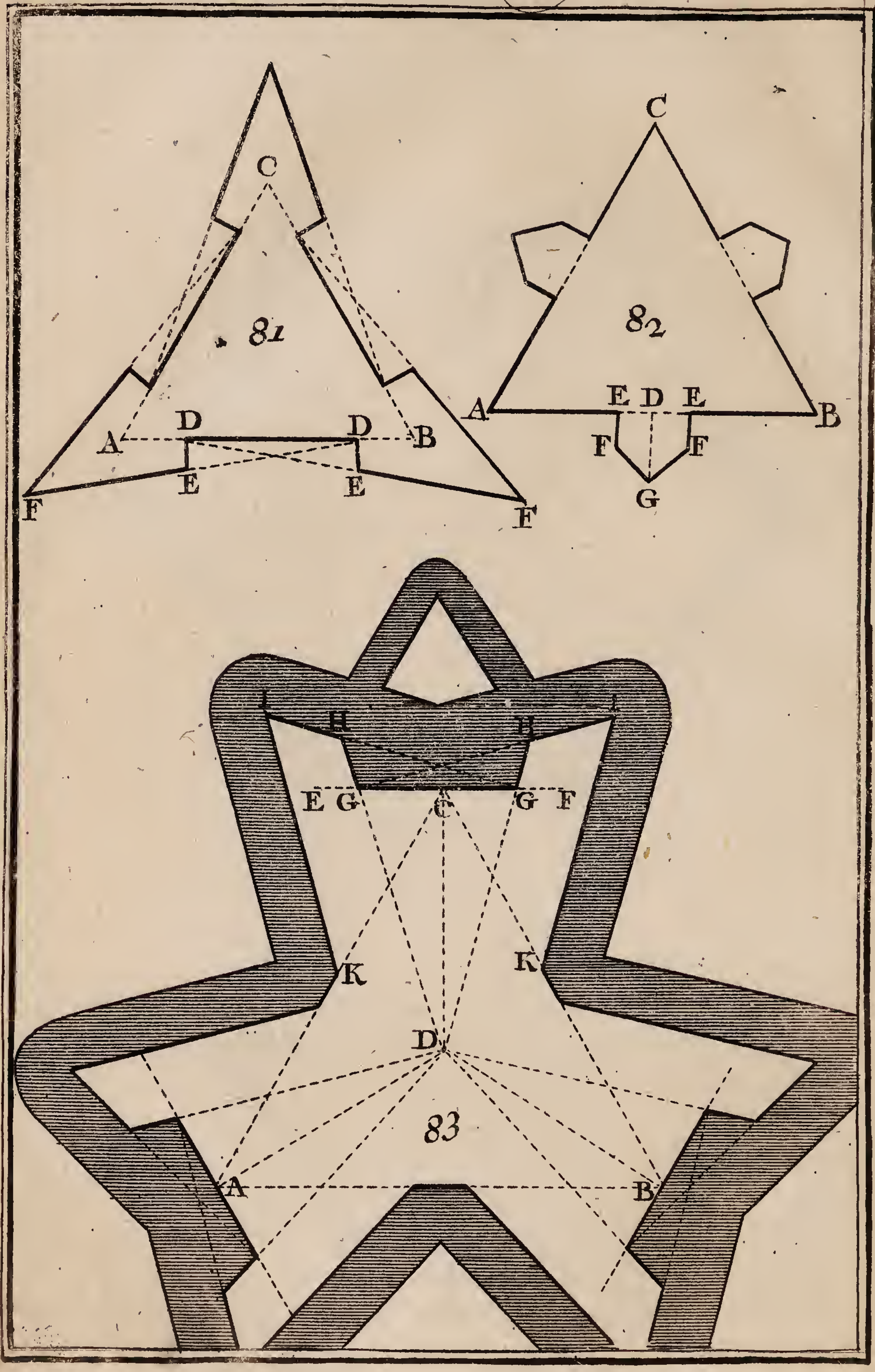
15
20
25
30
35
40

Fig: 80.


C
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E G
C
G F
K
K
D
83
A
B

may be varied as the Engineer pleaſes, according to the quality of the Ground, and the importance of the Fort.

Divers Ways of Fortifying an Equilateral Triangle.

BEcauſe the Equilateral Triangle is too imperfect, not having Extent enough to be well Fortified, we have not treated of it in Regular Fortification, tho' it may be Regularly Fortified ſeveral Ways, as you ſhall ſee.

The First Way.

FIRST, if you wou'd Fortify an Equilateral Triangle, as ABC, with Baſtions, allow the fifth Part of the ſide AB for each of the Demi-gorges AD,BD, and half of the Demi-gorge for each Flank DE, which muſt be perpendicular to the Curtain, then the Point F of the Baſtion, will be found by the interſection of the Raſant-lines. *Plate* 38. *Fig.* 81.

As one cannot avoid having the Faces too long, and the flank'd Angle too acute, a Ravelin may be made before the Curtain, if the Plan be of any conſequence, as for Example, if it encloſes an Iſland; for if it is but a Field-Fort, which endures no Siege, the Ravelin will be uſeleſs.

The Second Way.

YOU may alſo Fortify an Equilateral Triangle, as ACD, by erecting a flat Baſtion upon the middle of every ſide, allowing the fifth Part of AB, or BC for each Demi-gorge DE, and perpendicular Flank EF, and the Capital DG muſt be equal to the Gorge EE. It is plain that a Triangle thus Fortified is only fit for a Field-Fort, becauſe the Faces of its flat Baſtions are ſcarce defended at all. *Fig.* 82.

The Third Way.

WHEN an Equilateral Triangle is deſign'd for a Field-Fort, as ABC, it is uſually Fortified with Demi-Baſtions; which may be done by making each *Plate* 39. *Fig.* 84.

 Demi-

Demi-gorge, as CD equal to the fifth Part of its side BC, and the perpendicular Flank DE equal to half of that Demi-gorge; and lastly, lengthning the side AC to F, that CF may be equal to CD, to draw the Face EF, *&c.*

The Fourth Way.

Fig. 85. THE same Equilateral Triangle ABC may also be Fortified with *cut Bastions*, call'd also (in *French*) Bastions *accolez*, making the Demi-gorge CD equal to the fourth Part of the side AC, or BC, and the perpendicular Flank DE equal to half the Demi-gorge CD: Then draw a Rasant-line, to take upon it the Face EF equal to the line EC, and to joyn the Tenaille CF, *&c.*

The Fifth Way.

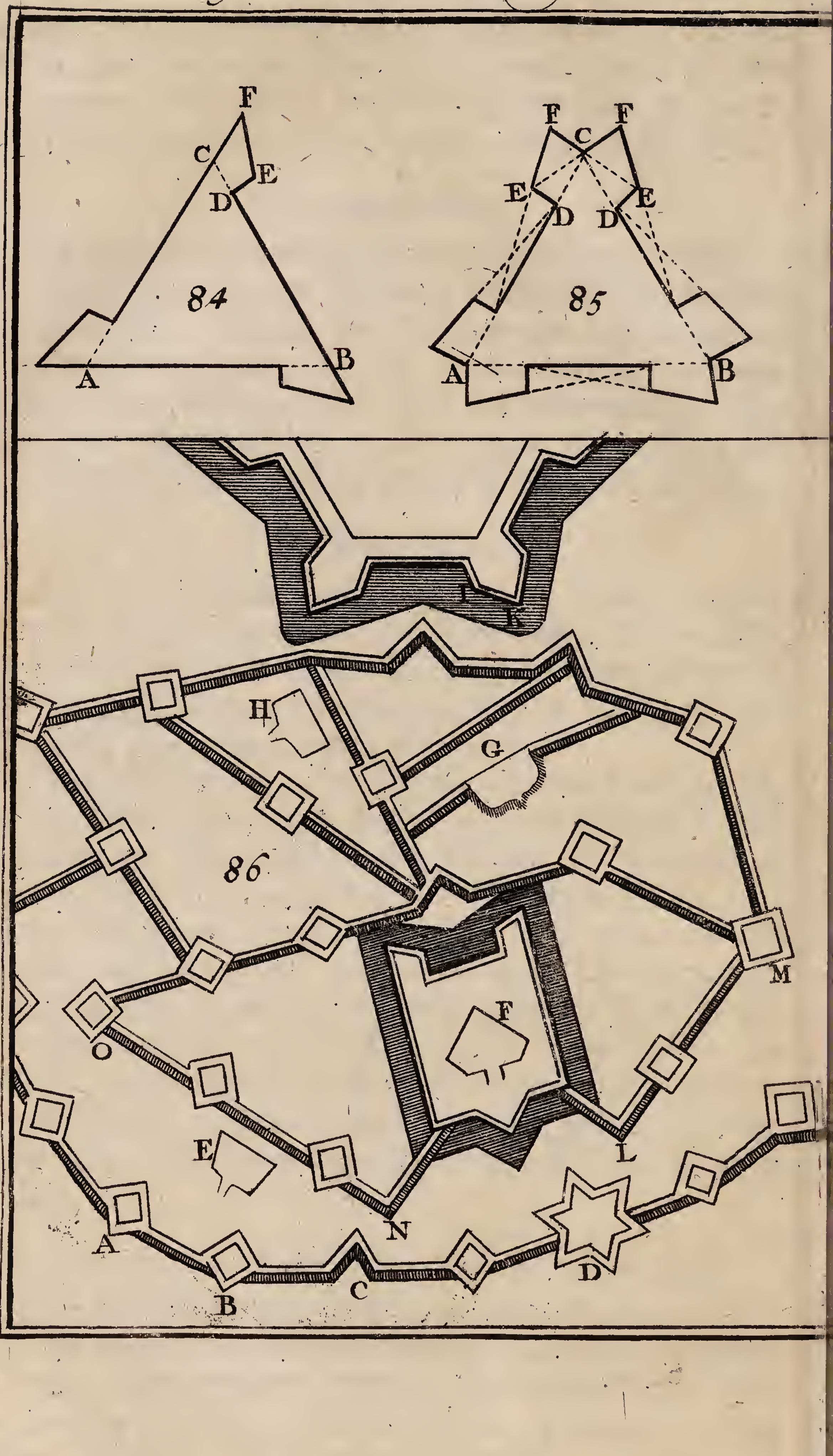
LAstly, An Equilateral Triangle, as ABC, may be well enough Fortified with a Re-inforc'd Tenaille, which may be made upon its Angles, when there is room enough in the Field, or time to spare, or the Prince will go to the Charge of it; thus,

Plate 38. Fig. 83. Having drawn from the Center D, the Radius DA, DB, DC, draw thro' each Angle to each Radius a Perpendicular, as EF equal to the Radius, so that each of the lines CE, CF, be equal to half the Radius CD.

Make the Demi-gorges EG, FG, each equal to the fifth Part of the side EF, and draw from the Center D, the Flanks GH, equal to the Demi-gorges. Last of all, draw the Rasant-lines GHI, to take upon them the Faces HI, each equal to the third Part of EF, and to draw thro' the points I, the Wings IK parallel to the Flanks GH, and the whole Triangle will be of good Defence.

The Fortification of one of the Quarters of an Army.

AS an Army Besieging a Town cannot Encamp all in one Place, it is divided into several Parts or Bodies of Troops, call'd *Quarters*, as well as the Ground which they are Encamp'd in, whose Figure is that of a long Square, which is more or less Fortified, as the Enemy


F
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is more or leſs to be fear'd, which we ſhall firſt ſpeak of, without any regard to the diſtributions of the Quarters and Lodgments, which is call'd *Pitching a Camp*, (in *French* Caſtrametation) becauſe that does not belong to Fortification, which we only pretend to treat of here.

Having diſpos'd the firſt Quarters in the moſt Advantageous Poſts, for Forage and Water, and the leaſt expos'd to the Enemy, and eſpecially in Places where Water is, if poſſible; ſuch Places as are too much expos'd to the Enemy muſt be Fortified with Redents, or Advances, and encompaſs'd with a Ditch, at leaſt 8 or 10 Foot wide, and 5 or 6 deep, with a Parapet likewiſe 5 or 6 Foot high, or higher, as of 8 or 9 Foot, and 8 or 10 Foot thick, if the Enemy be fear'd. This Parapet ought to have Two or Three Banquettes, to raiſe the Souldiers high enough when there is no Rampart, for ſometimes there is One made 4 or 5 Foot high, and 2 or 3 Toiſes thick: Care muſt be taken that the Ditch be always towards the Enemy, and the little Rampart upon its edge towards the Beſiegers.

Each Angle of this long Square may receive a Baſtion, and a flat Baſtion between, or only Advances, or Saliant Angles, taking care that all thoſe Works be not above 50 or 60 Toiſes from one another, that their defence may be the more certain, by reaſon of their ſmall height: And to be the more ſecure, towards ſuch Places as the Enemy is fear'd from, ſome Fort may be Erected to cover thoſe Works and that Quarter: Such a Fort may be like one of thoſe whoſe Conſtruction we have taught; and all the Quarters muſt always have betwixt them and the Enemy the Circumvallation, which we are going to ſpeak of.

Of the Circumvallation.

THE Quarters being order'd, you muſt conſider how many Forts will encloſe the Town, and form the Circumvallation, if you would have one; for as it is very Expenſive, and requires a deal of Time to be *Plate 39. Fig. 86.*

rais'd, it is usually neglected, unless there be a Strong Garrison in the Place, or the Enemy has an Army Strong enough to succour the Place.

The Figure of the Circumvallation depends upon the Quality of the Ground, and it ought to have as little Compass as possible, that it may be the easier Defended, by making the best use of the Nature and Disposition of the Ground. Forts must be stronger or lighter, as the Enemy is more or less fear'd.

Plate 39. *Fig.* 86. These Forts, if possible, must not be above 240, or or 250 Toises distant from one another, that is, not above twice the reach of Musket-shot, that the Musketteers from Two of them may reach that distance, those of each Fort shooting half-way: And if you shou'd be oblig'd to have them farther distant, then Tenailles must be made between, or Redoubts, as A,B, which must always be square, or in a Lozenge-form, and one of their sides must face the Field, or else one of their Angles, that the sides, which make those Angles, may be defended by the Line of Circumvallation.

These Forts being built in the most Advantageous Places, and the Redoubts and Tenailles, where there is need for them, the *Line of Communication* must be rais'd, which is so call'd, because it makes a Communication between all those Forts; Redoubts and Tenailles, being a continued Ditch, 12 or 15 Foot broad, and 5 or 6 deep, whose Earth is thrown up towards the Enemy to cover one's self, and by intervals made into Redoubts, or little Advances shap'd like Ravelins, as C, to flank the better.

Thus will the Circumvallation be finish'd, which, when a Succour only is fear'd, must be erected against that Succour, which is the most usual, with the Ditch towards the Field, and the Parapet towards the Place Besieged, upon the edge of the Line of Circumvallation. The Musketeers shoot over that Parapet, which for that reason must have One or Two Banquettes; or else it may have Embrasures of Brick, a Foot and a half wide on the outside.

But

But if there be a Strong Garriſon, or an Army ſhelter'd in the Place Beſieg'd, you muſt do the reverſe, and raiſe the Circumvallation againſt the Town, which, in ſuch a caſe, is call'd, *Contravallation*, and *Counter-Line*, as in the Figure. If the Enemy be fear'd on all ſides, a Second Circumvallation muſt be made, whoſe inward Line or Contravallation is of uſe againſt the Town, to prevent Sallies, and the outward Line is Fortified againſt the Field, to reſiſt the Enemy, who might advance to raiſe the Siege.

Betwixt the Circumvallation and the Contravallation, that is, betwixt the inward and outward Circumvallation, a Space muſt be left large enough for a Place of Arms, that when a Succour comes, there may be room enough to make the Baſtions deſign'd to ſuſtain the Enemy's ſhock, who might come as well from the Field as from the Place Beſieg'd.

When the Garriſon is Strong, the Beſiegers break Ground firſt by the Contravallation, to make it afterwards a Circumvallation. The Lines which run from one Work to another, are call'd, *Lines of Communication*, and the Ditch of the Contravallation, which is towards the Town, to hinder Sallies, is call'd, *Line within ſide*; and *Line without ſide* is the Ditch of the Circumvallation, which is towards the Field, to hinder Relief. Laſtly, The Line of Circumvallation is call'd, *Defenſive-Line*, becauſe it is only made to defend and cover one's ſelf: And the Line of Contravallation, which is between the Camp and the Place, and which ſecures the Beſiegers againſt the Town, is call'd *Offenſive-Line*, becauſe it affords a means to Attack the Place, by *opening the Trenches*, that is, beginning to work upon the Trenches, which have their *Tail* or beginning, always turn'd towards the Beſiegers.

The Conſtruction of the Forts is eaſy from what has been taught, and their Figure is a Square, or Pentagon, or at moſt an Hexagon, either with Baſtions, or only in a Star-form, as we have already ſaid; the Front ought to have at leaſt 10 Toiſes, and may be bigger, of 20

 Toiſes

Toiſes if you think fit, according as the Ground is, and the Number of Men that are to be a Guard in them, and then it muſt be conſider'd whether the Place be expos'd to the Enemy, or whether there be any likelyhood of his giving Attack that Way; for if it be ſo, the Fort muſt be made greater, and to hold more Men.

Lines of Circumvallation and Contravallation are but like a Trench, carried on from one Fort to another, and they muſt fall in about the middle of a Fort, to be *enfiladed* and defended by it. Thoſe of the inward Circumvallation, which are neareſt to the Place, muſt always be out of Cannon-ſhot, unleſs when you have *Rideaux* and cover'd Poſts; for then you may come nearer: And thoſe of the outward Circumvallation ought not to be very far, except, when you wou'd take in Eminences, from which the Enemies might Infeſt the Quarters, and Force the Lines.

Of Bridges for the Communication of the Quarters.

WHEN a great River paſſes thro' a Town Beſieg'd, Forts muſt be built on each ſide, eſpecially where the River comes into the Town, which place is higher than the Town, to cover the Bridge of Boats that is made over it, to go from one Fort to another, and that the Quarters of the Army may help one another: For without a Bridge and a Communication, the Army wou'd be too much divided, tho' but into Two Bodies, which the Enemy might more eaſily Attack and Defeat.

We have ſaid that this Bridge ought rather to be upon the upper, than upon the lower part of the River, becauſe a Bridge below, might be burn'd or deſtroy'd by a Boat fill'd with Fire-Works, or laden with great Stones, and ſent ſwiftly down from the City. This may be remedied ſeveral Ways, eſpecially by looſing the Cables which hold the Bridge, where you ſee a a Boat come, which puſhing the Bridge on one ſide will go by, and be of no effect.

This Bridge ought to be well guarded, Day and Night, and its breadth must be 20, or 25 Foot, that Two Carts a-breast may pass over it, with room to spare besides for the Foot. The Boats which sustain it, ought to be all of the same height and length, that the Bridge may be even, and of an equal breadth all over: They must be in a Right-line, and may be 12, or 15 Foot from one another. They are usually of Wood, sometimes of Copper, and joyn'd with Beams, which are cover'd with strong Planks.

Of Batteries.

WHEN you begin the Circumvallation, you must raise some Batteries against the Besieg'd, to support the Pioneers that work in the Trenches, behind which yon must have other Batteries nearer the Place Besieg'd, as G,H, to break its Defences, and dismount the Pieces. These ought not to be above 160 Toises distant from the Place, because otherwise their Guns wou'd not have strength enough to break the Defences, and ruin the Parapets. But they ought not to be too near the Place, least their shot shou'd graze over the Parapets, and be made upwards too obliquely. *Plate 39 Fig. 86.*

There are several sorts of Batteries, which we have spoken of elsewhere, and therefore shall not now Describe: We shall only say that buried Batteries are the most us'd, in carrying on Approaches, and ruining the Parapets and Defences of a Place; and that high Batteries are seldom us'd, except when you can play *de Revers* upon the Enemy's Fortifications, which pesters him sadly, and often obliges him to Surrender, as it happen'd at the Citadel of *Besancon*, besieg'd and taken by the King, in the Year 1674.

And that *Cross-Batteries*, are Two Batteries which play a-thwart one another upon the same thing, forming there a right Angle, or one pretty near it, and beating with more Violence and Destruction, because what one Bullet shakes, the other beats down. Pretty near such are the Two Batteries G,H, to make a Breach in the opposite Face KI.

And laſtly, that buried Batteries are made by digging in the Ground a place for the Cannons, and throwing up the Earth on either ſide towards the Town Beſieg'd. Theſe Batteries cannot be ſeen by the Enemy's Cannon, as the others are; but then they are not ſo effectual, becauſe they can only diſcover the tops of the Parapets rais'd above the Rampart.

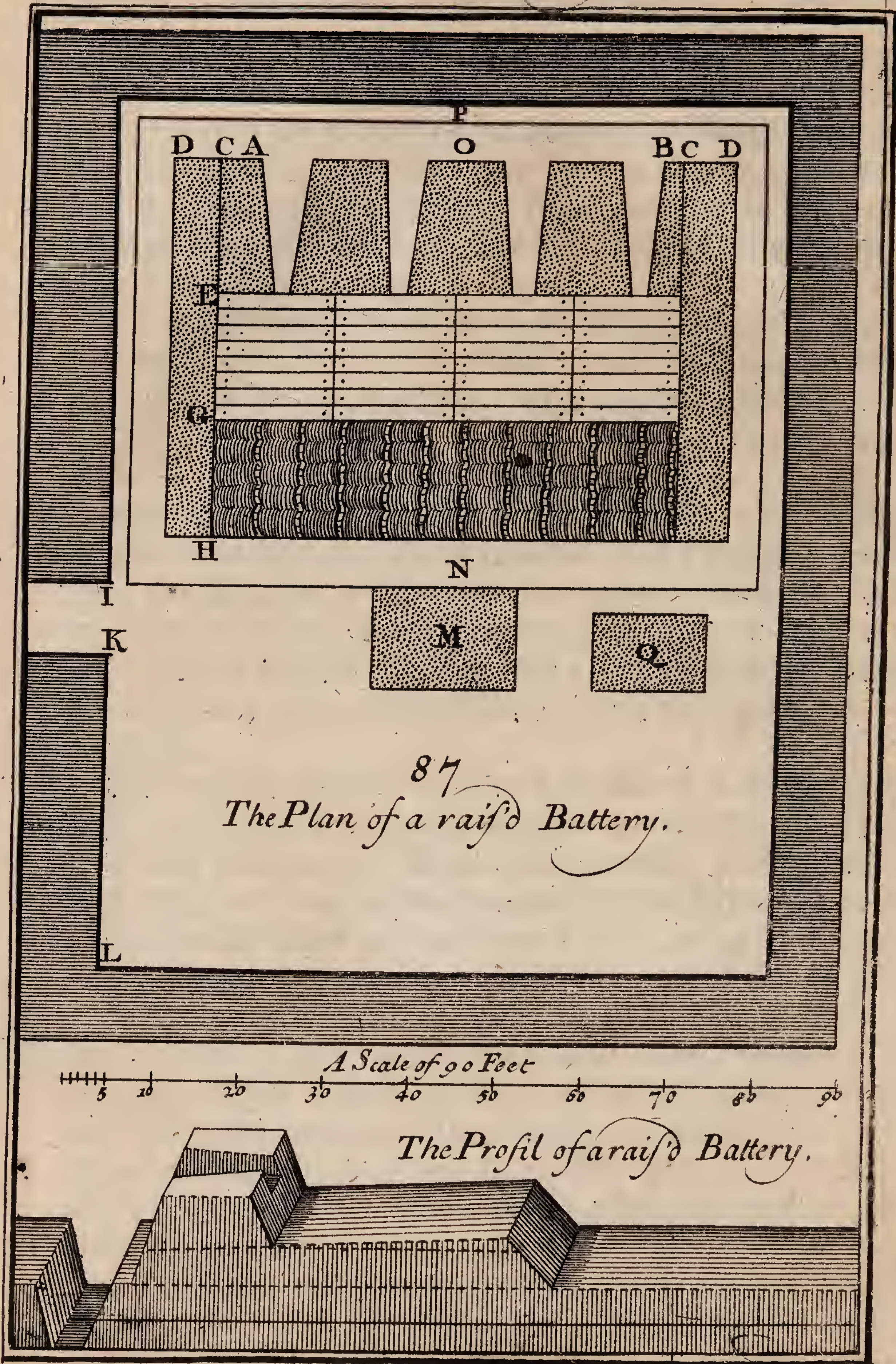
Batteries ought not to be very far from the Quarters, or Forts which defend them, that they may be ſafe againſt the Enemy's Sallies, who might come and *Nail up the Cannon*, that is, make it uſeleſs, by driving a great Nail, or Pebbles, into the Touch-hole.

When the Trenches are made firſt, the Battery ought to be betwixt the Trenches and a Redoubt, as E, that its Entrance may be the better defended. You may alſo ſhut up the Batteries in a Horn-Work, as F, or in any other Work that has a good Ditch, and may be defended by the Musketeers from the Sallies of the Beſieg'd.

One cannot exactly fix the breadth of a Battery, becauſe it depends upon the number of the Pieces which you wou'd ſet in it. When the Number is determin'd, multiply it by 12 for the breadth of the Battery, becauſe the Cannon muſt be 12 Foot from one another, that there may be room to ſerve and charge the Cannon.

Thus for 4 Pieces of Cannon, AB, the breadth of the Battery muſt be of about 48 Foot, without the length of the lines AC, BC, which is about 5 Foot, and CD the breadth of the Parapet, which is but of 6 Foot, becauſe it is on the ſide: For EC the breadth of the Parapet which covers the Cannons, ought to be of 15 Foot at leaſt. *Plate* 40. *Fig.* 87.

But EH the depth of the Battery, is always Thirty Foot over, or thereabouts, becauſe the Cannons mounted upon their Carriages, are uſually 15, or 18 Foot long, and it muſt have 10, or 12 Foot to recoil in. The breadth EG of the Floor is of about 15 Foot, and conſequently, GH the remaining part, is alſo of Fifteen Foot. The Entrance IK, is of Twelve Foot, and


P
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87
The Plan of a rais'd Battery.
A Scale of 90 Feet
5 10 20 30 40 50 60 70 80 90
The Profil of a rais'd Battery.

and the empty ſpace remaining, of about Thirty Five Foot, as KL. *Plate* 40. *Fig.* 87.

We had already ſaid that EG the Floor, or Bed of the Battery muſt be made of good Oaken Planks, nail'd a-croſs Beams, to hinder the Wheels of the Carriages from ſinking into the Ground; and that this Floor or Platform muſt incline a little, as about One Foot, towards the Parapet, to check the recoiling of the Pieces, and that they may eaſily be reſtor'd to their places, and made ready to fire.

Here you muſt Note, that GH, the remaining part of the Battery, is cover'd with ſtrong *Hurdles*, which are made of Boughs of Trees, ſtrongly Work'd together, that one may walk upon them without being dirtied by the Earth, which is very troubleſom, when it is Clay.

A Bridge, or Way IK, is made in the Ditch, to go into the Battery, about 12 Foot wide, and behind the Battery a ſpace Q is digg'd in the Earth, to put Ammunition in: It is cover'd over with Leather, or Hair-cloaths, to keep fire from the Powder, which muſt be there but in a little quantity at a time.

M, Repreſents the Steps to go up to the Platform of the Battery. The reſt may be eaſily conceiv'd by a ſight of the Figure, whoſe meaſures as well as thoſe of the Profil, at bottom, may be known, if you meaſure upon the Scale, which is common to the Plan and Profil, N, the breadth of the inward Talu, O, the breadth of the outward, P, that of the Berme, or Liziere, *&c.*

The Parapet which covers the Cannons muſt have as many Embraſures as Pieces, as here Four Embraſures for Four Cannons. The Parapet is Six Foot high, the Embraſures Three, which are ſhut up with great Planks Musket-proof, that the Enemy may not ſee what is done in the Batteries: And to deceive him the better, you may make more Port-holes than they have Cannons.

To batter a Flank, it is uſually oppos'd by as many more Cannons as thoſe in it: And becauſe Batteries, made for that deſign, are oppoſite to one of the Flanks of

Plate 40. Fig. 87. of the Place, they ought to have a little Parapet on that side, as AC, or BC, which we have made but Six Foot wide, and it ought to be of the same height, to secure those that look after the Cannons.

Sometimes we are oblig'd to make high Batteries, either by the Situation of the Ground, or the Enemy's Works, or to play upon him *de Revers*, which, in such a case, are call'd Cavaliers, or Platforms: To work at them, pieces of Cloath must be extended upon upright Poles, to hide the Work-men; and you may hang up several pieces where you don't design to Work, to deceive the Besieg'd.

Of Trenches.

THE Circumvallation being ended, and furnish'd with Forts, Redoubts, Fortins, Sconces, *&c.* If the Place is to be taken by Famine, there is nothing more to do, but to dispose the Artillery in the most convenient Posts, and to give orders for the Subsistence of the Army; but if the Place is to be forc'd, Trenches must be carried on, which are also call'd *Lines of Approach*, or only *Approaches*, which you must begin 140, or 150 Toises off of the Counterscarp, to avoid Musket-shot, and this must be done in the Night, when the place has been pitch'd upon by Day; and, in such a case, a good Geometrician has a great Advantage, because when he has well observ'd the Situation, by the help of a Magnetick Needle, he may make his Trenches in such manner, that they shall not be *enfiladed*, which is the greatest fault in Trenches. Besides, because Souldiers are to mount the Guard in the Trenches, they ought to be full Six or Seven Foot deep, and Ten or Twelve broad, and the Earth digg'd out of them must be thrown up towards the Enemy, to make a kind of Parapet. The Two chief Trenches here are LM, NO,
Plate 39. Fig. 86. between which there is an Horn-Work built for the Souldiers to assemble in.

It is proper to make upon the Wings of each Trench, towards the Field, *Lodgments* or *Epaulements*, after the

ne manner of Traverses, as well to keep in the Be- *Plate* 39.
eg'd, as to favour the carrying on of the Trenches, *Fig.* 86.
y supporting the Pioneers. These Lodgments or Epaulements are small Trenches, which face the place Besieg'd, and have one end in the great Trenches.

Platforms for Batteries are made behind the Trenches, s we have said elsewhere: thus the first are at some istance from the Place, as E, and are only design'd gainst Sallies: Then as the Trenches are carried on nearer, Batteries are erected nearer, as GH, to ruin the *Defences*, that is, the Parapets, and to dismount the Artillery of the Place. Last of all, the Batteries to make a Breach, are such as are nearest to the Counterscarp, as we shall say hereafter.

Trenches that are carried on in winding lines, are good, but they are not so soon made as those that are continued in the same line. Their lines ought to be defended with Redoubts at every 100 Toises, into which one may retire, if repell'd by too vigorous a Sally. The Parapet of these Redoubts ought to be higher than that of the Trenches, and stronger, with a Ditch round about; and also 4 or 5 Foot higher than the level of the Field.

After the Redoubt, which is to flank Two Trenches, you may alter your direction, and turn to the right, if the Trench has been carried on to the left, as at O: Or turning to the left, if the Trench has been carried on to the right, as at N. If necessity obliges you to make any *enfiladed* Trenches, you must cover them from the Enemy, with Gabions, or Fascines: Or at least every now and then raise a Parapet to cover part of such a Trench.

These Fascines are rang'd and heap'd up upon Two *Plate* 41.
or more *Chandeliers*, which are upright Stakes, as AB, *Fig.* 89.
CD, rais'd upon a piece of Wood, as EF, to support Planks, Boughs, Fascines, and generally all that can cover the Besiegers, to hinder the Enemy from seeing what is done behind.

When

When the Trenches are at the Glacis, and face the Place Besieg'd, the Pioneers are cover'd above with *Blinds*, which are pieces of Wood laid a-cross over the Trench, to sustain the Bavins, or Hurdles, laden with Earth, which cover the Work-men, and defend them from the Fire-Works and Stones, which the Enemy might throw upon them.

Plate 41. *Fig.* 87. The little Trenches, which run from one Trench to the other, when there are Two, to defend with more ease that which shou'd be first Attack'd by the Besieged, are call'd *Boyaux*, which must have also Redoubts, or Demi-Redoubts at several distances. This *Boyau* ought to be carried on, getting Ground towards the Place: and as they might be *enfil'd* near the Counterscarps, they must be made parallel to the Curtains of the Place, to fire upon those that may appear upon the Rampart, and so favour the Sappe and the Lodging upon the Counterscarp.

Of Attacks of Approach.

AN *Attack of Approach*, or *Attack of a Siege*, is the carrying on of Trenches made to cover one's self in approaching a Place that one wou'd become Master of: Tho' you wou'd make but one Attack, yet several must be begun, to decieve the Enemy, divide his Forces, and favour the true Attacks, which are call'd *Right Attacks*, when they are made in form, that is, carried on with Regular Works, such as we shall speak of; other Attacks, which are feign'd, are call'd *False Attacks*.

The Attacks of *Fig.* 86. having been only lightly drawn, to shew you the way to carry on Trenches, and Forts which defend them, is not sufficient to give you a true Notion of a Right Attack; wherefore we will give you other Figures, where this way of Attacking will be explain'd more particularly, and exactly, according to the different Methods, which have seem'd best to me, and which are most esteem'd of.

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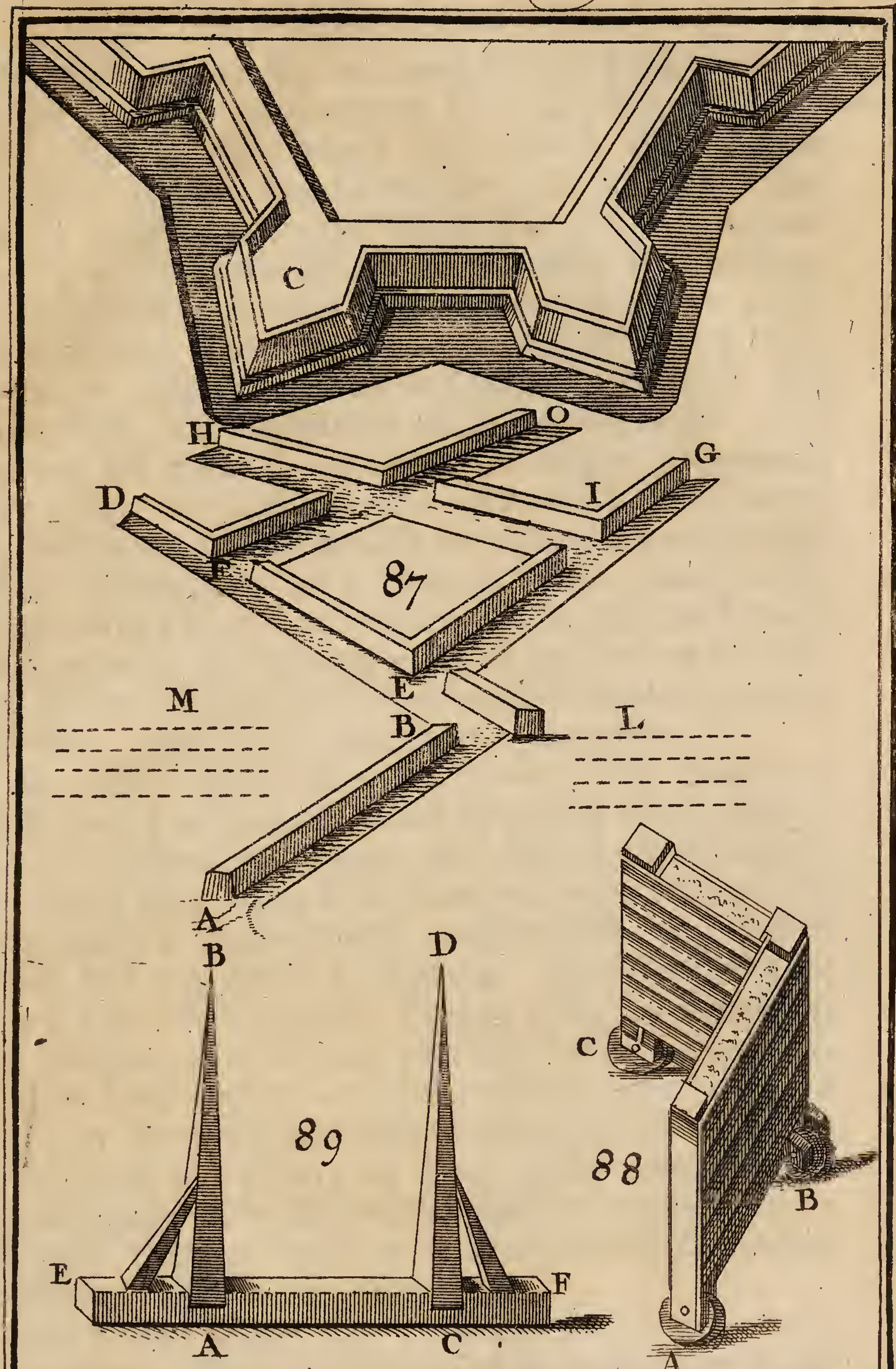
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The First Kind of Attacks.

WE ſhall begin by the moſt Simple Attack, which you may comprehend by barely ſeeing the Figure. Having then begun the Trench AB, within Muſket or Cannon-ſhot, or farther off, when you fear leaſt it ſhou'd annoy the Work-men, in ſuch manner that it be not *enfiladed*, or ſeen directly from the oppoſite Baſtion C, (otherwiſe its Parapet wou'd not cover the Souldiers, and it wou'd be impoſſible to ſtay in it) you may make at its end B a Redoubt to flank that Trench AB, and its turning off at E quite to D. This Trench AB muſt be begun in the Night, appointing 4 or 5 Foot for each Pioneer, who muſt there cover himſelf as faſt as he can with the Earth, which he muſt throw before him for a Parapet, which need not be higher than 3 Foot above the Ground, becauſe the Trench being as deep, the Parapet will be 6 Foot above the bottom of the Trench, and conſequently capable to cover the Men, wherefore it muſt have a Banquette. *Plate* 41. *Fig.* 87.

On either ſide of the firſt Trench AB, Bodies of Horſe and Foot as L,M, muſt be poſted about 200 or 300 in number, to ſupport and defend the Pioneers, who work thus; The firſt work upon their Knees, and only make a little Ditch, which thoſe that follow widen and dig down by degrees. The Work muſt be follow'd ſo cloſe that the Treneh with the Redoubts, and Lodgments in it, may be ready before Day, to defend, if need be, as well the Souldiers that work in them, as Batteries that may be made in them, to continue more powerfully by Day, againſt the Sallies of the Beſieged. The Work-men muſt be chang'd every Morning, that the freſh Pioneers may end by Day, what the others begun in the Night.

The Places where the Trenches are to be open'd, muſt be mark'd by the Marſhal of the Camp, or by the General: And if there be any Houſe which can cover from the Artillery of the Place, there the Trenches may be open'd, (if there be but a little ſpace of Ground

en-

enfil'd by the Besieg'd in your way to it) and the Pioneers sent under cover of *Mantelets*, which are great wooden Planks, usually of Oak, 5 Foot high, 3 Foot wide, and about 3 Inches thick, to be Musket-proof: And to make them stronger, their thickness is increas'd with Two or Three Planks bound together with Iron.

Plate 41. Fig. 88. These are call'd *single Mantelets*; for *double* ones, as ABC, are made with Planks that are double, and have Earth between them, and which are us'd in Approaches, and in making Batteries near the Place Besieg'd, these are upright upon Wheels, and Men wheel them before them, where-ever they please.

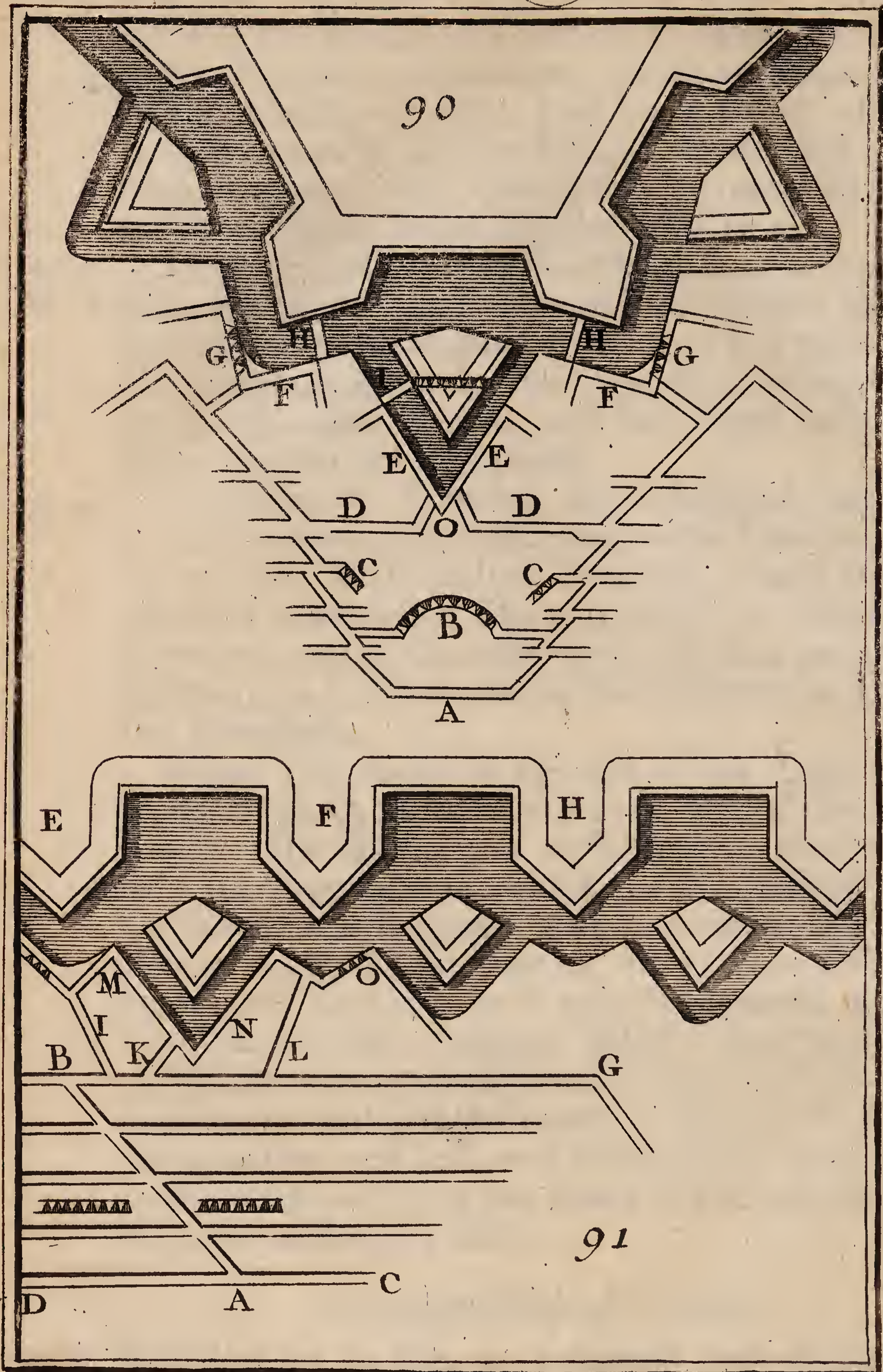
Fig. 87. By *Opening the Trenches*, we understand, beginning to work in them, and by *Carrying on the Trenches*, going on with the Work in the Trenches, whose end as B, which is towards the Place, is call'd the *Head of the Trenches*; and A, the other end, which is towards the Besiegers, is call'd the *Tail of the Trenches*, as we have said elsewhere.

When you come to the end of the Line of Approaches, as O, H, within 5 or 6 Foot of the Saliant Angle of the Glacis, you must begin to pierce or dig thro' the Counterscarp, which is call'd *Sapping*, to pass the Glacis, and so open a Way to come cover'd to the passage of the Ditch, after all the Enemies Efforts to hinder the Trenches have been surmounted, and that in spight of their frequent Sallies, they have been carried on as far as the Esplanade, which will be done so much more easily, if the Lines or Boyaux IH, FO, be made, and be well lin'd with Musketeers, who by their constant Fire may keep the Besieg'd from opposing your design of passing the Ditch.

The Second Kind of Attacks.

Plate 42. Fig 90. YOU have in *Fig.* 90. a Second Sort of Attacks, which is understood as soon as seen: Wherefore we shall use but few Words to explain them.

First, you must describe a great Place of Arms, as A, 60 Toises long, Musket-shot distant from the last Out-Works,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Works, and parallel to the Curtain which joyns the Two Baſtions, to which you wou'd carry on the Two Attacks, which this Place of Arms is common to, where you may place Two Battalions of Foot, One at each end, and in the middle a Squadron of Horſe. *Plate* 42. *Fig.* 90.

The Two Trenches are made at the end of the Place of Arms in a ſtreight line, or a little inclining to the Place Attack'd, according to the length of the outward ſide of the Place, yet in ſuch manner as not to be *enfil'd* from it, upon each of which are to be made little Places of Arms parallel to the firſt, which is the biggeſt of all, 25 Toiſes diſtant from one another, and 25 Toiſes long.

Between the Two firſt little Places of Arms there are Boyaux parallel to the great one, between which you ſee a great Battery B, capable of 10 Pieces of Cannon, made in an Arch, or in the form of part of a Circle, that it may play every way, and it has a Communication with the Two Boyaux, that you may go cover'd to it.

As the ends of the third Place of Arms within ſide, are Two little Batteries C, each capable of Two Pieces of Cannon, to break the Faces of the Ravelin, which may be done with the more eaſe the nearer they are to a parallel to thoſe Faces, becauſe Balls ſhot at right Angles do more Execution than ſuch as are ſhot obliquely.

At the ends of the fourth Place of Arms within ſide, Two long Boyaux D are drawn parallel to the Curtain, which come towards O in a Right-line, each of them of about 10 Toiſes, to cover the Musketeers, who, as we have ſaid elſewhere, muſt make a continual Fire, whilſt Lodgments are made upon the Counterſcarp, as E,F, which muſt have a Communication with the Place of Arms, as E, or with the Trenches, as F.

This continual Fire, with that of the four laſt Places of Arms of the Two Attacks, and the Three Batteries, will hinder the Musketeers of the Place from firing upon thoſe that Work, cauſing them to loſe their ſhots in the Air:

Air: Now, tho' the Cannon must Fire continually, yet it must not shoot *par Camarade*, that is, all the Pieces ought not to Fire at the same time, but one after another, otherwise their Fire wou'd not be continual, because there is some time requir'd to charge and restore them to their places.

To hinder the Lodgment upon the Saliant Angles of the Counterscarp from being seen and *enfiladed* of the Place, or from the Out-Works, they must be cover'd with good Epaulements Cannon-proof as H,I, and Two others are to be rais'd also, to facilitate the passing of the Ditch, in order to bring up the Miner, or Storm, or to *give Assault*, that is, to mount the Breach, or scale the Place. The Lodgments F, made upon the Saliant Angle of the Counterscarp of the great Ditch, ought to have Two Batteries G, of Two Pieces of Cannon each, to break the Flanks of the opposite Bastions, without which it is very hard to pass the Ditch.

While you are carrying on the Two Attacks, the Work must go on equally on each side, that the Two Attacks may be ready at one time, and that the Besieg'd by their Sallies may not take the weakest, which wou'd be that which is least advanc'd. All the Places of Arms must be 15 Foot broad, and 4 deep, with a Parapet 3 Foot, or more, above the level of the Field, and there must be in all about 9 Foot, from the bottom of the Place of Arms to the top of its Parapet, because the Earth cannot be press'd and beaten together so well as that of the Rampart and Out-Works of Places, which are made more leisurely and freely.

The Squadron of Horse, and the Two Battalions of Foot, which we have posted in the great Place of Arms A, serve to defend the Trenches: And to go on in order, as soon as the Two first little Places of Arms of the Two Attacks are made, half of each Battalion must be plac'd in them, who must stay there 24 Hours: And when the Two following Places of Arms towards the Town are made, these Two last Battalions must be posted in them, that their place may be supplied by those

those that were left in the great Place of Arms: And likewise when the two third Places of Arms are made, the Battalions of the two second must go into them, and their places must be supply'd by those of the two first, which in the mean-time must be taken up by half the Horse, and so on.

The Third Kind of Attacks.

THIS Third Kind of Attacks is good, when the Place to be Attack'd has a very long outward side, for in such a case you cannot Attack with Two straight Trenches, because it wou'd be hard to hinder One of them from being *enfil'd* by the Place; therefore One only shall be made in a straight line, as AB, along the middle of the first Place of Arms CD, and 100 Toises distant from the outward side, in such manner that the Trench AB be not *enfil'd* from the opposite Bastion E; the rest is easy to be understood by the Figure. *Plate 42. Fig. 91.*

The Two Batteries which are between the second and third Place of Arms, are each capable of 5 Pieces of Cannon. All the *Boyaux* betwixt each Place of Arms, which are equal parts of the Trenches, are, as before said, 25 Toises long, 6 Foot wide, and 4 deep. These first Batteries are usually rais'd upon some Eminence, if the Ground has any, because then they will play less obliquely, and almost level with the Parapet of the Rampart.

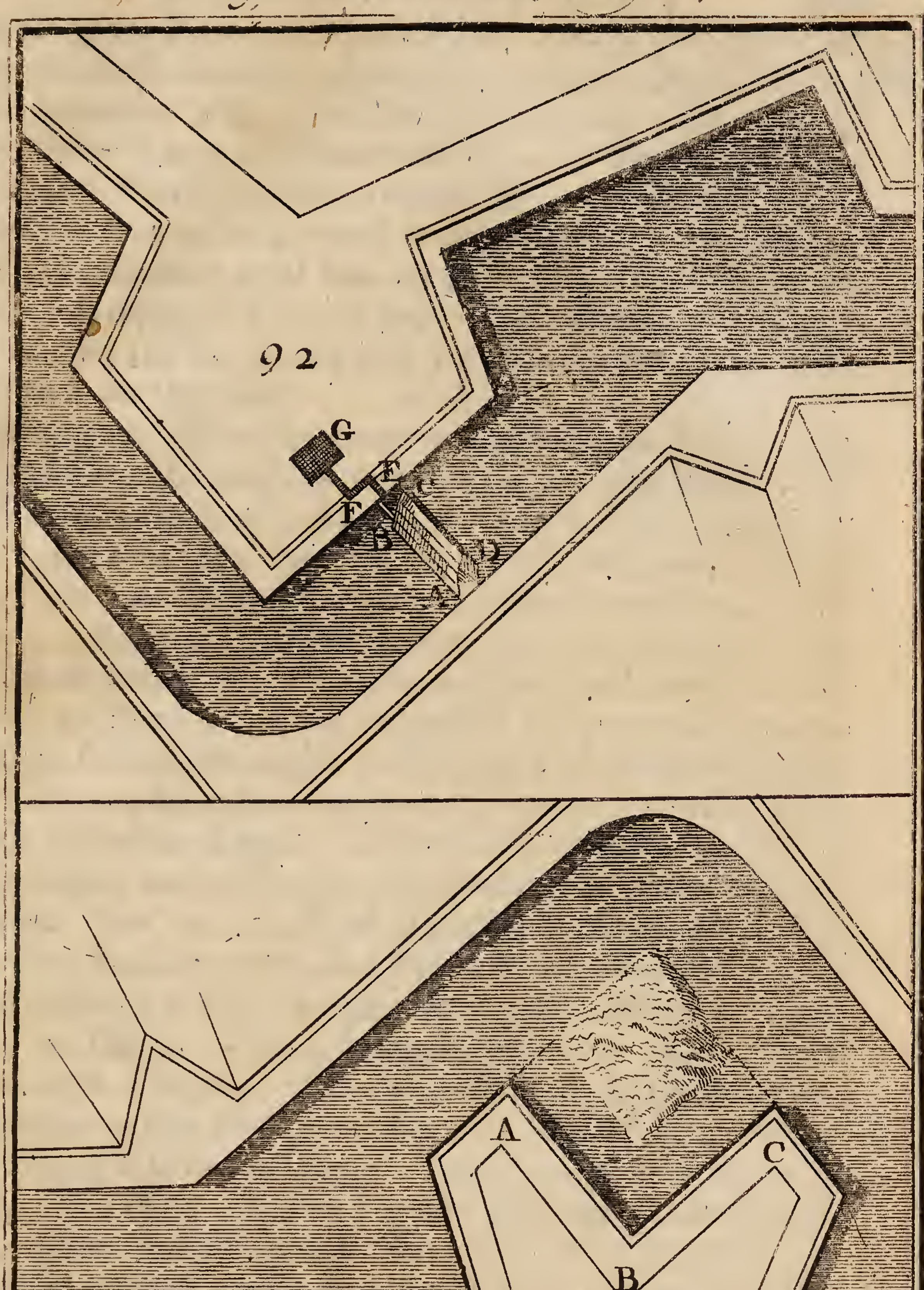
The Places of Arms, which are nearer the Place besieg'd, must be a little longer towards the Right-Hand than those that are farther off, so that the nearest and last GB be the longest of them, to be the better cover'd in approaching the Bastion F, without being *enfiladed* from the other Bastion H. Upon this Place GB must be made 3 Trenches, or *Boyaux* I,K,L, to communicate with the Lodgments M,N,O, which, as in the foregoing Attacks, are made upon the Three Saliant Angles of the Counterscarp, the Artillery of which Lodgments will serve to ruin the Defences of the Flanks,

and thence the Counterſcarp may be cut to go into the Ditch, to make a Gallery in it, and bring up the Miner to the Face of the Baſtion, as we ſhall ſhew more particularly.

Of the Sappe, Gallery, and Mines.

Plate 43. *Fig.* 92. WE have already ſhew'd what a *Mine* is, and here we ſhall tell you, that the *Sappe* is a deep Trench made in the Earth deſcending, begun within 5 or 6 Foot of the Saliant-Angle of the Glacis, to be cover'd in coming into the Ditch: And that the *Gallery*, call'd alſo *Traverſe*, when it is made to croſs the Ditch, as here, is a long Paſſage cover'd over with Earth, to keep off Stones and Fire-Works, and defended on each ſide with good thick Planks Musket-proof, made in the Ditch for the Miner to go thro', when the Defence of the Flanks of the oppoſite Baſtion are deſtroy'd, and there is no fear of Cannon-ſhot, as ABCD.

The Trenches then being carried on as far as the Counterſcarp, where the chief Trench is made, with ſeveral well cover'd Lodgments, the Beſieg'd muſt be driven from it, with *Fourneaux*, or otherwiſe breaking or cutting the Counterſcarp in ſome places, to lodge one's-ſelf in it. Then if you wou'd make a Breach to give Aſſault, play the Batteries: But if you wou'd blow up the Rampart with Mines, you muſt carry on a Gallery thwart the Ditch, and for that purpoſe a cover'd Deſcent is made under the Counterſcarp, cutting thro' it over-againſt the Face of the Baſtion, as privately as may be, then in the Night muſt be ſet up the firſt Piles of the Gallery, which are half a Foot thick, and Three Foot diſtant from one another, to faſten to them the Planks, which the Carpenters muſt have ready cut, of a due length; and to avoid Musket-ſhot, you muſt be cover'd with Mantelets, or elſe Earth and Faſcines muſt be thrown into the Ditch, ſo as to make a little kind of Mount, which muſt be divided by degrees for the place of the Gallery, which will be ſo much the Stronger, if you cover the Piles, or Joiſts, with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with good Planks on both sides, and fill the Space between with Earth, especially towards the Curtain; and because, when the Ditch is pretty wide, part of the Gallery may be discover'd from the Cannons of the opposite Flank, which cannot all be destroy'd, especially when it is cover'd with an Orillon; in such a case the Gallery must be cover'd with a Traverse, made like a Parapet, Two or Three Toises thick, which may be built with the Earth and Fascines, that have been thrown on that side. *Plate 43. Fig. 92.*

Going on after this manner every Night, and by Day if possible, the Gallery must be fix'd to the Face of the Bastion, which is rather to be attack'd than the Curtain, because the Curtain is better defended, being between two Bastions which Flank it, and also its defence is shorter, and consequently more dangerous for the Besiegers. The Gallery must be well cover'd with Turfs or Tin, to secure it against Fire-works; and to hinder such Fire-works or Stones, which the Besiegers may throw, from staying upon it, its Roof must be made with an acute Angle. This Gallery must be 7 or 8 Foot high, and as broad, or broader, to have the freer passage thro' it. Last of all, care must be taken that all the Planks towards the Flank be Musket-proof.

A Gallery is fitter to pass over a dry Ditch than one that is full of Water, because Water is often pass'd over with a Bridge without a Gallery, little hurt being done to the Besiegers, because the Water hinders the Besieg'd from Sallying. This Bridge must be made solid, by filling the Ditch up to the top of the Water or higher with Fascines, that must have great Stones in them to make them sink; and this will be the safest Bridge. This must be done whilst the Miners, who can easily pass the Water, are at work in the Mine, which as soon as it is sprung, you go over this Bridge to give Assault; upon this Bridge, may be set a Gallery of Boards, or of Boughs only, that the Besieg'd may not see you.

Plate 43. *Fig.* 92. To paſs a full Ditch, ſometimes the Courſe of the Water may be turn'd, and when the Field is lower than the Ditch, it may eaſily be drain'd; in doing which you muſt have a care that you don't drown the Lines. When the Ground will bear it, the Ditch may be paſs'd by a Gallery or Mine under Ground, and ſeveral other ways which one may invent, according to the Nature of the Place, and quality of the Ground.

When the Gallery is fix'd to the Baſtion, to baulk the Beſiegers Aim, another Gallery may be made at the Foot of the Scarp, and carried on towards the Point of the Baſtion, that they may not gueſs where-about the Rampart is pierc'd thro' for the Mine; or elſe to pierce it in ſeveral places, by ſeveral Mines, or little Fourneau, made one beyond another, to gain, by degrees, the Retrenchments of the Beſieg'd, and thus become Maſter of the whole Baſtion. Whatever place the Rampart is pierc'd thro' in, the * *Head* E muſt be drove very narrow, as the breadth of 4 Foot only, but juſt large enough for one Man at a time to roll a Barrel of Powder thro', and its height muſt be of about 4 Foot, ſo that Men muſt work in them upon their Knees, and the Earth muſt be put into little Paniers, which the Miners reach to one another between their Legs, to take it out with all the ſpeed poſſible.

The Head of the Mine being continu'd about four or five Foot in a ſtraight line, you muſt turn it off upon the right or left, as EF, which ſpace muſt be of 10 Foot, winding ſtill at every 10 Foot, till you come to the place, which you intend to make the Chamber for the Powder: and as ſoon as you are come to the Earth of the Rampart, you muſt ſet it over head and on the ſides with Timber, to keep it from falling in. Theſe Windings muſt be ſhut as ſoon as the Mine is charg'd, to hinder the force of the Powder from ſpending it ſelf that way.

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* *Driving a* Head, *Signifies making a Paſſage under Ground.*

There

There need be no Windings if you drive your Head from a *Cascan*, which is a Hole digg'd in the Ground in the form of a Well, where the Head or Paſſage to the Mine begins. Counter-mines to give Air to the Enemy's Mines, are alſo made thus:

When you are far enough in the Rampart to make the *Chamber* or *Fourneau*, which is uſually a Cube in ſhape, as G, make it 6 Foot high, about 4 broad and 4 long. When you fear leaſt the Enemy ſhou'd give Air to the Mine, make it in the form of a Croſs, or a Gallows, that the fire may have a Paſſage upwards. If its ſhape be Cubical, it will be able to contain 4000 or 5000 pounds of Powder, tho' there is ſeldom ſo much put into a Mine, and commonly not above 1200 or 1500 pounds, which is enough to blow up a Rampart 12 Toiſes thick, or thicker, if the Mine be about the Middle of its Solidity, or ſomething nearer the Ditch, and level with the bottom of the Ditch, if poſſible. *Plate* 43. *Fig.* 92.

You muſt not forget to ſupport the upper part of the Chamber with a *Sommer*, or *Madrier*, and to ſet it round with Timber, leaſt the Earth ſhou'd fall in and fill it: For that Sommer, which is a ſtrong Beam, by its reſiſtance will help to blow up the Rampart; but you muſt leave a little Paſſage for the *Saucidge*, which is a long piece of Cloth, ſew'd round in the form of a Gut, and dipp'd in Pitch, of about Two Inches Diameter, and fill'd with fine Powder, which reaches from the Fourneau to the beginning of the Mine, where it muſt be Fir'd to be Sprung.

If the bottom of the Chamber be Watery, as it commonly happens, make a Floor for it, with Planks, to keep the Powder from being wet, which you muſt put in but juſt before you ſpring the Mine, becauſe if it ſhou'd lie long there, the moiſtneſs of the Earth might weaken it, or the Enemy might find it and take it away, or ſpoil it.

The Powder that the Chamber of the Mine is charg'd with, muſt be in pitch'd Bags, or Barrels, that muſt

be open in ſome places as well as the Bags, and a great deal of Powder is to be ſcatter'd about them, that all may fire at once, by means of the Saucidge.

Having then charg'd the Fourneau with as many *Caiſſons*, or Barrels of Powder, as are needful, its Mouth or Entrance muſt be well ſtop'd with ſound Earth, ſupported by croſs ſtrong pieces of Timber, having firſt fill'd all the empty ſpace in the Fourneau with Stones, or great pieces of Timber, forc'd and wedg'd in; but you muſt always leave room for a wooden Pipe, which you muſt prime as you ſtop up the Entrance: At the end of this Pipe towards the Ditch, when it's time, thruſt in a ſlow Fuſe, or a Match, long enough to laſt a quarter of an Hour, or as long as one muſt be a getting away after it is lighted.

When the Mine is Sprung, if it makes a good Breach, you muſt give Aſſault, to lodge your ſelf in it, poſting there as many good Souldiers as it will hold; that is, in caſe that you hope for a Capitulation: For otherwiſe it wou'd be better to give a warm Aſſault during the Fright of the Beſieg'd, without allowing them time to Defend and Retrench themſelves.

A *Mine-Dial* is us'd to drive a Mine under Ground to the Place propos'd, after you have with an Univerſal Inſtrument, or ſome other way, taken the Diſtance from the Place where you begin to drive your Head, to the Place that you wou'd blow up. Several difficulties may be met with in digging a Mine, but thoſe that are us'd to it will eaſily maſter them, without any further Directions.

The

The Sixth Part.

OF

Fortification Defensive.

THO' what has been hitherto ſaid concerning the Fortification of Places, ſeems to be the Defenſive part of Fortification, yet it ſhou'd rather be call'd the *Conſervation of Places*, as being only a diſpoſition towards Defence. Therefore we ſhall call *Defenſive Fortification* or the *Defence of Places*, the Way to defend ones-ſelf and oppoſe the Enemy, who wou'd make himſelf Maſter of a Strong Place: which is done by building Works like, or different from thoſe hitherto mention'd, as Retrenchments, Counter-trenches, Counter-mines, *&c.* which we ſhall ſpeak of in a few Words.

Of Retrenchments.

A Retrenchment is any Work whatever, made uſe of *to Retrench*, that is, to cover ones-ſelf, and Fortify within the Town the neareſt place to that which is attack'd: this Work is uſually a *Retirade*, that is, a Ditch border'd with a Parapet, whence we call *Retrench'd Quarter*, that place which is Fortified with a Parapet and Ditch.

Retrenchments are either *General*, when you retrench a whole Piece of Fortification where the Enemy has made a Lodgment, and make new Fortifications to cover your ſelf againſt the Enemy, and diſpute every Foot of Ground, in expectation of Succour: or *Parti-*

Plate 43. Fig. 93. *cular*, when you only recede from that part which has been taken, to make a new Retrenchment at a reaſonable diſtance, to withſtand and repel the Enemy: as ABC, which is alſo call'd a *Cut* and *Retirade*, becauſe it is made with a re-entrant Angle or Tenaille.

Retrenchments are the laſt Remedy us'd, when you are forc'd to give way and forſake the Head or ſide of a Work, which you muſt do but upon the laſt extremity, for all the means imaginable ought to be us'd to prevent the Enemy from lodging in it, as Paliſſado's, Faſcines, Barrels of Earth, or Sacks of Earth, or any thing which may cover the Musketeers, who are to ſtop the Enemy when the Defences are broken: which is it ſelf a kind of Retrenchment or *Barricado*, which defends thoſe that fight with the Enemy, whilſt a good Retrenchment is making, which muſt always have a Ditch towards the Enemy, and be open towards the Town, from ſome part of which it muſt be Flank'd: As
Plate 44. Fig. 94. AB, and CDE, the firſt of which is made in a Baſtion, and the other in a Ravelin, where you may alſo retrench your ſelf with a little Ravelin, when the firſt is very large.

Fig. 95. Or elſe as AB, CD, which is in a Horn-Work; where you may Retrench ſeveral ways; the Retrenchment may be made like the Piece taken, and Two or Three may be made after one another, being always careful leaſt the Enemy ſhou'd attack you behind, that is, attack the laſt Retrenchment inſtead of the firſt. It is well to make that Retrenchment before-hand, as Count *Pagan* does, who upon the Baſtion makes a *double Baſtion*, that is, a little Baſtion within the great one, becauſe ſuch a Retrenchment having been made at leiſure, is much Stronger than one made in haſt, which cannot reſiſt ſo well, becauſe the Earth is not ſettl'd; beſides a Retrenchment made before-hand, requires no more Souldiers to defend it, than if the Work was not Retrench'd: becauſe the Retrenchment is only defended when the main Work is taken.

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We ſhall not here teach how to make theſe Retrenchments, becauſe whoever comprehends what we have hitherto ſaid concerning Fortifications, and eſpecially ſuch as are skill'd in the practical part, will eaſily find means to retrench, according to the quality of the place attack'd, and the ſpace that is left to retrench in: beſides Nature teaches any body how to retrench and cover himſelf againſt the Enemy. Therefore I think it uſeleſs to add new Figures, and be too tedious about the different ways that a Retrenchment may be made; I ſhall only ſay that the Profils of the Retrenchments are moſt commonly like thoſe of the Out-works, and that if you have not time to make thoſe Retrenchments according to the meaſures that belong to the Profils of the Out-Works; that is, if you can't make the Ditch ſo deep, and the Parapet ſo high, you muſt cover your ſelf with Gabions, Sacks of Earth, or Dung, with Beams, Hogſheads, Carts, and any thing that may bear a ſhock. But to avoid thoſe Inconveniences, the Retrenchments ſhou'd be begun betimes, as ſoon as you can foreſee which ſide will be attack'd.

The Retrenchment muſt be rais'd as high as poſſible; and if you can, make Terraſſes juſt by to throw Bombes, Carcaſſes, Granadoes or Fire-pots, *&c.* upon the Enemy from them; or to have two or three Pieces of Cannon there, laden with Musket-balls to Fire at the very moment of the Aſſault. It will not be amiſs to make Fourneau, or Fougades, under ſuch a Retrenchment, to blow it up as ſoon as the Enemy is lodg'd in it. And if in the end you are forc'd from the laſt Retrenchments, you may yet make another kind of Retrenchment in the Town, by ſhutting up the Streets with Chains and Barricadoes, making little Flanks along the Houſe ſides, whoſe Doors and lower Windows muſt be ſhut; and laſt of all, throwing Stones and Fire-works from the upper Windows upon the Enemies, when they endeavour to break open the Doors below; thus to make them pay dear for their Conqueſt.

There

There are *cover'd Retrenchments*, which are properly the ſame as we have call'd *Caponnieres*, for they are very low Parapets, behind which there is a kind of a Ditch 4 or 5 Foot deep, and 7 or 8 wide, where the Souldiers are, as it were, buried, and fire thro' *Meurtrieres* which are in the Parapet, that is, but two Foot high; and theſe *Caponnieres* are cover'd over with ſtrong Oaken Planks, leaving a Hole to let out the Smoak of the Powder. Theſe Retrenchments, as well as the Coffers, are very good to defend the Ditch, and they are uſually made before a Flank.

Of Counter-Trenches.

C*Ounter-Trenches*, or *Counter-Approaches*, are Trenches made againſt the Beſiegers, to ſtop their going on, and keep them as far from the Place as poſſible, it being certain that the farther the Enemy is from the Place, the leſs he can annoy it; and to *Scoure the Trench*, is to make a vigorous Sally againſt thoſe that Guard the Trench, make them *give way*, that is, quit their Poſt or loſe Ground, and put the Pioneers to flight: but to *Mount the Trenches*, is to go upon Duty in them; and to *deſcend the Trenches*, to come back from guarding them.

As contrary things have contrary effects, ſo it is eaſy to know that a Counter-Trench muſt have its Parapet turn'd towards the Beſiegers, that the Beſieg'd may be cover'd in it: and that it muſt be enfil'd from ſeveral Places of the Town, leaſt the Enemy ſhou'd cover himſelf with it when he has taken it. It is to be carried on to ſuch places as are advantageous for the Town, and prejudicial to the Enemy, ſo that it may be defended from the Out-works, and not *enfiladed* or commanded by any Height, which the Enemy is poſted in.

Laſtly, The Counter-Approaches, ought to have Advances or Demi-Redoubts, whoſe Points muſt be turn'd towards the Enemy in ſuch manner, that they may be open inwards towards the Town, and enfil'd within and without, to drive the Enemy from them.

Of

Of Counter-Batteries, and of the Defence of Breaches.

A *Counter-Battery*, is a Battery which the Besieged oppose to the Enemy's Battery to ruin it, especially when he is got pretty near to the Counterscarp, and is raising Batteries to ruin the Flanks, which the Defenders most vigorously oppose, by a continual Fire upon the Work-Men, with Cannons charg'd with Stones or old Iron, which may also be done by Night, if you observe by Day after what manner the Cannon is levell'd, to shoot against the Place where the Enemy has begun his Works, which may also be discover'd by Night, by shooting red-hot Bullets to enlighten the Field. These *Red-Bullets* are Cannon Balls heated in a Forge till they have a Flame-heat, which are also us'd by the Besieg'd to defend Breaches.

Flaming-Barrels, or *Fire-Barrels* are also of use in defending Breaches, which are also call'd *Thundering-Barrels*, being fill'd with Granadoes, and *Fire-Pots*, that are Pots fill'd with fine Powder and a well charg'd *Granado*, which is a hollow Globe, usually of Metal fill'd with fine Powder, that takes Fire by means of a slow Fuse at its Touch-hole, which reaches down to the Powder. Some of them are made of Past-board, to be thrown upon Fascines to burn them, or into the Trenches to drive the Souldiers from them.

The Besiegers use also Granadoes, Bombs and Carkasses to beat down the Houses of the Town, and Fire the Magazines of Powder, as we did at *Nice* in this last War. A *Bomb* is a great hollow Ball of cast Iron fill'd with Powder, Nails and Fire-works, and *Carkasses* are Boxes full of Powder made with Iron hoops, fill'd with Granadoes and charg'd Pistol-Barrels, which together with the Granadoes are wrapp'd in oil'd Tow, and other combustible matter.

The Granado, being very little, is thrown by hand, but the Bombs, Carkasses, Red-Bullets, Fire-pots, Flam-

Flaming-Barrels, and Stones are thrown out of a *Mortar-Piece*, which is a ſhort ſtrong Piece of Ordnance, of a large *Calibre* or Diameter. It is not levell'd Point-blank as the Cannon; but with the Muzzle upwards, very high towards the place where you wou'd ſend the Bomb.

To avoid the effect of the Bomb when it falls by you in an open place, you muſt fall flat upon the Ground, or hide your ſelf behind Traverſes, or Parapets of Earth, made for that purpoſe: Or elſe the Streets muſt be unpav'd, ſo that there may be only the ſoft Earth left, which you may alſo cover with Dung, where the Bomb falling, ſinks in by its own weight and loſes its force. A Vault Bomb-proof muſt be Five or Six Foot thick at leaſt, and to ſecure it the better, you may cover it with a great many Bruſh-Faggots, or Vine-Twiggs, or with Five or Six Foot of ſoft Earth, which will deaden the Bombs and render them ineffectual.

Breaches are alſo defended by *Chauſſe-trappes*, or *Col-trops*, which are Iron Inſtruments with 4 or more Spikes, made after ſuch a manner, that whatſoever way they fall, one Point will always lie uppermoſt like a Nail, which being thrown up and down, are very troubleſome to the Foot that mount the Breach, or to the Enemy's Horſe, that wou'd paſs ſome narrow place: there are three ſorts of them, *ſmall ones*, whoſe Points are about Three Inches; *middling ones* with Spikes Four Inches long; and *great ones* with Spikes of Five Inches.

To diſturb the March of the Horſe in a Camp, or that of the Foot in a Breach, *Herſes* or *Herſillons* alſo are us'd, which are Planks 10 or 12 Foot long, with Iron Spikes all over on both ſides: or elſe *Chevaux de Friſe*, which are Beams with Spikes all over. Much ſuch a Machine is us'd, ſupported and turning upon a Pivot, and ſet up in a paſſage which muſt be often open'd and ſhut; and in ſuch a caſe it is call'd an *Heriſſon*.

The

The Counter-Batteries or Cavaliers, which serve to Fire upon the Enemies that attack the Retrenchments, must not be very high, so that they may be cover'd by the Parapet of the Rampart, or of the Bastion: but such as are design'd to defend the neighbouring Hills, ought to be rais'd higher, that they may play upon those Eminences without being cover'd by the Parapet of the Place.

Of Counter-mines.

WHEN the Besieg'd, in Spight of all their resistance, have not been able to hinder the Enemy from passing the Ditch, and making under the Rampart a Mine, which is the most powerful Engine for the taking of Towns, because it can in a little time make a Breach large enough to lodge in and give Assault; they must speedily remedy it with Counter-mines, which are made to discover and overthrow the Besieger's Mines.

To discover these Mines, Cascans or Wells are made obliquely in the solid part of the Terrass, where a Miner is suspected to work: and when these Wells are judg'd to be lower than the Besiegers Mines, little Heads or Channels are driven every way to find the Enemy's Mine, or to enlose it, and render it useless, by cutting the Train to prevent the Enemy from Springing it, taking the Powder away, or spoiling it, by pouring in a great quantity of Water.

These Heads must be driven perpendicular to the Capital of the Bastion, when you fear a Mine in its Point; and parallel to the Face, if you fear a Mine there; and if the Ditch be dry they must be driven under it; because the Enemy then may have a Gallery under it also.

As the Ancients made Mines to surprise Towns, they also knew how to discover them. *Vitruvius* in his *last Chapter of his last Book*, says, that *Marseilles* being besieg'd, the Inhabitants suspecting the Enemy's Mines, digg'd

digg'd the Ditch deeper all round the Town, and by that means found in the Ditch the Avenues of above 30 Mines, which the Enemy had prepar'd to surprize them.

An *old fashion'd Counter-mine*, is a Vault made before-hand round the Faces of a Bastion, Six or Seven Foot from the Wall, with several Air-holes, which come up quite to the Top of the Bastion. It is so call'd, because it is now out of use, by reason of its making a Rampart too weak.

Mines are also discover'd by boring with Rods or great Augars till you perceive the light of the Enemies, then if the Earth be but 6 or 7 Foot thick betwixt you and them, all that Earth must be thrown against the Miner with a *Petard*, which is a hollow Engine of Metal, usually shap'd like a Hat or a Cup, with handles, which is fill'd with fine Powder, about 7 Inches deep, and 5 Inches over at the Mouth.

Some are larger at the Breech than at the Mouth, but they are not so effectual, and are more apt to burst. Both kinds are us'd in Counter-mines to break into the Heads or under-ground Galleries of the Enemy, to give his Mine Air: they are also us'd to break Bridges, Gates and Barriers, in Towns that you wou'd take by Storm.

When it is us'd against Mines, the Mouth of the Petard is fix'd to a Beam, arm'd on both sides with cross plates of Iron nail'd over it, after it has been well stopp'd and charg'd as high as within two Fingers of its Mouth: then the Petard, which must be fasten'd to the Beam by its Handles, is set against the place where the Enemy's Mine is, in such manner that the Beam be parallel to the Horizon, when the Petard is level with the Mine; otherwise the Petard must be directed against the Mine.

When it is us'd to break Bridges, Gates, Batteries, Chains, and whatever may be an obstacle in a Surprize, Turrels, or Screws, or strong Hooks are fix'd in such a Gate,

Gate, or Bridge, which I ſuppoſe acceſſible, to faſten the Petard, ſo that its Beam or Madrier may joyn cloſe to it; for the cloſer the Madrier is, the more effectual will the Petard be.

If the place to be Petarded be inacceſſible, as Draw-bridges when they are drawn up, the Petard muſt be applied with a long Pole, at whoſe end the Petard muſt be faſten'd, and a Fuſe muſt run all along the Pole, quite to the Touch-hole of the Petard to Fire it: and if the Draw-bridge, being drawn up, does not lie cloſe to the Gate, inſtead of a Pole, you muſt uſe a Bridge moveable upon two Wheels, and thruſt it hard againſt the Draw-bridge.

FINIS.

AN

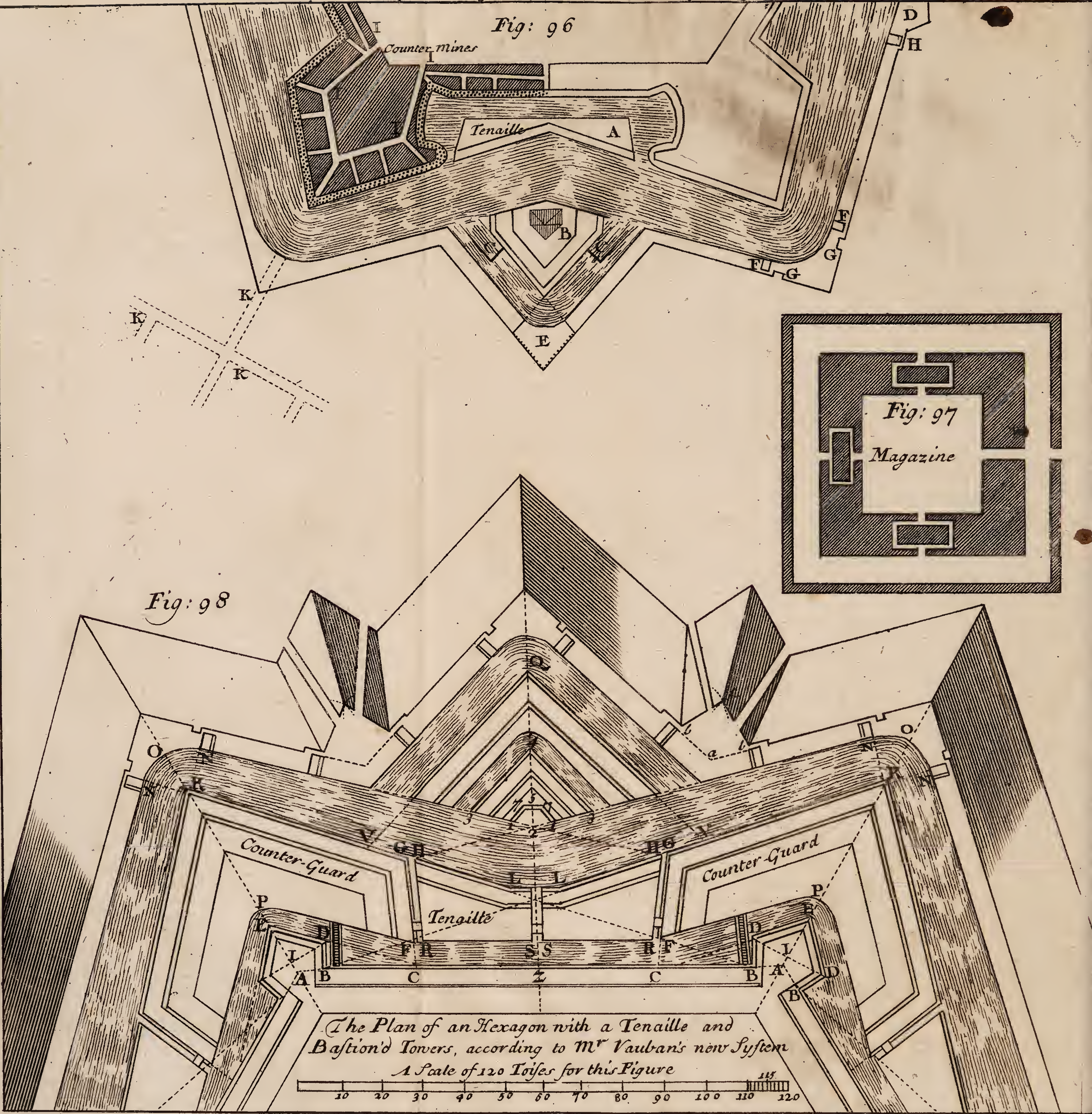
Fig: 96
Counter mines
Tenaille
Fig: 97
Magazine
Fig: 98
Counter-Guard
Counter-Guard
Tenaille
The Plan of an Hexagon with a Tenaille and
Bastion'd Towers, according to Mr Vauban's new System
A Scale of 120 Toises for this Figure

AN

APPENDIX,

CONTAINING

A True and Short Account of Mr. *Vauban*'s Manner of *Fortifying*: Taken from a French Book, publish'd by the Abbot *Du Fay*, with Mr. *Vauban*'s Approbation.

With his New System of *Bastion'd Towers*.

MR. *Vauban*'s Fortification has been so Universally approv'd, that his Method only has been made use of in all the Towns that have been Fortify'd since he first was employ'd in the *French* King's Service. This is what no Military Man is ignorant of; and whoever has seen the Fortresses in *Flanders* will be convinc'd, that the following Manner is truly Mr. *Vauban*'s.

Not to repeat any thing that has been said in the foregoing Treatise, we shall omit the Description of several Warlike Machines; and only mention such Works as are peculiar to Mr. *Vauban*, of which Mr. *Ozanam* has given but an imperfect Account.

Mr. *Vauban's* Manner of drawing the Bastions, Casemates and Orillons has been shew'd, *Page* 114, only we must take notice, that his Orillon is Square on the inside, for the conveniency of the Musketeers; and that of his Four Flanks (*viz.* that of the Place, that of the Orillon, that of the Tenaille, and that of the Caponiere) the two last are the best; because they Command without being Commanded. *Plate* 54. *Fig.* 96.

His Flank of the Place is better than the Flank of such Engineers as allow second Flanks; not only

becauſe it holds as many Muſketeers as both their Flank and ſecond Flank in the Curtain; but becauſe the Shots made from it are *right*, and it is only ſeen *obliquely*: Beſides the Beſiegers have a great Front againſt a Second Flank, which holds but few Muſketeers for the defence of the Ditch.

The *Reinforc'd Tenaille*, or Tenaille with Flanks in the Ditch, is alſo deſcrib'd; and one ſort of *Simple* Tenailles for the ſame purpoſe is to be ſeen, *Plate* 44. *Fig.* 94.

Plate 45. *Fig.* 96. The other ſort, mark'd A, has its Rampart parallel to the Curtain of the Place; and then its narroweſt part ought to be about 3 Toiſes broad.

The Caponiere ought to have on each ſide a Banquette; and the Parapet of the Place two or three, for Men of different Stature, that they may all fire from the ſame level. The Half-Moon has ſometimes Flanks, and within, a Reduct, as B, whoſe Walls have Battlements.

Places of Arms, as C, are made in the Ditch of the Half-Moon perpendicular to the Faces, near the Angles of the Epaule, when they have Flanks, to hinder the Paſſage of the ſaid Ditch.

When ſuch Places of Arms are made in the Cover'd-Way, if in the re-entrant Angles, as D; their Demi-gorges are of 10, and their Faces of about 12 Toiſes; if in the ſaliant Angles, they are deſcrib'd by the length-en'd Faces of the Baſtions, or of the Half-Moons, and the round part of the Ditch, as E. Theſe Places of Arms, which muſt have Paliſſado's, are ſhut up with Traverſes, which are Parapets 5 or 5½ Foot high on that ſide where their Banquette is, and 3 or 4 towards the Cover'd-Way, which they ſcoure.

Theſe Traverſes are 4 Toiſes and 1½ Foot long at the ſaliant Angles coming from the Ditch: the deſign of them is to hinder the Cover'd-Way from being enfiladed, and to defend their Place of Arms.

The ſpace of 4½ Foot, which is between the Traverſe and the Parapet of the Cover'd-Way, is ſhut up with a little Merlon, rais'd within the Place of Arms, 4½ Foot diſtant from the Traverſe, that the Souldiers may have

room

room to paſs by. The Traverſes are mark'd F, and the Merlons G. *Plate 45. Fig. 96.*

At the re-entrant Angles the Traverſes ſcoure or *enfile* the Cover'd-Way, and are 5 Toiſes long; that is, the whole Breadth of the Cover'd-Way; and then a Cut as H, muſt be made in the Parapet of the ſaid Cover'd-Way for the Souldiers to paſs by.

There ought to be no ſuch Cuts at the Saliant Angles, becauſe they wou'd not be ſeen by the Beſieged, and ſo might be of uſe to the Beſiegers; whereas theſe are flank'd from the Half-Moon, or from the Lunettes, which have been mention'd Page 115.

A Table to Draw the Maſter-Line *of a Regular Place.*

	Of the Square.	Of the Pentagon	Of the Hexagon.
Outward Side.	180. T.	180. T.	180. Toiſes.
Perpendicular.	22. T.	25. T.	30. T.
Face.	50. T.	50. T.	50. T.
Radius.	127. T.	152. T. 3. foot.	180. T.

Radius	Of the Heptagon.	206. T. 3. foot.
	Of the Octogon.	234. T. 3. f.
	Of the Enneagon.	262. T. 2. f.
	Of the Decagon.	291. T.
	Of the Hendecagon.	314. T.
	Of the Dodecagon.	346. T. 4. f.

An Ichnographical Table of Mr. Vauban's *Method.*

Of the Body of the Place.	Base of the Rampart.	11 T.
	Base of the Parapet.	3 T.
	Breadth of the Ditch.	20 T.
Of the Tenaille.	Distance from the Orillon of the Bastion.	3 T.
	Base of the Rampart of the Face and Flank.	7 T.
	Base of the Rampart of the Curtain.	5 T.
	Base of the Parapet.	3 T.
Of the Half-Moon.	Base of the Rampart.	10 T.
	Base of the Parapet.	3 T.
	Breadth of the Ditch.	12 T.
Of the Cover'd Way, Traverses, and Places of Arms.	Breadth of the Cover'd Way.	5 T.
	Demi-gorge of the Places of Arms at the re-entrant Angles.	10 T.
	Faces of the Places of Arms.	12 T.
	Length of the Traverses at the re-entrant Angles.	5 T.
	Length of the Traverses at the Saliant Angles.	4 T. $1\frac{1}{2}$ foot.
	Base of the Traverses.	3 T.

When

When the Ditch is fac'd, Steps are made at all its Angles for the ſervice of the Counterſcarp. Each detach'd Piece ought alſo to have a pair of Stairs to lead up to it.

The Countermines I of a Place are built under the Terre-plain of the Rampart level with the Ditch, ten Foot diſtant from the *Revêtement* or Facing of the Ditch, to which they are parallel, having a Communication with it thro' ſmall *Heads* or Paſſages vaulted. *Plate 45. Fig. 96.*

From the Countermines of the Place you go down into the Caponieres, and then go up into the Countermines of the Cover'd-Way, from whence Heads, as K, are drove on under the Field, to make *Fourneaux* or little Mines, to blow up the Works of the Beſiegers, and hinder their Approaches.

Where-ever there are Countermines there ought to be Miners conſtantly; or at leaſt Miners ought to be ſent for when the Town fears a Siege.

When an Eminence falls gradually from its higheſt part down to the Glacis of a Place, Works muſt be made one before another, with their Flank'd Angles rais'd. The moſt diſtant ought to cover all the reſt, and draw its defence from them; and they ought all to be built in ſuch manner, that the Enemy may not be able to make a Retrenchment in the Firſt, without undergoing the Fire of the Second.

The Glacis ought as often as poſſible to be of Pebbles, or of Stones cover'd with Turf; for the Beſiegers can work but ſlowly in ſuch Ground: and ſuch Parapets, as the Pioneers make with what they dig up, are often the occaſion of their being Kill'd or Lamed, becauſe the Cannons of the Place, ſhooting againſt thoſe Parapets, cauſe the Stones to fly about dreadfully.

Mr. *Vauban* obſerves, that the Bridges of the Curtains do not hinder the effect of the Fire of the Flanks along the Faces of the Baſtions in the Ditch.

He likes that Ditch beſt, which by the means of Sluces may be fill'd, or emptied at pleaſure.

The

The Paſſage out of the Common-Shores ought to be in the middle of the Curtain, Grated with Iron, and guarded by a Sentinel.

In Sea-Port-Towns a Falſe Braye is of great uſe, becauſe Cannon Balls, ſhot from it, raſe the Water and pierce the Ships dangerouſly.

Magazines for Powder ought to be well ſecur'd from Fire, and floor'd with Oak that they may not be moiſt, and ſo Vaulted as to be Bomb-proof. See *Page* 188. Their Walls ought to be very thick, and the Paſſage for Light ought to be winding, and ſo narrow as not to admit a Rat, which is ſometimes made uſe of by the Enemy to ſet a Magazine on Fire, by tying a lighted Match to his Tail.

Plate 45. *Fig.* 97.

To have ſuch a Light, a Pilaſter ought to be built in the middle of the Wall, with a ſmall Paſſage round it, which muſt be ſhut up on the inſide with a fine Grate of Wire: This Magazine ought to have a Wall about it at ſome diſtance, which alſo muſt have no other Building near it.

The laſt Places that Mr. *Vauban* Fortified, are done after a new Manner, which may be ſeen at *Landau* and *Beffort*, which Towns are Fortified with Baſtion'd Towers, that have Counter-guards or detach'd Baſtions before them. This Way of Fortifying is beſt for a Town ſeated in a Plain, where there are Eminences or Commanding-Grounds near the Place; becauſe you are not ſo much ſeen upon the Back, or *de revers*, in ſuch Works as in ordinary Baſtions; and ſuch Towers ſerve both for Magazines and Cavaliers.

The

The Method of Fortifying Places, according to this New System.

ALlow to the Side of the inner Polygon AA, 120 Toises, to the Demi-gorges AB 6; at the points B raise Perpendiculars equal to the Demi-gorges, which will describe the Flanks BD; to finish the Flank'd Angle, draw thro' the points D, a line DD, which you must divide into two equal parts at I, to set off from I to E the distance ID; from the point E, draw the lines ED, which will describe the Faces, and make the Flank'd Angle a right One. *Plate 45. Fig. 98.*

Then to make the Counter-guards, or the detach'd Bastions, Set off upon AO the Capital lengthen'd 39 Toises, from the Flank'd Angle E to K, and from the point K, draw the line KB, to the Angle of the Flank, upon which set off 56 Toises from K to G, for the length of the Faces.

You will have the Flanks of the Counter-guards by drawing lines from G to C, which is the point where the Defence of the Face of the Bastion'd Tower begins; the length of the Flanks will be terminated at the point F, which is the place where the Lines of Defence KB, and GC meet.

For the Ditch of the Bastion'd Tower, with the distance B D, which is the length of the Flanks of the Tower, describe from the Flank'd Angle E, the Arch P, for the Rounding of the Ditch, to which you must draw the line PF, which will terminate its breadth.

For the Construction of the Tenaille before the Curtain, having left a Ditch before the Flank of the Counter-guard 2 Toises broad, set off upon the Line of Defence beyond this Ditch from the point H to L, 24 Toises for the Faces of the Tenaille; then draw through the points LL, the Curtain LL, which will be of 10 Toises, which you must cut in the middle by

a Ditch,

a Ditch where the prick'd line QZ cuts the Tenaille, which Ditch is to be of the ſame breadth with that of the Flank: the Ditch between the Curtain of the Place, and this Tenaille, is made by drawing a line through the points FF, which muſt be parallel to the Curtain; then joyn the Ditches of the Flanks of the *Counter-guards*, and that of the middle of the Tenaille, at the points R, S; S, R.

To draw the great Ditch of the *Counter-guards* and of the Tenaille, ſet off upon its lengthen'd Faces, 12 Toiſes, from K to N, N; draw from theſe points N,N, lines to the Angles of the Epaule or Shoulder G,G, which will determine the breadth of the Ditch, whoſe Extremities muſt be made round, by means of the Arch N,N, whoſe Center is at K.

To make the Half-Moon before the *Curtain*, raiſe from the middle of the Curtain Z, a Perpendicular line of 72 or 73 Toiſes to Q, from which draw lines to the points V,V, which will be 10 Toiſes above the Angle of the Epaule, upon the Faces of the *Counter-guards*; the Demi-gorges will be terminated by the lines which make the Ditch.

The Ditch of this Half-Moon muſt be 10 Toiſes broad, and parallel to its Faces.

To make the Retrenchment, or the little inner Half-Moon, ſet off upon the Demi-gorges of the great Half-Moon 14 Toiſes from 2 to 3: then ſet off 18 Toiſes from 2 to 4 upon the Capital, and drawing lines from 4 to 3, you will deſcribe the Faces of your little Half-Moon, before which you muſt make a parallel Ditch 5 Toiſes broad.

In the re-entrant Angle of this Retrenchment is made a little Harbour to cover the Boats, that carry the Soldiers to the Out-works; theſe little Harbours are built after the following manner:

Set off 5 Toiſes upon the Demi-gorges from 2 to 1, and as many upon the Capital from 2 to 5, which you muſt cut at this point by a line parallel to the Curtain, and ſet off upon this line each way from 5 to 7 one Toiſe


A Scale of 100 Toises
5 20 40 60 80 100
10 30 50 70 90
Glacis
Cover'd Way.
Lunette
Lunette
Half moon
Redoubt
Place of Arms
Ditch
Barbette
Gorge
Caserns
Caserns
Caserns
Magazin

Toiſe 2 Foot, and draw the line, 1 7, and by this means your little Harbour will be finiſhed. *Plate* 45. *Fig.* 98.

For the Cover'd-way, draw lines parallel to the *Counterſcarp* 5 Toiſes diſtant; then to form the Places of Arms, ſet off from the points where the lines of the Cover'd-way make the re-entrant Angles, towards the Half-Moon or other Out-works, as from each ſide of *a* to *b*, *b*, 10 Toiſes; then with the diſtance 1 3 deſcribe from the points *b*, *b*, Arches interſecting at *c*, draw from this point the Faces of the Saliant Angle *c b*, in the middle of which make cuts of 9 or 10 Foot wide for Sallies. Traverſes are made in the Cover'd-way 3 Toiſes thick, to keep the Men from being Enfiladed; they are built perpendicular to the *Counterſcarp*, and there is reſerved a Space of 4 Foot, fit for the paſſage of one Man at a time.

Whatever elſe we have mention'd in this Appendix as peculiar to Mr. *Vauban*, may be ſeen at one view in the laſt Plate.

The Explanation of It.

This Figure repreſents the Fourth Part of a Regular Hexagon, Fortify'd according to Mr. *Vauban*'s Method. *Plate* 46.

You may ſee in it a Baſtion, a Demi-Baſtion, a whole Curtain, half a Curtain, a Half-Moon, and half of a Half-Moon: two great Lunettes, a little Lunette, and a Ravelin: a Cover'd-way, Places of Arms, Traverſes, a Paliſſade, Merlons, a Caponiere, and half of a Caponiere: Barbettes, Guerites, Steps, *Rampes* or Ways cut aſlope; Countermines, a *Head*, or Rameau, a Gate, a falſe Gate, Bridges, a Magazine, a retrench'd Corps de Guard, a Redoubt, two *Sorties*, or Sallies, an High-Way, a Watering-Place, and Cazerns.

The Profil of this Figure is perpendicularly cut upon the middle of the Curtain; whence it happens that the Profil of the Rampart of the Half-Moon, is wider than the Profil of the Rampart of the Place; becauſe the Half-Moon is cut along its Capital.

The prick'd lines, which joyn the Works to their Profil, ſhew what part of the Profil is repreſented by each Geometrical Line; and the two little Vaults which are in the Profil of the whole Work, repreſent the Countermines of the Rampart, and of the Cover'd-Way. The Countermines of the Rampart have a Rampe, or Steps, to go down into them; and from the Countermines of the Cover'd-Way there is a *Head*, or Rameau, which goes under the Field.

Beſides this general Profil, each detach'd Piece has its Profil deſcrib'd.

On the inſide of the Place along the *Enceinte*, you may ſee three lengths of Cazerns, built to lodge the Garriſon.

The *Enceinte* of the Place is drawn in Six Lines; the Firſt repreſents the inward *Talu* of the Rampart; the Second the Height of the Rampart; the Third the *Talu* of the Parapet and its Banquettes; the Fourth the inward Height of the Parapet; the Fifth its outward Height, and the *Cordon* or Maſter-line; the Sixth the outward Height of the Rampart, or the Scarp.

The Baſtion and the Demi-Baſtion are part of that *Enceinte*.

The whole Baſtion is a full Baſtion, and has a Barbette, and three Guerites, one at its Flank'd Angle, and one at each of the *Epaules;* in its Parapet a Way is cut before the Guerites for Centries to get to 'em: in the Baſtion near one of the Orillons, there is a little pair of Stairs to go down to the Ditch, into which one may enter thro' a Poſtern, which is on the outſide of the Orillon, near the concave part of the Flank. The Demi-Baſtion is empty, and has a Guerite and half a Guerite, with a Magazine.

In the middle of the Curtain within the Place, there is a Vault for Carts to go thro'; there are alſo three little Vaults, which lead to the Countermines: Two of theſe little Vaults are at the Gorge of the Baſtion, and the third at the Demi-gorge of the Demi-Baſtion.

Near each of theſe little Vaults there is a Slop'd way

way made to go up to the Rampart: when there are no Steps they make uſe of theſe Slop'd ways, where-ever they have occaſion to aſcend; in this Plate they are only in the Ramparts and the Barbettes.

Before the Demi-Curtain there is a Demi-Tenaille, with its Parapet: half a Caponiere whoſe Paliſſade is repreſented in its Profil: half of a Half-Moon, with Flanks, with its Rampart, its Parapet, and Half of its Guerite.

Upon the Epaule or Shoulder of this half of a Half-Moon there is a little Lunette, with its Parapet, its Barbette, and its Guerite.

Before the Curtain there are Two Demi-Tenailles of different Conſtruction, each with its Parapet: theſe Two Demi-Tenailles are ſeparated from each other by a Way, one end of which is at the little Bridge betwixt the Rampart and theſe Two Demi-Tenailles; and the other end of it is at the great Bridge, which Croſſes the Caponiere length ways, and which joyns theſe Two Demi-Tenailles to the Half-Moon. This Half-Moon is without Flanks, with a retrench'd Corps de Guard, a Rampart, a Parapet, and a Guerite.

The Two great Lunettes which Cover its Faces, have each of them a Rampart, a Parapet, a Barbette and a Guerite. The Ravelin which Covers the Flank'd Angle of this Half-Moon has only a Parapet.

At one of the Shoulders of the Half-Moon the Rampart is cut, to continue the way, at the end of which there is a Bridge to go to one of the great Lunettes, whoſe Rampart is alſo cut for the ſame purpoſe. Afterwards you ſee a Bridge, which joyns this Lunette to one of the Places of Arms of the Counterſcarp.

Every Particular Bridge has its Draw-bridge mark'd with Two croſs lines.

The Counterſcarp or Cover'd-way encompaſſes the Place and its Out-works. At all the Angles of the Counterſcarp or Cover'd-way, except before the Ravelin, there are Places of Arms Shut up with Traverſes, broader at the re-entrant Angles, and narrower at the Saliant Angles, where their little Enfilade is Cover'd

with

with a Merlon. There are alſo Traverſes in all the Ditches of the Out-works, except in the Ditches of the Demi-Tenailles.

The Ditches of the Place, and thoſe of the Demi-Tenailles, are of an equal depth. The Ditches of the other Out-works are not ſo deep as that of the Place; but they are of equal depth one to another.

Round the Counterſcarp is the Glacis, in which there are ways cut call'd *Sorties*, (Sallies) theſe *Sorties* are over againſt the end of the Faces of each great Lunette at their Flank'd Angle.

In one of the Faces of the Places of Arms, where the Bridge of the Lunette ends, there is a Way cut, which leads to the Watering-place, near which there is a Redoubt to ſecure the Horſes from the inſults of the Enemies. This Redoubt has its Parapet, its Banquette, its Guerite, its Ditch, and its Glacis.

A TABLE of the TERMS Explain'd in the *FORTIFICATION.*

Bar-

Com-

He-

S.

T.

V.

W.

FINIS.